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Papers

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Note.—Part 2 of this Journal is the 1961-2 Newsletter of the Hydraulics Division.

DISCUSSION

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HYDROGEOLOGIC NATURE OF STREAMFLOW ON SMALL WATERSHEDS

By J. L. McGuinness,[1] L. L. Harrold,[2] F. ASCE, and C. R. Amerman[3]

SYNOPSIS

The importance of watershed area in depth-area-duration-frequency relationships on streamflow from small watersheds was investigated. Size of area was found to be an index of geologic and geomorphologic properties of the watersheds. Area index was related to annual streamflow and also to flows of as short as 2- or 8-day duration. A reconnaissance survey of base flow and a preliminary geologic investigation in one of the watersheds helped clarify the interrelationships between the geologic and hydrologic aspects of the study.

INTRODUCTION

Various watershed characteristics have long been recognized as major factors in streamflow relationships. The investigations reported herein were aimed at assessing the importance of a watershed area index value in depth-area-duration-frequency relationships of streamflow on watersheds ranging in size from 0.5 to 17,540 acres. Although watershed area was the characteristic used, it was not in itself a causal factor in the relationship. In east-central Ohio, increased drainage area goes hand-in-hand with increased re-

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lief. As drainage area enlarges, the stream channel cuts deeper and deeper into the geologic column. Interspersed through this column are aquifers of varying strengths. Interception of these aquifers may be the dominating factor in the depth-area relationship. Until the present program of ground-water investigation provides a more adequate definition of geologic factors, watershed size is used as an index of the aquifers intercepted and referred to as the "area index." Precisely, area index is the common logarithm of the number of acres in the watershed. It may be plotted as the number of acres if a logarithmic scale is used.

EXPERIMENTAL AREA

In the late 1930's, the Soil Conservation Service established the Coshocton Hydrologic Research Station about 10 miles northeast of Coshocton, Ohio. Gaging equipment was installed on watersheds ranging in size from less than 1 to more than 4,500 acres. Gaging of a 17,500-acre watershed which includes part of the research area is done by the U. S. Geologic Survey. The area is typical of the general farming area of the Allegheny-Cumberland Plateau physiographic province[4] (Fig. 1), where the valleys are narrow with steep side slopes and the streams have moderate to steep gradients. The drainage pattern is dendritic and quite distinct, even on the small watersheds.

The bedrock of the area consists of cyclic deposits of sandstone, shale, limestone, coal, and clay which are in the upper Pottsville, Allegheny, and lower Conemaugh series of the Pennsylvanian system. Horizontal and vertical variation occurs in several of the rock units. The regional dip is very slight and to the east,[5] but within the experimental area minor local folds are found. This same general geologic pattern occurs in much of the Allegheny-Cumberland Plateau province. A generalized section of the strata underlying the Coshocton watersheds is given in Fig. 2.

The soils are of the residual Muskingum-Keene complex and associated series. Topsoil is generally 6-8 in. thick and the internal drainage of the soil profile is rapid to medium. Silt loam texture and crumb structure predominate. The soil mantle is shallow, bedrock, generally being from 5-8 ft below the surface. For the watersheds used in this report, land uses include rotation crops, grass, and woods.

The climate of the Station is characterized by annual averages of 50° temperature and 38-in. precipitation. Storms of high intensity rainfall occur mostly in the May-October growing season. They typically cover limited areas and are of short duration. Dormant-season precipitation is usually of low intensity, long duration, and covers large areas. About 70% of the yearly stream flow occurs during the dormant period.

MEAN ANNUAL STREAMFLOW

Nearly 20 yr of streamflow data have been compiled at the Coshocton Station. A recent publication[4] listed the monthly precipitation and runoff on the Coshocton watersheds for the period of record through 1955. A plot of the

4 "Monthly Precipitation and Runoff for Small Agricultural Watersheds in the United States, U. S. Dept. of Agric., Washington, D. C., 1957.

5 "Geology of Coshocton County," by R. E. Lamborn, Ohio Dept. of Nat. Res. Bulletin, 53, 1954, 245 pp.

area index-streamflow relationship for the 12 watersheds in the 29- to 17,500-acre size range appears in Fig. 3. In this report, streamflow includes all flows past the gaging site and is expressed in inches depth over the watershed area.

Nine of the watersheds, shown in Fig. 3 by open circles, were in mixed cover and were managed under a system of improved-type farming. Of the three watersheds shown by closed points, the 43.6-acre area was in a complete woodlot cover. The 74.2- and 303-acre areas were both in mixed cover but managed under a prevailing practice system of farming. Data on these 12 watersheds were all for the period 1940-1958 inclusive. A definite pattern

FIG. 1.—SITE MAP OF COSHOCTON AND THE ALLEGHENY-CUMBERLAND PLATEAU PHYSIOGRAPHIC PROVINCE

was evident in the depth-area index relationship. As watershed area increased somewhat above 20 acres, the soil, practice, and land use differences tended to average out or become obscured by increasing groundwater flow.

Average annual streamflow increased from 7 in. to 12 in. as area increased from 30 acres to 1,000 acres. Above the 1,000-acre size, the curve in Fig. 3 indicates much smaller streamflow increases for rather substantial increases in the area index.

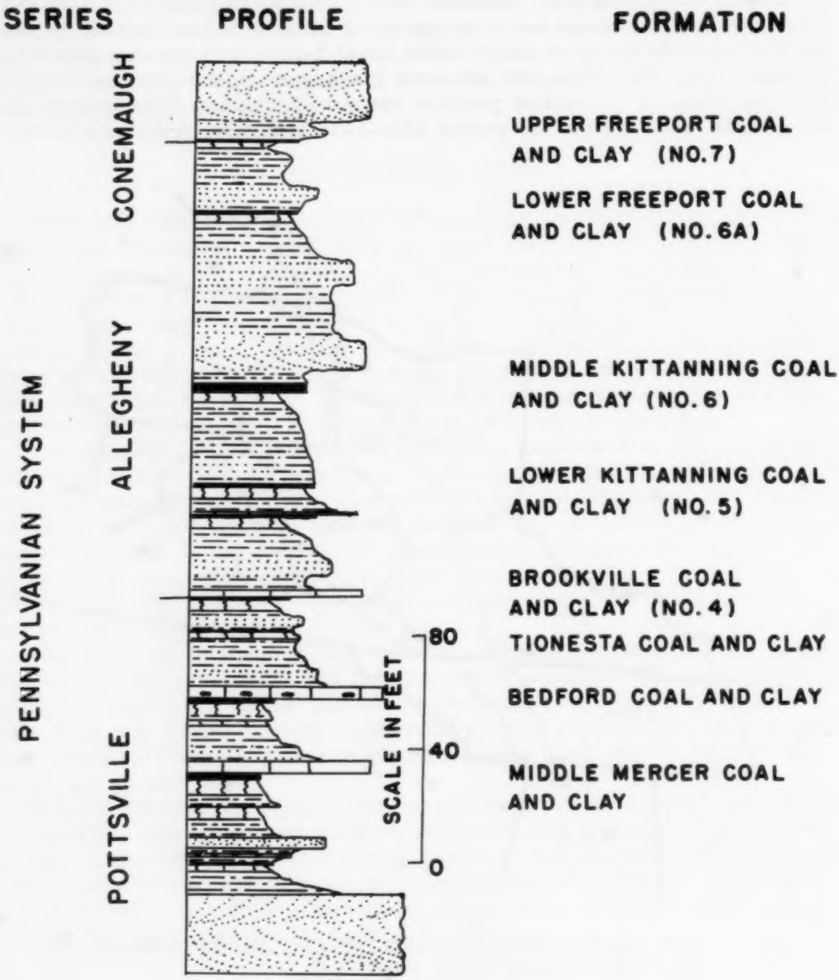

FIG. 2.—GENERALIZED GEOLOGIC SECTION OF STRATA UNDERLYING THE COSHOCTON WATERSHEDS

Although watershed treatment[6] influences streamflow, some of the erratics in the relationship shown on Fig. 3 are a reflection of geologic differences. The watershed of 75.6 acres with 7.9 in. annual flow falls a little below the average trend. A strong spring occurs about 100 ft downstream from the gage of this watershed. Moving the gage site downstream below the spring would increase the watershed area very slightly but would significantly increase the average annual flow. The 43.6-acre wooded watershed and the 303-acre prevailing-practice watershed plot well above the general trend line. Here, too, it seemed that geologic conditions played the major role. Further discussion of data from these two watersheds is deferred to a later section.

MINIMUM AND MAXIMUM ANNUAL FLOW

The minimum annual flows expected from watersheds of varying size also have the tendency to increase with increasing area index. L. L. Harrold[7] gave an analysis of the minimum-flow relationships on the Coshocton watersheds. The bottom curve drawn on Fig. 3 shows the annual minimum flow expected once in 10 yr as taken from his analysis. The relationship between low-flow volume and area index was quite similar to the average flow-area index pattern.

The top curve of Fig. 3 shows the maximum annual flow expected once in 10 yr. The line was fitted by eye through the points calculated from frequency analyses of the data for each watershed. This curve, although it had a somewhat greater slope, followed the pattern of the average- and minimum-flow plottings. Thus it seems probable that the same factor affects the entire range of the annual flow-area index relationship.

FLOW FROM VERY SMALL AREAS

Monthly precipitation and runoff amounts for 25 watersheds in the size range of 0.6 to 7.6 acres were also listed[4] for the period of record through 1955. The average annual runoff, primarily storm flow, varied from 0.2 to 4.8 in. The generally implied relationship between size of watershed and surface inches of runoff on these small areas was obscured by land use and treatment influences and soil type effects. However, when the watersheds were placed in homogeneous soil type, land use, and treatment groups, a covariant analysis showed that the relationship between area index and runoff was statistically significant. Thus, the association of increasing runoff with increasing watershed size was found to hold even on these very small areas.

SHORT DURATION STREAM FLOW VOLUMES

Data for this study were taken from another publication[8] which includes values for the maximum runoff volumes for 1-, 2-, 6-, and 12-hr and 1-, 2-, and 8-day periods for each year of record up to and including 1956. In addition

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6 "The Influence of Land Use and Treatment on the Hydrology of Small Watersheds, at Coshocton, Ohio, 1938-1957," by L. L. Harrold, D. L. Brakensiek, J. L. McGuinness, C. R. Amerman, and F. R. Dreibelbis, U. S. D. A. Tech. Bulletin (in press), 1960.

7 "Minimum Water Yield from Small Agricultural Watersheds, by L. L. Harrold, Transactions, Amer. Geophysical Union, Vol. 38, 1957, pp. 201-208.

8 "Annual Maximum Flows from Small Agricultural Watersheds in the United States," U. S. Dept. of Agric., Washington, D. C., 1958.

to the published data, the 1957 values were also used so as to include in this study data from the extremely high floods in June of that year. The frequency of these maximum runoff volumes was estimated by extreme value methods using the fitting procedure proposed by D. L. Brakensiek.[9] Since the data are distributions of independent observations each of which is an extreme value, this frequency model seemed most appropriate. Inspection of the 238 fitted curves (7 durations times 34 watersheds) indicated a reasonably satisfactory fit to the data.

Fig. 4 shows an area index plotting of the maximum annual flows expected for a 2-yr return period for nine larger watersheds for durations of 1 hr, 1 day, and 8 days.

The wooded (43.6-acre) and two prevailing practice (74.2 and 303-acre) watersheds were omitted from this analysis. As frequency comparisons were to be made, it was necessary to select those watersheds which had a uniform treatment program. Preliminary statistical analyses of the data indicated that the three watersheds in question had different time trends than the nine improved practice watersheds. This precluded their use in this frequency analysis.

For 8-day volumes of flow, the relationship as shown in the top plotting of Fig. 4 was similar to that for average annual flow given in Fig. 3. In the Coshocton area, this maximum 8-day runoff volume was generally associated with the late winter and early spring season, the end of the groundwater recharge cycle. During this period, the soil mantle was nearly saturated and groundwater discharge was near its maximum. Rainfall during this season was of low intensity and long duration. The total stream flow would then include a substantial amount of base flow. Under these conditions, the influence of the underlying geological strata could be quite significant.

The bottom plotting of Fig. 4 shows the depth-area index relationship for 1-hr durations. These maximum 1-hr volumes were primarily surface and quick return flow. They were associated with the high intensity, short duration, small-area coverage, convective-type summer rainfall. Some storage space was available in the soil mantle. Groundwater discharge was slight. The 1-hr duration plotting showed a trend toward decreasing flow (in inches) with increasing area index. This was partly due to the limited storm area in this season and the probability that only portions of the watersheds contributed large volumes of flow as the watershed area became larger. Differences in times of concentration and channel storage would also be a factor.

The middle plotting of Fig. 4, maximum 1-day duration flow volumes, show practically no trend of change with watershed area-index. Their occurrence was distributed rather evenly throughout the year. They were caused by both winter and summer storms. In some cases aquifer flow was strong. In some it was weak.

The depth-area index relationships for 6- and 12-hr and 2-day durations were similar to those for 1-day flows shown on Fig. 4. Slight downward trends may be present in the 6- and 12-hr relationships and a slight upward trend in the 2-day plotting. The trends, however, were very poorly defined and may well be fortuitous. For 2-hr durations, there was evidence of a downward trend, similar to that of the 1-hr plotting on Fig. 4.

It seems clear that geologic differences associated with increasing watershed size exerted a noticeable influence on stream flow relationships even

[9] "Fitting a Generalized Log-Normal Distribution to Hydrologic Data," by D. L. Brakensiek, Transactions, Amer. Geophysical Union, Vol. 39, 1958, pp. 469-473.

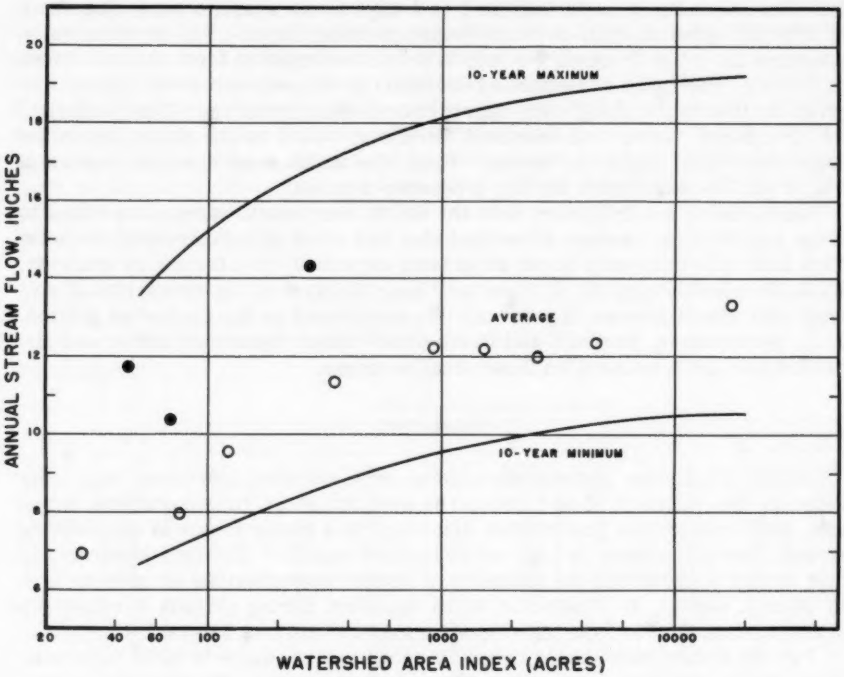

FIG. 3.—AVERAGE AND EXPECTED MAXIMUM AND MINIMUM STREAMFLOW FOR WATERSHEDS LARGER THAN 10 ACRES

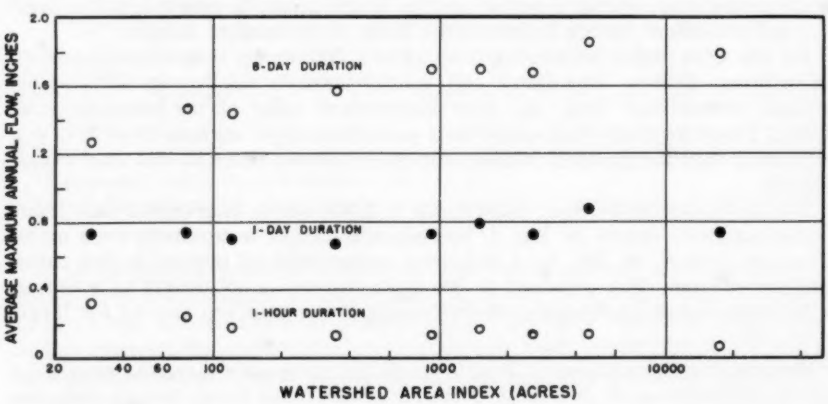

FIG. 4.—LARGEST FLOW EXPECTED IN 8-DAY, 1-DAY, AND 1-HOUR PERIODS FOR 2-YEAR RECURRENCE INTERVAL ON WATERSHEDS LARGER THAN 10 ACRES

for such short duration periods as 2 to 8 days in the average year. The same is true for greater-than-average return periods. Depth-area-duration relationships for other frequencies than 2-yr can be computed from the data given in Table 1. The mean and standard deviation of the extreme-value curves are given in this table along with appropriate frequency factors. The product of the frequency factor and standard deviation, added to the mean, yields the expected runoff depth in inches. Thus, the depth-area-duration curves of Fig. 4 can be constructed for any frequency desired.

Analyses of the frequency data for the 25 very small watersheds failed to show any relation between watershed size and short duration runoff amounts. This lack of relationship is not surprising especially in a frequency analysis. F. R. Dreibelbis and W. H. Bender[10] have illustrated the interaction of soil type and runoff amount for two of the soils found on the Coshocton Station. J. L. McGuinness, Harrold, and Dreibelbis[11] found significant influences due to land use and treatment on these smaller areas.

COMMENT

On the Coshocton watersheds stream-flow volumes increased with area index in the range of 10 to 1,000 acres even for short-time durations. It has long been recognized that aquifer discharge is a major factor in maintaining stream flow when there is little or no surface runoff.[12] The data presented in this report suggest that the influence of aquifer contributions on stream-flow is strong enough at Coshocton to be apparent during periods of relatively heavy surface flow and during storm periods as short as 2 days.

For the upland watersheds under 10 acres in size, there is little entrenchment of the drainage system into the underlying strata. These areas are located high on the land slope near the ridges which form larger watershed divides. This places them high on the geologic column, usually above the lower Kittanning coal. It can be seen from Fig. 2 that the possibility of one of these small areas intercepting a major aquifer is not great. In general, soils, land use, and treatment effects dominate the flows from smaller areas.

As the area index increased from 10 to 1,000 acres, the entrenchment of the drainage system also increased. As the channels cut through successive geologic formations (Fig. 2), they intercepted many of the water-bearing strata. These aquifers then made their contributions to stream flow. It is not surprising that the greatest increase in streamflow occurs in this size range (Fig. 3).

At the Coshocton Station, watersheds of 1,000 acres intercepted flow from all the aquifers shown in Fig. 2. Increasingly larger watersheds were made up of a number of 10- to 1,000-acre watersheds all incised in this same geologic column. The gradient of the main channel is not nearly as steep as in the hilly areas upland and proportionately fewer aquifers are cut for large

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10 "A Study of Some Characteristics of Keene Silt Loam and Muskingum Silt Loam," by F. R. Deibelbis and W. H. Bender, Journal, Soil and Water Cons., Vol. 8, 1953, pp. 261-266.

11 "Some Effects of Land Use and Treatment on Small Single Crop Watersheds," by J. L. McGuinness, L. L. Harrold, and F. R. Dreibelbis, Journal, Soil and Water Cons., Vol. 15, 1960, pp. 65-69.

12 "The Relation of Geology to Dry-Weather Stream Flow in Ohio," by W. P. Cross, Transactions, Amer. Geophysical Union, Vol. 30, 1949, pp. 563-566.

increases in size. Thus, the average annual flow volume for larger and larger watersheds approaches a maximum quantity. Of course, this maximum may not be reached when withdrawals and channel losses occur.

Table 2 gives data on the entrenchment process for the 12 watersheds of Fig. 3. Both the relief and relative relief of watershed 172, complete wooded cover, were quite high. The deep entrenchment in this small area had exposed several of the aquifers. Contributions from these aquifers made up a large part of the base flow, estimated as about 70% of the annual flow from this watershed. It was not surprising to find this 43.6-acre area plotting well above the average trend on Fig. 3.

TABLE 1.—MEAN ($\bar{Q}$), STANDARD DEVIATION (σ), AND FREQUENCY FACTOR (K), FOR COMPUTATION OF EXPECTED RUNOFF DEPTH IN INCHES FOR SELECTED AREA-DURATION-FREQUENCIES[a]

Duration	$\bar{Q}$ or σ	Watershed size, in acres								
		29.0	75.6	122	349	920	1,520	2,570	4,580	17,500
1 Hour	$\bar{Q}$	0.43	0.34	0.24	0.18	0.16	0.21	0.17	0.18	0.09
	σ	.56	.55	.34	.27	.17	.24	.18	.20	.13
2 Hour	$\bar{Q}$	.52	.45	.34	.27	.26	.34	.29	.31	.17
	σ	.65	.60	.45	.33	.25	.37	.29	.35	.25
6 Hour	$\bar{Q}$	.67	.61	.51	.43	.45	.55	.49	.60	.43
	σ	.64	.60	.54	.40	.35	.53	.42	.62	.50
12 Hour	$\bar{Q}$	.75	.70	.64	.57	.62	.72	.65	.77	.66
	σ	.65	.60	.55	.44	.41	.62	.52	.62	.61
1 Day	$\bar{Q}$	.84	.84	.80	.76	.81	.91	.83	1.02	.86
	σ	.66	.57	.58	.49	.48	.70	.59	.81	.68
2 Day	$\bar{Q}$	1.01	1.05	1.00	.99	1.08	1.12	1.09	1.27	1.12
	σ	.69	.59	.64	.60	.63	.77	.67	.85	.75
8 Day	$\bar{Q}$	1.41	1.59	1.58	1.71	1.86	1.87	1.83	2.16	2.02
	σ	.81	.74	.83	.84	.91	1.00	.88	1.77	1.36

Recurrence Interval	2-Year	5-Year	10-Year	20-Year	25-Year	50-Year
Frequency Factors	-0.17	0.73	1.30	1.87	2.01	2.58

[a] Expected runoff depth in inches = $\bar{Q} + \sigma K$

The prevailing practice watershed 196 also had above-average flow. The relief of this watershed is somewhat greater than would be expected from its area of 303 acres (Table 2), although the relative relief[13] was more in line in this respect. Some additional factor besides land use and treatment effects and greater than average relief is apparently operating here. This will be discussed later.

Table 3 lists the simple linear correlation coefficients for the data of Table 2. Area index was found to be significantly correlated with average

[13] "An Analysis of the Relations among Elements of Climatic, Surface Properties, and Geomorphology," by M. A. Melton, Office of Naval Res. Tech. Report No. 11, Dept. of Geology, Columbia Univ., N. Y., Vol. 102, pp. 1957.

annual runoff and with relief, relative relief, and the circularity ratio. Runoff itself was found to be as well correlated with relief as with the area index. Although runoff was not significantly correlated with either relative relief or circularity ratio, the coefficients were fairly high in both cases. This table confirms the view that area is just an index. Watershed area reflects influences of both geology and geomorphology. Additional mapping of the watershed areas is now being considered to enable further study of these relationships.

TABLE 2.—RELATION OF WATERSHED SIZE TO ANNUAL RUNOFF AND OTHER WATERSHED PHYSICAL CHARACTERISTICS

Watershed number (1)	Size, in acres (2)	Area[a] index (3)	Average annual runoff, in inches (4)	Relief[b] in feet (5)	Relative[c] relief (6)	Circularity[d] ratio (7)
169	29.0	1.46	6.93	153	3.61	0.88
172	43.6	1.64	12.03	284	5.14	.78
183	74.2	1.87	10.57	215	2.87	.72
177	75.6	1.88	7.96	243	3.16	.70
10	122	2.09	9.59	227	2.42	.76
196	303	2.48	14.64	280	1.78	.67
5	349	2.54	11.37	253	1.73	.90
92	920	2.96	12.27	327	1.34	.84
94	1,520	3.18	12.21	390	1.18	.76
95	2,570	3.41	12.01	425	.88	.60
97	4,580	3.66	12.37	482	.74	.59
994	17,540	4.24	13.35	530	.41	.58

[a] Common logarithm of the size in acres.
[b] Elevation difference between highest and lowest points in watershed as read from topographic map.
[c] 100 (relief in feet)/perimeter in feet.
[d] Ratio of area of watershed to area of circle with same perimeter length as the watershed.

TABLE 3.—LINEAR CORRELATION COEFFICIENTS BETWEEN AREA INDEX, ANNUAL RUNOFF AND OTHER WATERSHED PHYSICAL CHARACTERISTICS

Watershed physical characteristics (1)	Average annual runoff (2)	Relief (3)	Relative relief (4)	Circularity ratio (5)
Area index	0.64[a]	0.95[b]	-0.90[b]	-0.63[a]
Average annual runoff	...	0.65[a]	-0.52	-0.45
Relief	...	...	-0.73[b]	-0.73[b]
Relative relief	...	...	...	0.49

[a] Significant at 5% level.
[b] Significant at 1% level.

The data of Fig. 4 are for periods of relatively high flow and short time durations. Yet, even under these conditions, there is a definite relationship between area index and streamflow amounts. The geological differences associated with increasing area are pronounced enough to exert a measurable

influence on storm flows even in 2-day storm events. The trends are no longer evident for shorter durations though there probably is still some small influence.

In order to test some of the above concepts, a brief examination was made of stream flow in a 303-acre watershed (No. 196) on the Coshocton Station. A reconnaissance survey of the quantity of flow found in the stream channels during a 4-day period was made in July, 1959. During this period and for some weeks preceding, there was no surface runoff (overland flow). Thus, only waters properly classified as ground water were in the drainage channels of the watershed.

Stream flow was gaged at numerous points throughout the drainage system of this watershed by means of a precalibrated 0.6-ft HS flume.[14] The results are portrayed schematically in Fig. 5. In this drawing, the channel at any point was drawn with a width proportional to the strength of the flow at that

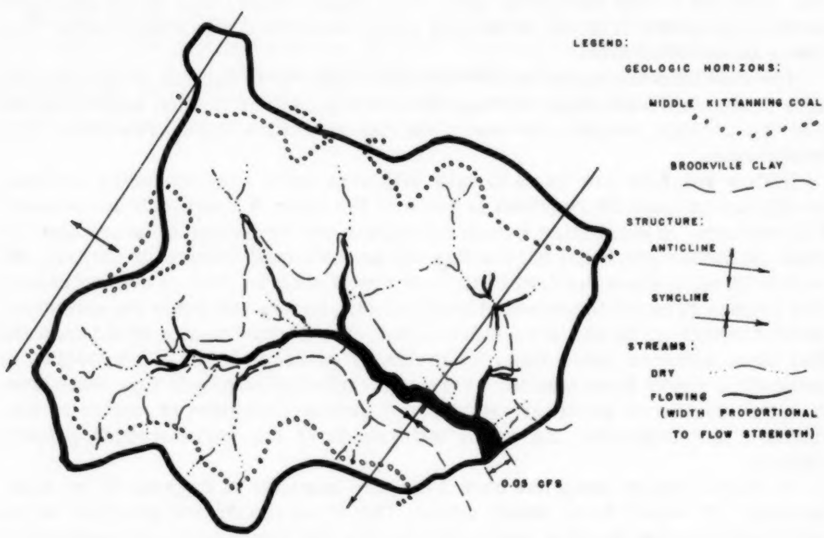

FIG. 5.—WATERSHED 196. GENERALIZED STRATIGRAPHY, GEOLOGIC STRUCTURE, AND STRENGTH OF BASE FLOW

point. Broken lines are an indication of no flow. The finest solid lines represent trace flows, and so forth up to the thickest line at the watershed gage which represents a flow of 0.05 cfs.

No flow was found in the higher and outermost reaches of the watershed drainage system. As the tributaries became more deeply incised, they intercepted an aquifer capable of supplying some flow. If the flow was weak, the succeeding strata pervious, or the attitude of the strata dipped away from the tributary, the flow might again disappear from the surface. In general, however, the flow became greater as the channels cut into the geologic section

14 "Devices for Measuring Rates and Amounts of Runoff," by L. L. Harrold and D. B. Krimgold, Soil Cons. Serv. Tech. Publ. 51, rev. 1948, 42 pp.

and intercepted additional water-bearing strata. The pattern is quite clear even on this relatively small area. The strata intercepted by the drainage system of the watershed shown on Fig. 5 are given as a generalized geologic section in Fig. 2. The coal and limestone strata are usually well fractured and water bearing whereas the clays are practically impervious.

Following the base flow survey, an intensive investigation was made of the geologic outcrop conditions in this same watershed. As shown in Fig. 5, the Middle Kittanning coal outcropped in the upper reaches of this watershed. There was a definite relationship between the outcrop line of this strata and the heading of drainage channels. Further downslope, there was another relationship between the outcrop line of Brookville clay and the place in the channels where base flow was first noted. Although not shown on the map, there were two additional aquifer outcrops further downslope which coincided with stream reaches where significant increases in base flow were also noted.

The geological investigation also provided a possible explanation for the very high annual runoff from this 303-acre watershed (Fig. 3). A syncline was detected in this watershed which may funnel aquifer flow in the direction of the gaging site (Fig. 5). Base flow in this watershed was estimated as 75% of the annual total flow.

The results of the base-flow survey and geologic investigation in this limited area tended to substantiate the hypothesis advanced earlier that entrenchment into the geologic strata was one of the causal factors in the area index-flow relationship.

If flow volumes are found to vary with area index over extensive regions, modifications may be required in some of the current hydrologic techniques. For instance, in estimating a peak runoff rate, one technique assumes that the peak (in inches per hour) varies directly as the runoff volume (in inches) and inversely with some function of time of runoff such as time of concentration. The volume of runoff is estimated from rainfall, cover, and other parameters—without reference to any area-index factor. The Coshocton data would indicate that area index or some other criterion of geology should be considered in estimating runoff from smaller areas especially for the longer flow durations. A better means of quantitatively defining geologic features is needed to distinguish hydrologically the small watersheds of the various physiographic regions.

In recent years there has been a definite increase in interest in the management of water from small areas. The small watershed program of the Soil Conservation Service under Public Law 566 has done much to stimulate this interest. The individual farm owner in those sections of the country where ground water resources are inadequate has a vital interest in these programs.[15] Yet it is for these smaller areas that reliable hydrologic information is most often unavailable.[16]

It is recognized that a knowledge of geology is vital to dependable stream-flow evaluations in this part of the country. The interrelationships between geology and hydrology can be quite complex.[12] The role of the geologist in the solution of many of our water problems here and elsewhere can not be overlooked. R. G. Kazmann[17] has shown how a knowledge of geology and published hydrologic records can be combined with a few isolated stream-flow records

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15 "Indiana Water Resources," Indiana Water Resources Study Comm., 1956.

16 "An Engineering Appraisal of Hydrologic Data," Comm. on Hydrological Data of Hydr. Div., Proceedings, ASCE, Vol. 85, No. HY 7, July, 1959.

17 "The Utilization of Induced Stream Infiltration and Natural Aquifer Storage at Canton, Ohio," by R. G. Kazmann, Economic Geology, Vol. 44, 1949, pp. 514-524.

for the solution of a hydrogeologic problem. J. K. Searcy[18] discusses the influence of geology on low flows of streams in Mississippi.

More intensive geologic studies are planned for the Coshocton watersheds. It is expected that these studies will aid in the interpretation of watershed-management hydrologic research programs now active on the Station. Furthermore, they will provide more useful guidance for future watershed-management action programs. The results of the investigation reported herein proved useful in planning the geological and ground-water investigations now under way at the Coshocton Station. The methods of analysis used may be of value in other sections of the country in the investigation of hydrogeologic problems. In any area where geologic conditions exert an influence on stream flow, some preliminary information on the geology and ground-water contributions may be obtained from analysis of the stream flow record.

ACKNOWLEDGMENTS

These studies were conducted by the Agricultural Research Service, Watershed Technology Research Branch, in cooperation with the Ohio Agricultural Experiment Station. Grateful acknowledgment is made of the assistance of J. B. Urban, Geologist, in the latter phases of this study.

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18 "Flow-Duration Curves," by J.K. Searcy, U. S. Geol. Survey Water Supply, Paper 1542-A, Washington, D. C., 1959.

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FREQUENCY OF NATURAL EVENTS

By H. C. Riggs,[1] M. ASCE

SYNOPSIS

Magnitude-frequency relationships are used frequently in design problems, yet the precise meaning of the relation is not widely understood. This paper begins with the development of a cumulative frequency curve and its statistical interpretation. From the frequency curve, a relation between magnitude, design period in years, and probability of not exceeding that magnitude in the design period is derived. The relation is presented graphically for easy use, and the applicability of the general procedure is shown by sampling from a 1,023-yr period of tree-ring indexes.

INTRODUCTION

An estimate of the frequency of an event, such as a flood or a drought, usually is obtained from a cumulative frequency curve. That curve, based on observed events, and constructed by one of several standard methods, relates the magnitudes of events to mean recurrence intervals or to probabilities. Both the magnitude and the recurrence interval in such a plot are subject to sampling errors. The sampling error of the magnitude can be reduced only by increasing the sample size. A sampling error of the recurrence interval is present because the recurrence interval is not a fixed value but is the mean length of the intervals between events that exceed a given magnitude. The two sampling errors are dependent and a procedure for combining them has not been

Note.—Discussion open until June 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 1, January, 1961.

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developed except by non-parametric methods. However, a relation between magnitude, probability of exceedance (the occurrence of an event greater than that magnitude), and design period (defined later) will allow the variability of the recurrence interval to be assessed and will therefore provide more complete information than is given by the conventional frequency curve. Such a relation is developed in this paper.

The theory used and the results obtained are not original. Beard,[2] Davenport,[3] Thomas,[4] Court,[5] Gumbel,[6] Kendall,[7] and others have made similar analyses. However, the method of presenting the results is new, and the importance of the theory would seem to justify its wider notice.

The construction of a cumulative frequency curve from a probability density function and from a small sample of observations is described first. Next is given the statistical interpretation of the cumulative frequency curve and the derivation of the relation between magnitude, probability, and design period. Finally, the theoretical results are substantiated by sampling from a list of 1,023 tree-ring indexes.

CONSTRUCTION OF CUMULATIVE FREQUENCY CURVES

Consider the histogram of Fig. 1 that shows the frequency of events for several ranges of magnitude. If the number of observations is allowed to approach infinity at the same time that the class interval (width of the rectangles) approaches zero, the enveloping line of the histogram will approach a smooth curve. Then if the ordinate values are divided by a number, such that the area under the curve becomes one, the resulting curve is a probability density curve, also shown in Fig. 1.

For the theoretical development of the cumulative frequency curve, assume that the probability density curve is known to be that of Fig. 1. By definition, the probability of a random event falling in any particular interval is the ratio of the area under the curve within that interval to the total area under the curve. The hatched area under the curve of Fig. 1 is one-tenth of the total, and by the preceding definition the probability is 0.1 that a random event will be greater than E. There is no probability associated with the exact event E. Probabilities in continuous distributions refer only to an event being within a certain range or of being larger or smaller than some magnitude E.

In hydrology it is conventional to interpret the cumulative frequency curve as giving the probability of occurrence of an event "equal to or greater (less) than." The "equal to" portion of the statement is not supported by theory, has no practical meaning, and, therefore, is not used in this paper.

If the area under the curve of Fig. 1 is divided into many vertical strips, the relative area of each determined and these relative areas plotted cumulatively

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2 "Statistical Analysis in Hydrology," by L. R. Beard, Transactions, ASCE, No. 108, 1943, pp. 1110-1121.

3 "Discussion of Characteristics of Heavy Rainfall in New Mexico and Arizona by Luna B. Leopold," by R. W. Davenport, Transactions, ASCE, No. 109, 1944, pp. 877-878.

4 "Frequency of Minor Floods," by Harold A. Thomas, Jr., Journal, Boston Soc. of Civ. Engrs., Vol. 35, No. 4, 1948, pp. 425-442.

5 "Some New Statistical Techniques in Geophysics," by Arnold Court, Advances in Geophysics, Academic Press, Inc., New York, Vol. 1, 1952, pp. 45-85.

6 "The Calculated Risk in Flood Control," by E. J. Gumbel, Applied Science Research, The Hague, Holland, Sect. A, Vol. 5, 1955, pp. 273-280.

7 "Statistical Analysis of Extreme Values," by G. R. Kendall, First Canadian Hydrology Symposium, Natl. Research Council of Canada, November 4 and 5, 1959.

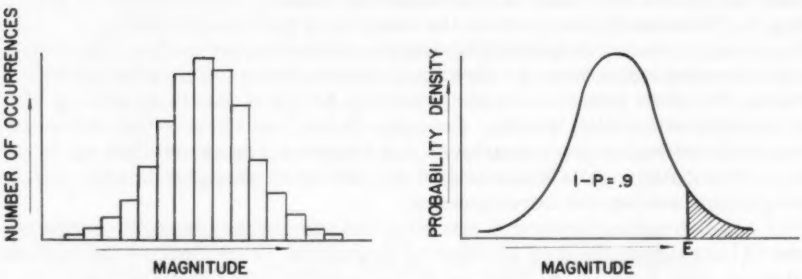

FIG. 1.—HISTOGRAM AND PROBABILITY DENSITY CURVE

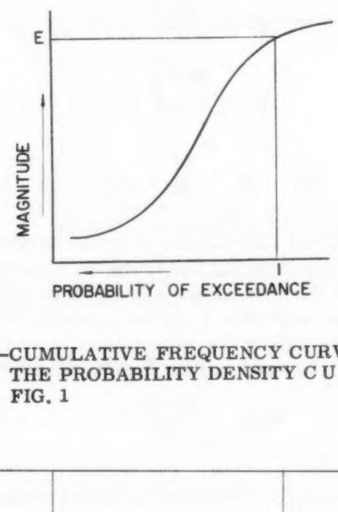

FIG. 2.—CUMULATIVE FREQUENCY CURVE FROM THE PROBABILITY DENSITY CURVE OF FIG. 1

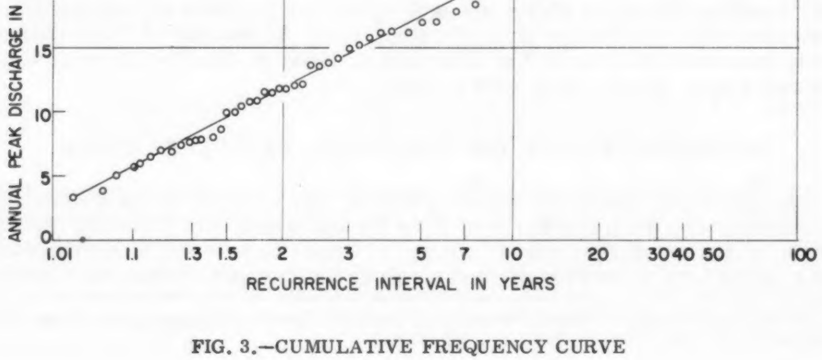

FIG. 3.—CUMULATIVE FREQUENCY CURVE

against magnitude, the result is a cumulative frequency curve such as is shown in Fig. 2. The cumulative curve is the integral of the density curve.

In practical work, the probability density curve is never known. The cumulative frequency curve must be developed directly from the data by one of two methods. The first requires the mathematical fitting of data to an arbitrarily-selected theoretical distribution. Procedures are described in the literature. The second method is semi-graphical and requires no assumptions as to the type of distribution. It is described in the following paragraphs as an aid in understanding subsequent developments.

The semi-graphical method of obtaining the cumulative frequency curve requires (1) arranging the data in order of magnitude, (2) computing the plotting position of each item, (3) plotting each item against its corresponding plotting position on probability paper, and (4) fitting a line to the plotted points.

Plotting positions may be computed by one of several formulas. The more common ones are

$$p = \frac{M}{N+1} \qquad (1)$$

$$p = \frac{M - \frac{1}{2}}{N} \qquad (2)$$

and

$$p = \frac{M}{N} \qquad (3)$$

in which p is the probability of exceedance, N is the number of events (or years, for annual events) used in preparing the frequency curve, and M is the order number of an event when the events are arranged in order of magnitude from the largest to the smallest, with M = 1 for the largest. More commonly the reciprocal of the probability, called the return period or recurrence interval, is plotted, and that approach is used in this paper. The formula used here is

$$\text{R.I.} = \frac{N+1}{M} \qquad (4)$$

in which R. I. is recurrence interval. The following developments are not affected by the equation chosen to compute the recurrence intervals.

Only three types of probability plotting paper are commonly used; normal, log-normal, and extreme value. The frequency curves given herein are plotted to a recurrence interval scale based on an extreme-value distribution proposed by Gumbel.[8] When graphic interpretation of the plotted points is used the resulting line is not always straight; if not, the line does not represent an extreme-value distribution of the Gumbel[8] type. An example of a cumulative frequency curve is given in Fig. 3 for annual floods on John Day River at McDonald Ferry, Oregon, from 1905 to 1948.

INTERPRETATION OF THE CUMULATIVE FREQUENCY CURVE

As previously stated, the density curve of Fig. 1 defines the probability of a random event being smaller than E as 0.9 and larger than E as 0.1. Similarly, on the cumulative frequency curve of Figure 2, the event E corresponds to a probability of exceedance of 0.1. Thus, the cumulative frequency curve

8 "On the Plotting of Flood Discharges," by E. J. Gumbel, Transactions, American Geophysical Union, Part II, 1943, pp. 699-719.

shows the probability that a single random event will exceed a given magnitude. If the cumulative frequency curve is based on annual events (that is, only the largest event per year is used), then the probability given is that of an annual event exceeding a certain magnitude. Now suppose the abscissa scale of Fig. 2 is changed to a recurrence-interval scale by taking reciprocals of the probabilities. The 0.1 probability becomes a 10-yr recurrence interval, and E is called the 10-yr event. This means that the average time between annual events that exceeds E is 10 yr. This is illustrated in Fig. 4 by considering the

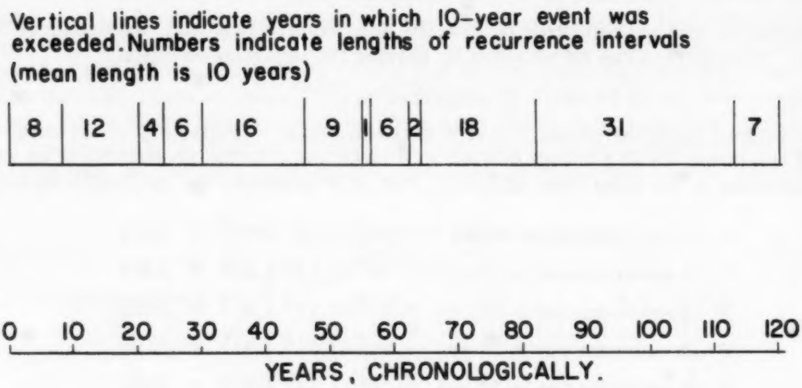

FIG. 4.—HYPOTHETICAL SEQUENCE OF RECURRENCE INTERVALS DURING A 120-YEAR PERIOD

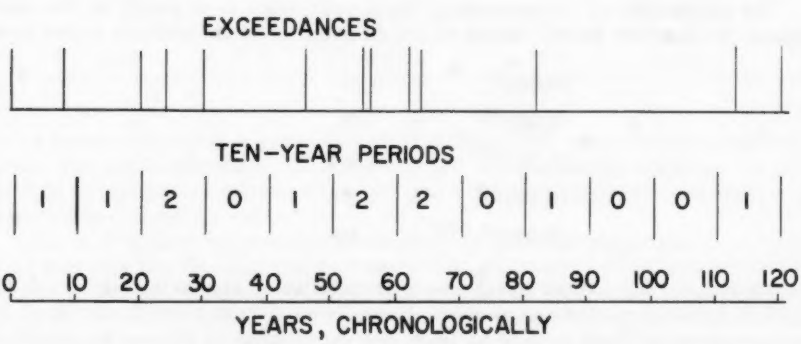

FIG. 5.—RANDOM SEQUENCE OF RECURRENCE INTERVALS (FROM FIG. 4), SHOWING THE NUMBER OF EXCEEDANCES IN EACH 10-YEAR PERIOD

random sequence of recurrence intervals in 120 yr. Although these intervals between exceedances range from one to 31 yr the average is 10 yr. Therefore, the event exceeded could be called a 10-yr event. Neglecting one of the two end exceedances, there are 12 exceedances in 120 yr for a probability of exceedance in any one year of 12/120 or 0.1, as previously defined. If both or neither of the end exceedances were neglected, the computed probabilities would approximate the 0.1 value.

Recurrence interval is an average value, thus, the n-year event will be exceeded at intervals averaging n years in length, but will also be exceeded in more than half of a series of n-year periods. If clarification is needed, refer again to Figs. 1 and 2. The probability of not exceeding E is 0.9, Fig. 1, and of exceeding E, 0.1, Fig. 2. If E is assigned a value equal to the 10-yr event, the probabilities remain the same. The probability of 0.9 applies to one annual event not exceeding E. The probability of not exceeding E in 10 yr is, by the multiplicative law

$$(0.9)^{10} = 0.35$$

Similarly, the probabilities of exactly one, exactly two, etc., exceedances in the 10-yr period can be obtained by solving the binomial equation

$$f(x) = {}_{n}C_{x}\, p^{x}\, (1-p)^{n-x} \qquad (5)$$

in which $f(x)$ is the probability of x exceedances in n trials, ${}_{n}C_{x}$ is the number of combinations of n things taken x at a time, and p is the probability of an exceedance in one trial (see Mood[9]). For this problem, the probabilities are:

$$P\ (\text{1 exceedance in 10 yr}) = {}_{10}C_{1}(.1)\ (.9)^{9} = .3874$$
$$P\ (\text{2 exceedances in 10 yr}) = {}_{10}C_{2}(.1)^{2}\ (.9)^{8} = .1935$$
$$P\ (\text{3 exceedances in 10 yr}) = {}_{10}C_{3}(.1)^{3}\ (.9)^{7} = .0576$$
$$P\ (\text{4 exceedances in 10 yr}) = {}_{10}C_{4}(.1)^{4}\ (.9)^{6} = .0105$$
$$P\ (\text{5 exceedances in 10 yr}) = {}_{10}C_{5}(.1)^{5}\ (.9)^{5} = .0015$$

The sum of the probabilities of 0, 1, 2, ... , 10 exceedances (in a 10-yr period) equals one.

The probability of not exceeding the n-year event in n years is, for other values of n and for known values of the probability of exceedance in one year:

$$(0.80)^{5} = 0.33$$
$$(0.95)^{20} = .36$$
$$(0.98)^{50} = .364$$
$$(0.99)^{100} = .366$$
$$(0.999)^{1,000} = .368$$

Therefore, the n-year event has a probability of approximately 1 - 0.36 = 0.64 of being exceeded one or more times in an n-year period; or 64 of 100 n-year periods would include at least one exceedance of the n-year event; or the n-year event will be exceeded at least once in about 64% of a series of n-year periods. The last is shown empirically in Fig. 5 in which the sequence of exceedances of Figure 4 is repeated. The 120-yr period is divided into 12

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9 "Introduction to the Theory of Statistics," by A. M. Mood, McGraw-Hill Book Co., Inc., New York, 1950, pp. 54-58.

ten-yr periods and 8 of the 12 periods (67%) include exceedances of the ten-yr event.

DESIGN-PROBABILITY CURVES

It has been shown that the probability of the n-year event (from a cumulative frequency curve of annual values) being exceeded in an n-year period is about 0.64. Additional information regarding the probabilities of exceedances in a definite period of years would be useful and can be obtained. Consider the use to which a frequency relationship is put. Design of a project logically might begin with selection of a design period, the number of years for which the project is expected to operate. Having fixed that period, the designer would inquire as to the probability of occurrence of damaging floods or of inadequate supply during the period. The conventional frequency curve cannot answer this inquiry adequately. A relation between (1) magnitude, (2) probability of not exceeding that magnitude, and (3) design period would supply the answer needed.

TABLE 1.—VALUES OF p AND R. I.

n (Design period) (1)	p (2)	R.I. = 1/p (3)
2	.293	3.4
3	.206	4.8
5	.129	7.8
10	.067	14.9
20	.034	29.4
30	.023	43.0
40	.017	59.0
50	.0138	72.0
100	.0069	145.0
1,000	.00069	1,450.0

Such a relationship can be obtained by modifying the conventional frequency curve. The modifications to be described are not confidence limits on the position of the frequency curve. They provide a more complete interpretation of the frequency curve as defined by the data.

Assume it is desired to define a relationship between magnitude of an event E and a design period such that there is a 0.5 probability of not exceeding the event in the design period. Further assume that the magnitude-frequency relationship is exactly defined to large recurrence intervals and that the events are randomly distributed in time. Let 1-p equal the probability of not exceeding E in one year and let n equal the number of years. Then

$$(1 - p)^n = 0.5$$

from which the values of p and R. I. given in Table 1 are computed.

These results indicate that the 29.4-yr event from the cumulative frequency curve is the one that has a 0.5 probability of not being exceeded in a single 20-yr period. Likewise it is the 72-yr event that has an even chance of not being exceeded in a 50-yr period.

These results may be used to modify a frequency curve by plotting the magnitude of the 3.4-yr recurrence interval event at the 2-yr design period, the

7.8 at 5, the 14.9 at 10, and so forth. Similar adjustments for different probabilities of not being exceeded can be computed by substituting the desired probability instead of 0.5 in the above formula and recomputing p and R.I. Results for the probabilities of 0.25 and 0.75 are given in Table 2.

The conventional frequency curve of Fig. 3 and the data in Tables 1 and 2 are used as the basis for constructing the curves of Fig. 6. Fig. 6 shows the design-probability curves for annual floods on John Day River at McDonald Ferry, Oregon (based on the curve of Fig. 3). For example, the magnitude of the event having 0.5 probability of not being exceeded in a 10-yr design period is that corresponding to the 14.9-yr recurrence interval (see Table 1) on Fig. 3. Other points are obtained similarly. Notice that the abscissa on Fig. 6 is the length of the design period and not a recurrence interval. These curves are named "design-probability curves" to distinguish them from the conventional cumulative frequency curve. It should be noted that the middle curve of Fig. 6, that for P = 0.50, is essentially the same as the curve obtained by the Beard[2] method.

TABLE 2.—PROBABILITY OF NOT EXCEEDING IN n YEARS

Design Period, n, in years (1)	0.25		0.75	
	p (2)	R.I. (3)	p (4)	R.I. (5)
2	.5	2.0	.134	7.5
3	.37	2.7	.092	10.9
5	.242	4.1	.056	17.8
10	.130	7.7	.028	35.7
20	.067	14.9	.0141	71.0
30	.045	22.2	.0095	105.0
40	.034	29.4	.0072	139.0
50	.0273	36.6	.0057	175.0
100	.0138	72.5	.0029	345.0
1,000	.00138	725	.000283	3,500.0

To further interpret the curves, consider a 20-yr design period on Fig. 6. The probability is 0.5 that a flood of 26,700 cfs will not be exceeded in the 20-yr period. Likewise, the probability is 0.25 that a flood of 23,000 cfs, and 0.75 that one of 31,000 cfs will not be exceeded in the 20-yr period. The latter discharge is based on an extension of the frequency curve (Fig. 3). Design-probability curves for small probabilities of exceedance are thus seen to be limited in extent by the length of the frequency curve from which they are obtained. If the frequency curve of Fig. 3 were plotted on Fig. 6 it would correspond to a probability of about 0.36.

This type of plot (Fig. 6) does furnish many answers needed by the designer. Further, it makes clear that no matter what design period is used, there is still an appreciable probability of experiencing an extremely large event in that period. This tends to be overlooked in interpreting the conventional frequency curve.

Although the probability of experiencing no exceedances is of primary interest, the probabilities associated with other outcomes help to complete the picture. These additional probabilities, computed as shown in the previous section, are given for several sizes of events and for 5 design periods in Table 3. Minor inconsistencies in the table are due to the limited number of significant

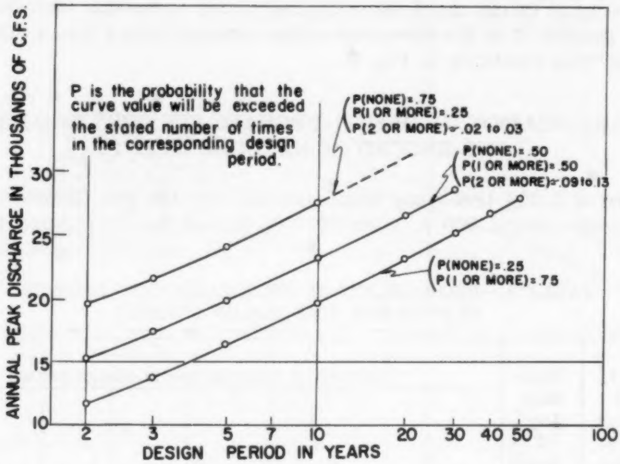

FIG. 6.—DESIGN-PROBABILITY CURVES BASED ON THE CURVE OF FIG. 3

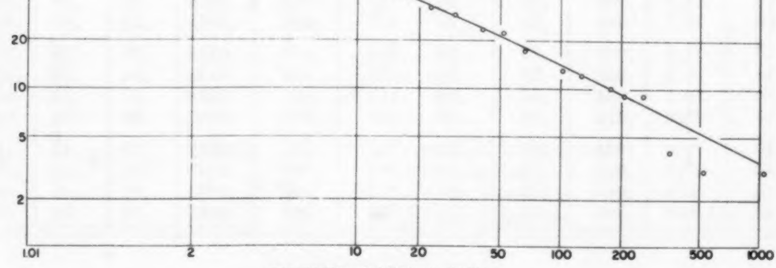

FIG. 7.—CUMULATIVE FREQUENCY CURVE OF TREE-RING INDEXES FOR RIO GRANDE AREA, NEW MEXICO, 908-1930 A. D.

figures used. For a given probability of no exceedances, the probability of one or more exceedances is fixed (because the sum of the two must equal one), but the probability of two or more exceedances varies somewhat with the length of the design period. It is for this reason that a range rather than a unique value is shown for this condition in Fig. 6.

APPLICATION OF DESIGN-PROBABILITY CURVES TO A LONG RECORD OF NATURAL EVENTS

A record of 1,023 tree-ring width indexes for the Rio Grande area, New Mexico, for the period 908 A. D. to 1930 A. D. has been compiled by Smiley, Stubbs, and Bannister.[10] The record is a composite each part of which has been adjusted for growth trend. The annual growth of a tree (as measured by the tree-ring width index) in certain locations in the arid West is very sensi-

TABLE 3.—PROBABILITY OF EXCEEDANCE OF VARIOUS EVENTS FOR FIVE DESIGN PERIODS

Design period, in years	R. I. of event from frequency curve	Probability of exceedance in one year	Number of exceedances in design period							
			0	1	2	3	4	1 or more	2 or more	3 or more
2	2.0	.500	.25	.50	.250	...	...	0.75	0.25	...
2	3.4	.293	.50	.41	.086	...	...	.50	.09	...
2	7.5	.134	.75	.23	.018	...	...	.25	.02	...
5	4.13	.242	.25	.40	.255	.082	.0130	.75	.35	.095
5	5.0	.200	.33	.41	.205	.051	.0064	.67	.26	.055
5	7.8	.129	.50	.37	.110	.016	.0013	.50	.13	.020
5	17.8	.056	.75	.22	.026	.002	.0001	.25	.03	.004
10	7.7	.130	.25	.37	.250	.100	.0273	.75	.38	.130
10	10.0	.100	.35	.39	.194	.057	.0112	.65	.26	.066
10	14.9	.067	.50	.36	.116	.022	.0028	.50	.14	.024
10	35.7	.028	.75	.22	.028	.002	.0001	.25	.03	.002
20	14.9	.067	.25	.36	.245	.105	.0319	.75	.39	.145
20	20.0	.050	.36	.38	.189	.059	.0133	.64	.26	.071
20	29.4	.034	.50	.35	.118	.025	.0028	.50	.15	.032
20	71.0	.014	.75	.21	.030	.003	.0002	.25	.04	.010
40	29.4	.034	.25	.35	.242	.110	.0352	.75	.40	.158
40	40.0	.025	.36	.37	.185	.060	.0143	.64	.27	.085
40	59.0	.017	.50	.34	.118	.026	.0041	.50	.16	.042
40	139.0	.007	.75	.21	.030	.003	.0002	.25	.04	.010

10 "A Foundation for the Dating of Some Late Archeological Sites in the Rio Grande Area, New Mexico," Bulletin, University of Arizona, Vol. XXIV, No. 3 (Laboratory of Tree-Ring Research Bulletin, No. 6).

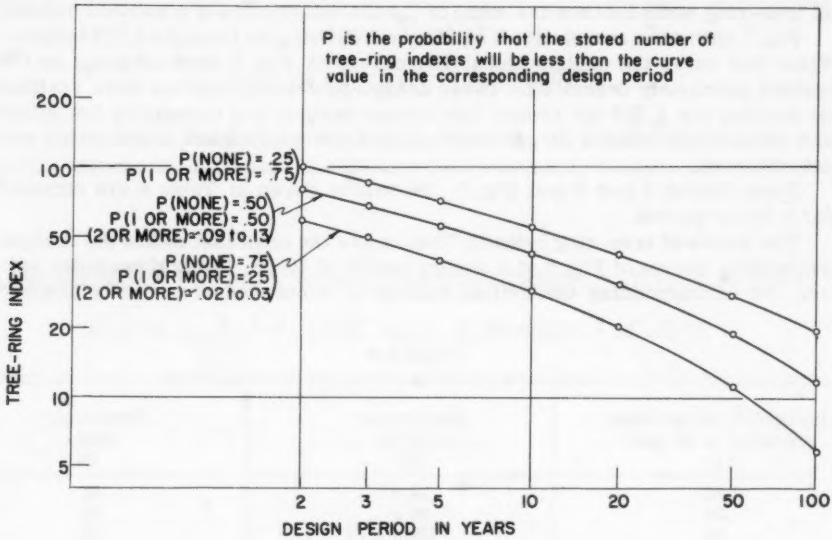

FIG. 8.—DESIGN PROBABILITY CURVES OF TREE-RING INDEXES (BASED ON CURVE OF FIG. 7)

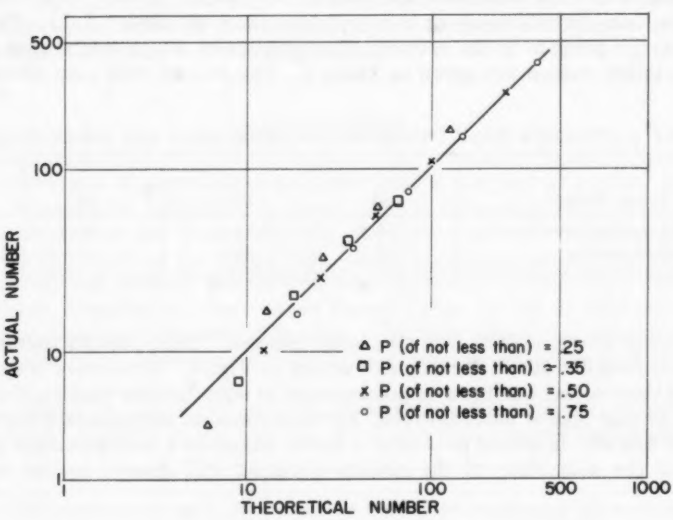

FIG. 9.—COMPARISON OF ACTUAL AND THEORETICAL NONEXCEEDANCES FROM ANALYSIS OF A 1,023-YEAR RECORD OF TREE-RING INDEXES

tive to weather conditions. As previously proposed by Douglass[11] the record of tree-ring width indexes has some of the characteristics of a weather record.

Fig. 7 shows the cumulative frequency curve obtained from the 1,023 indexes. From that curve the design-probability curves of Fig. 8 were obtained by the method previously described. These design-probability curves were verified by dividing the 1,023-yr record into n-year periods and comparing the actual and theoretical number of periods in which the appropriate index value was not exceeded.

From Tables 1 and 2 and Fig. 7, the values shown in Table 4 are obtained for a 20-yr period.

The values of tree-ring index in Table 4 are the ones that define the design-probability curves of Fig. 8 at a design period of 20 yr. Using these index values, the corresponding theoretical number of nonexceedances can be checked

TABLE 4

Probability of not being exceeded in 20 years (1)	Recurrence interval (2)	Tree-ring index (3)
0.25	14.9	41
.36	20.0	35
.50	29.4	29
.75	71	17

by counting the number of 20-yr periods in the original record in which these index values are not exceeded (not exceeded means "greater than" here because the cumulative frequency curve gives values of "less than"). There are 51 twenty-yr periods in the record. The number of 20-yr minimums greater than the index values are given in Table 5. The record was also divided into

TABLE 5.—TWENTY YEAR MINIMUMS GREATER THAN THE INDEX VALUES

Index Value	41	35	29	17
Theoretical Number	12.8	18.4	25.5	38.2
Actual number	17	20	25	38

2, 5, 10, and 40-yr periods and similarly studied. These results together with those obtained for 20-yr periods are shown in Fig. 9. Agreement between actual and theoretical numbers is good enough to substantiate applicability of the method to this type of natural event. For this example the cumulative frequency curve is known. In actual practice, a curve based on a small sample must be used and the reliability of the results obtained will depend on the sampling error.

11 "Climatic Cycles and Tree Growth: A Study of the Annual Rings of Trees in Relation to Climate and Solar Activity," by A. E. Douglass, Publication No. 2890, Carnegie Inst., Washington, D. C., 1928.

Journal of the
HYDRAULICS DIVISION
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UNDERGROUND OUTLET WORKS FOR COURTRIGHT AND WISHON DAMS

By J. B. Cooke,[1] F. ASCE, and J. E. Schumann,[2] M. ASCE

SYNOPSIS

The diversion and outlet tunnel for Courtright Dam is in competent granite. It is unsupported and unlined. The outlet tower and trashrack are submerged. The outlet control is located in the tunnel at the axis of the dam and consists of: (1) a pipe embedded in a plug, (2) a butterfly valve in a dry chamber, (3) a cone-dispersion valve in a discharge chamber, and (4) an air-vent access shaft. Wishon Dam outlet works are similar.

INTRODUCTION

Courtright and Wishon Dams and Reservoirs are part of Pacific Gas and Electric Company's 313,000 kw Kings River Hydroelectric Development, which is located in the Sierra Nevada Mountains east of Fresno, Calif. The overall development of the Kings River is multipurpose including irrigation, flood control, and power. The principal cooperating agencies are the Kings River Water Association, the United States Army Corps of Engineers, and the Pacific Gas and Electric Company. The new roads and Courtright and Wishon Reservoirs have greatly increased the recreation potential of the area.

Fig. 1 shows the plan and profile of the power development. The Wishon and Courtright rockfill dams and the Haas and Balch power plants were all

Note.—Discussion open until June 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 1, January, 1961.

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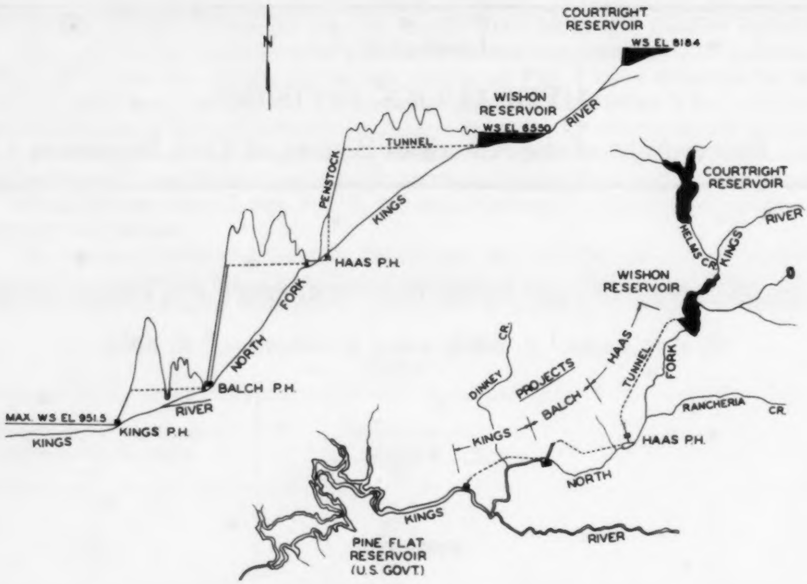

FIG. 1.—MAP AND PROFILE - KINGS RIVER DEVELOPMENT

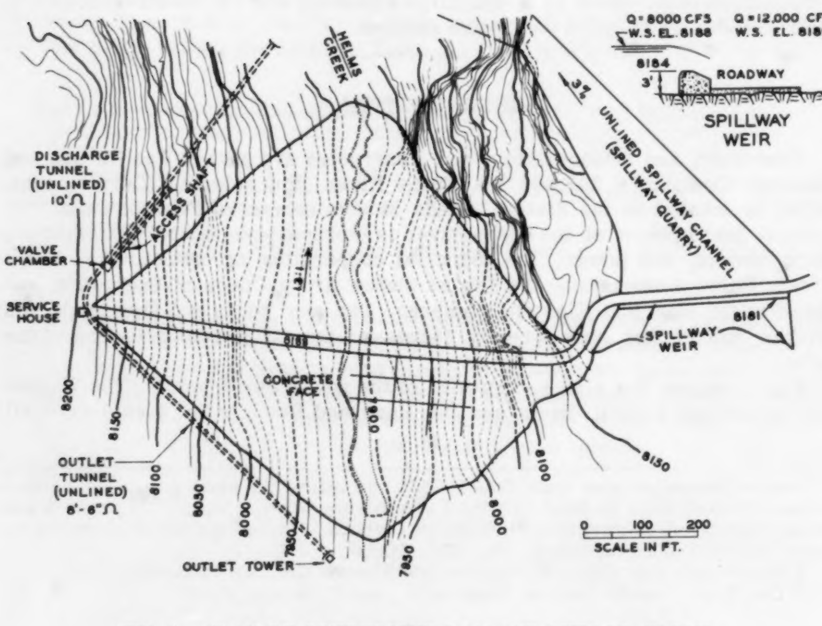

FIG. 2.—PLAN OF COURTRIGHT DAM AND OUTLET WORKS

completed in 1958. The Kings River Plant is under construction and is scheduled to be completed in June, 1962.

The storage reservoirs, Courtright and Wishon, have normal maximum water surface elevations of 8,184 and 6,550. Precipitation at these elevations is primarily in the form of snow. Runoff from melting snow occurs in late spring and early summer, followed by very low flows during late summer and fall and most of the winter. Annual runoff may vary considerably from year to year, some dry years yielding as little as 60% of the average. Power demand, on the other hand, is generally greatest in the low runoff months of August, September, and December. Because the natural streamflow characteristics are not suited to power development, the high elevation storage reservoirs are necessary to provide annual and cyclic storage.

Water and power studies indicated about 250,000 acre ft of storage to be required. Only two natural reservoir and dam sites existed in this steep granite country, and economic and water study analysis indicated that the storage should be shared about equally between the two sites (Courtright 121,300 acre ft and Wishon 128,400 acre ft). Unfortunately, the best site had the smallest drainage area. Of the total 178 sq mi drainage area of Wishon, only 39 sq miles are tributary to Courtright. However, by assigning most of the carryover storage to Courtright and drawing heavily on Wishon each year, it is usually possible to start each summer with both reservoirs full.

The design, construction, and performance of Courtright and Wishon Dams have been described in detail,[3] but the outlet works have been briefly examined.

GENERAL DESIGN CONSIDERATIONS - COURTRIGHT DAM

General.—A basic and economical decision was to permit flow through the concrete face rockfill dam during construction, and thus permit design of the permanent outlet for the smaller controlled flow. It was considered desirable that the outlet conduit be at stream bed level in order to be able to utilize all storage and to have future access to the concrete face of the dam. However, operation will normally maintain a substantial minimum pool. The alternative conduits considered were a steel lined reinforced concrete conduit in the stream bed and a tunnel. The geology of the site is such that a tunnel would be unsupported and could remain unlined. Though the concrete conduit was the shortest, it was estimated to cost more than an unlined tunnel. The unlined tunnel was adopted as being both economical and preferable from the standpoint of maximum safety.

Site Geology.—Bedrock at the damsite and in the surrounding area is granodiorite. The left bank is a granitic mass with no jointing other than "onionskin surface layers." The right bank, deeply jointed and steep, presents stepped cliffs in the pattern of the jointing.

Diversion.—The diversion tunnel size was established to meet permanent outlet requirements. Due to snow and freezing conditions at the high elevation site, over 8,000, the construction season is only about 5 months. During the first construction season in 1956 the tunnel was driven, streambed cutoff was constructed, and some rockfill was dumped. The 1957 runoff passed

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[3] "Rockfill Dams: Wishon and Courtright Concrete Face Dams," by J. Barry Cooke, Proceedings, ASCE, Vol. 84, No. PO 4, August, 1958.

through the rockfill and the tunnel. At the end of the 1957 construction season, the concrete face was poured to a uniform height, to distribute flow evenly into the dam in case the 1958 runoff exceeded the diversion tunnel capacity.[3] Actually, the runoff was low and the concrete face slab was not overtopped. In order to permit convenient and dry construction of the outlet works in the diversion tunnel, one bottom hinge slab of the concrete face rockfill dam was not placed, and the small summer flow was diverted through the rockfill in 1958 while the outlet works were being constructed and the dam was being completed.

Tunnel Location and Alinement.—The tunnel alinement is shown on the plan of the dam and outlet works (Fig. 2). A right abutment tunnel location was favored initially because it would be shorter and more suitable for a possible future tunnel and power plant between Courtright and Wishon Dams. However, the left abutment was chosen to avoid the deeply jointed granite and to be assured of the tunnel being unsupported, unlined, and watertight in the massive granite of the left abutment. The tunnel alinement enters the abutment parallel to the cutoff in order not to cross the cutoff grout curtain until adequate rock cover, for an unlined pressure tunnel under the dam, was obtained. An acceptable criterion would have permitted the tunnel to cross the cutoff where rock cover equalled about 0.7 times static head, but in order to accommodate a vertical float well, for determination of reservoir level, it was alined to pass under the end of the dam where a drill hole could be driven from the service house to the tunnel. The cover of 0.7 static head is based on the buoyant weight of rock being equal to the static head.

Operating Requirements.—Courtright Reservoir water is released to the streambed and flows to Wishon Reservoir. Runoff from the Courtright drainage area is stored until sometime in July when the Haas Power Plant begins to draw on Wishon storage. Courtright storage that is scheduled to be used that year is then released in order to put the water through the Haas plant at maximum head, that is, from a nearly full Wishon Reservoir level. The required outlet capacity was therefore the Haas Power Plant flow, 800 cfs. It is neither practical nor essential to release 800 cfs from Courtright when the reservoir is at a low stage and a minimum design head for the 800 cfs flow was established. Because reservoir capacity in the lower 100 ft depth of the 300 ft deep reservoir is only 2,000 acre ft, a practical and economical design criteria, 800 cfs discharge with 100 ft of reservoir head, was adopted.

Frequently a storage reservoir of this importance would have two similarly sized outlets in order not to rely on one only. At Courtright one regulating outlet with a small bypass was accepted. The small bypass was required to drain the full reservoir during four months of the low inflow period. Flows for stream maintenance and fish life are released by a smaller valve.

Outlet Tower and Shutoff Valve Location.—In a deep storage reservoir, with relatively small regulated outflow, a full height outlet tower and gate or valve is seldom used because of its high cost. A possible alternative is a gate or valve in a submerged outlet structure. At Courtright, the design went one step further and utilized a submerged outlet tower and trashrack with no shutoff gate or valve in the tower. The shutoff valve was located in a dry chamber at the axis of the dam, which has the desirable feature of making the valve and operating system accessible.

Adopted Basic Layout.—Having determined the outlet tunnel alinement and accepted a submerged outlet without shutoff valve, three alternatives for outlet valves and piping were considered: (1) a plug and shutoff valve at the axis

of the dam with a free pipe in the tunnel and discharge valve at the downstream portal; (2) a concrete embedded pipe in the tunnel from the axis of the dam to the downstream portal, with shutoff and discharge valves in a chamber at the portal; and (3) the adopted layout shown on Figs. 2 and 3. The adopted layout makes maximum economical use of the excellent granite and has advantages over other layouts, such as (1) lowest cost, (2) safety from freezing, and (3) minimum maintenance.

Economics.—In locating the shutoff and discharge valves adjacent to each other in the underground chamber, a 500 ft length of concrete backfilled pipe, and a valve chamber at the portal were eliminated. However, the cost of the access shaft and the additional cost of the underground chamber offset some of the saving. At this high elevation site, over 8,000, the air temperature remains below freezing several months of the year and temperatures get as low as 20° to 40° below zero. The dam is unattended and no power is available. The location of the valves and piping well underground is the best defense against freezing. The use of unlined tunnel in competent granite instead of 500 ft of pipe removes the danger of damage to such pipe and eliminates maintenance.

OUTLET WORKS DESIGN - COURTRIGHT DAM

General.—The basic engineering considerations resulting in the adopted arrangement (Fig. 3) have been outlined. The more detailed design will now be reviewed.

Outlet Tower.—The concrete tower (Fig. 3) with overall dimensions of 12 ft-6 in. square by 30 ft high is capped with a 12 ft square by 30 ft high galvanized steel trash rack. Two 24 in. square gates are located in the base of the concrete tower and are provided with a grizzly. In normal operation, the two small gates are closed and the outlet flow passes through the large area box trash rack. When the reservoir is nearly empty, the water surface may be brought down below the top of the concrete tower by opening the gates. Closing the gates permits inspection or maintenance of the unlined tunnel and the disk and seat of the shutoff butterfly valve, while inflow is stored below the top of the concrete tower. If the requirement for storage of inflow during a period of access to the tunnel had not governed, the top of the tower would have been set about 10 ft above ground line to protect the steel trash rack from possible slides and to provide storage for water-logged debris that slides down from the vertical rack bars.

The operating cylinders for the gates are located above the top of the concrete tower to permit occasional inspection and possible maintenance without complete unwatering of the reservoir. Operation is by a small portable hand pump, and the connections are at the top of the trash rack so that the gates may be operated whenever the water level is below the top of the trash rack. An important consideration in installing a hydraulic cylinder-operated slide gate is to grout the cylinder base plate in place when the gate is properly seated and the piston is against the bottom of the cylinder. This prevents subjecting the stem, the gate frame, and gate seats to the maximum operating cylinder force each time the gate is closed. This requirement is important, and is best spelled out on the installation drawing.

Similar minimum cost outlet towers have been used for some of the company's other reservoirs. The size of gates and minimum height of concrete

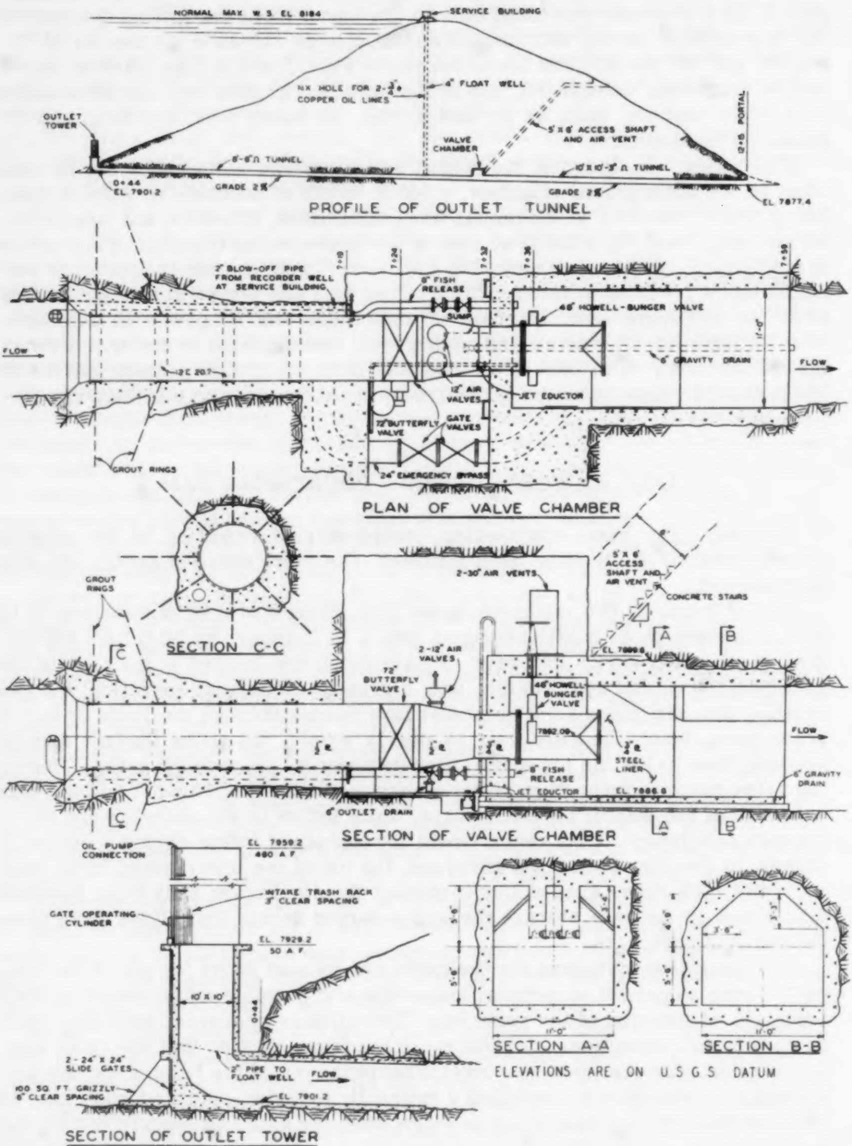


FIG. 3.—COURTRIGHT DAM OUTLET WORKS

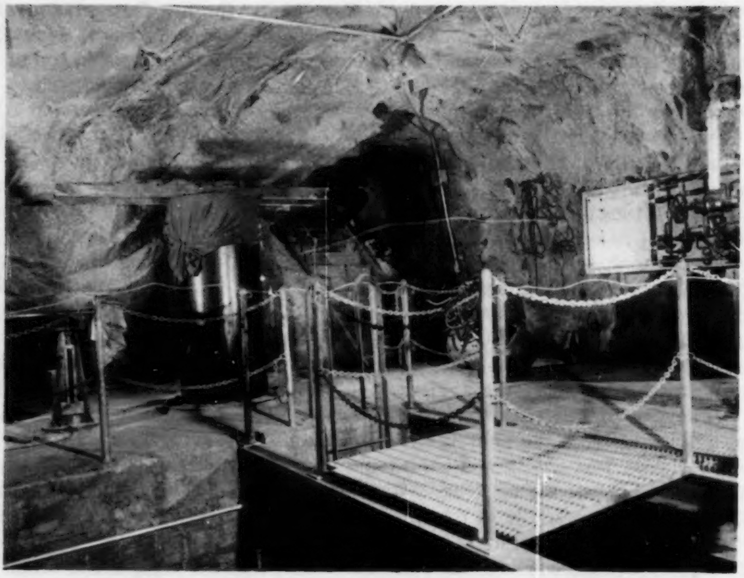
tower depend on the rate of reservoir inflow during the low flow season when the reservoir might be emptied. At Courtright, the inflow during the low flow season is only 2 cfs to 5 cfs and the 50 acre ft of storage below the top of the concrete tower is considered adequate to give about a week of dry access to the tunnel and to the shutoff butterfly valve. On the other hand, at Wishon 1,200 acre ft was considered necessary. Above the top of the concrete tower a steel frame with panels of rack bars provides the lowest cost trash rack.

Pressure Tunnel.—The tunnel between the outlet tower and the valve chamber is a 630 ft long unlined pressure tunnel under 300 ft head. It is an 8 ft-6 in. horseshoe that is a minimum practicable size for construction of the tunnel and for construction access to the valve chamber. Head loss is not of importance and the velocity in the unlined tunnel is 13 fps for the 800 cfs design flow. This velocity would move 12 in. rocks, and for that reason the invert was thoroughly cleaned in contrast to leaving all muck in the inverts of the company's power tunnels that operate at 5 to 5.5 fps velocity flow. The tunnel is all unsupported and is unlined, except for the arbitrary lining of 30 ft from the outlet tower to seal positively the tunnel from the reservoir. The acceptance of the unlined tunnel upstream from the shutoff valve to a major reservoir is evidence of the successful experience with and confidence in unlined tunnels.

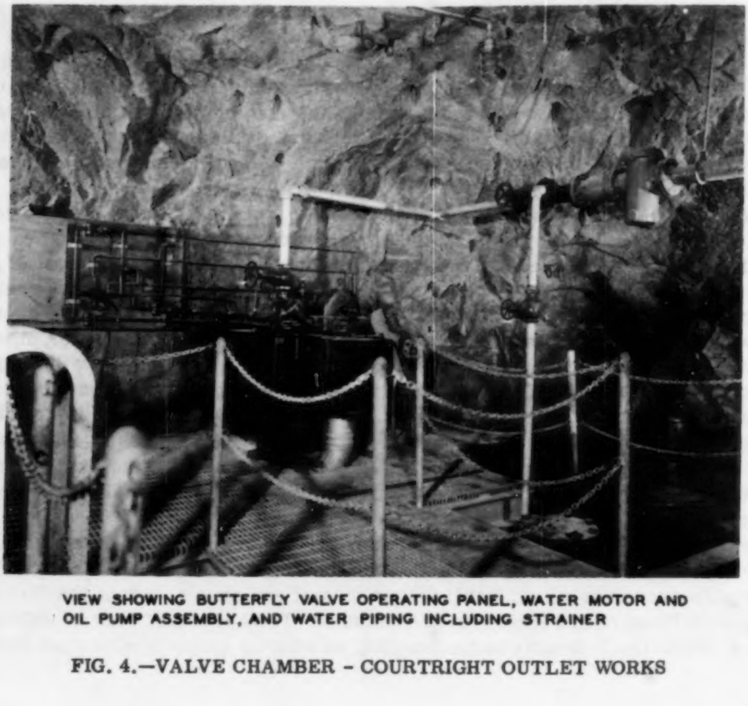
Plug.—The grouted plug (Fig. 3) is 25 ft long and is subjected to a 300 ft head. Grouted plugs of 20 ft to 30 ft length have given watertight and secure service at many of the company's dams and tunnel adits, operating at heads of 100 ft to 300 ft in unlined rock. The saw-tooth shape of the excavation might not be essential in rough rock excavation, but, as shown in Fig. 3, the drilling for two rounds of excavation is given intentional additional flare to provide a positive key. For convenient drilling and grouting after the pipe is concreted in place, 3-in. threaded plugs are furnished in the pipe. They are spaced at 45° in both of the grout rings. Drilling of EX (1-1/8 in.) holes and grouting are done through these holes. The channels stiffen the pipe and confine the high pressure grout, as well as function as a water-stop. The grouting between pipe and concrete is done through small 3/8 in. holes that are drilled and tapped at bottom and top of areas found to be hollow by sounding the pipe. The top holes are to release air and indicate return grout.

Pipe.—The 72-in. pipe embedded in the plug and the 72 to 48-in. pipe connecting the butterfly and Howell-Bunger valve are conservatively designed and carefully installed. The cost of the pipes is nominal compared to the cost of the high quality valves, and a rugged installation is important where high energy and possible vibration are involved. The steel used is A285B, which is ductile and weldable. The working stress is 9,000 psi in the 72-in. diameter pipe of 1/2-in. plate thickness. The 3/4-in. plate connecting to the Howell-Bunger valve is arbitrarily thick to hold rigidly on to the cantilevered free discharge valve. The two large pieces are fully shop fabricated, welds are radiographed, and the completed sections stress-relieved. The interior was shop coated with hot coal tar enamel and the exterior was shop primed for two field coats of cold applied coal tar. Flanges are riveted to the pipes after the complete assembly is made in the field.

Dry Valve Chamber.—Access to the dry valve chamber is by a concrete stairway in a 5 ft wide by 6 ft high shaft sloping at 45° and 200 ft long. The shaft also serves as an air vent. All valves and equipment, except the Howell-Bunger H-B) valve, are located in this chamber. The operating deck is shown in Fig. 4. While the H-B valve is discharging, an eductor pump is also operated to



VIEW SHOWING MANUAL H-B VALVE OPERATOR, AIR VENT, STAIRWAY IN ACCESS SHAFT, AND BUTTERFLY VALVE OPERATING PANEL



VIEW SHOWING BUTTERFLY VALVE OPERATING PANEL, WATER MOTOR AND OIL PUMP ASSEMBLY, AND WATER PIPING INCLUDING STRAINER

FIG. 4.—VALVE CHAMBER - COURTRIGHT OUTLET WORKS

keep the chamber dry. The ceiling and the walls of the chamber are unsupported and unlined in excellent granite as illustrated in Fig. 4. A 5 kw gasoline-electric generator is located in the service house and supplies power for lighting the access shaft and valve chamber.

Shutoff Butterfly Valve.—The decision to have only one regulating outlet to this important storage reservoir required a reliable and rugged shutoff valve; a butterfly valve was selected. A trouble-free service record existed for over 100 butterfly valves ranging in size from 3 ft to 13 ft in diameter. These valves are custom made to a conservative specification. The moving parts are few and they are rugged and simple. The seating is on a smooth rounded surface and not a groove in which material can be jammed. The size of the shutoff butterfly valve must be such that the nearby downstream free discharge valve is the positive free discharge control. This is equivalent to saying that there must be positive pressure between the two valves. This condition is obtained if flow of the butterfly valve under free-discharge exceeds that of the H-B valve. Meeting this requirement at Courtright results in a 72-in. shutoff butterfly valve teamed with a 48-in., more hydraulically efficient, H-B valve.

The butterfly is operated by an oil cylinder mounted on a trunnion. The valve is designed to close or open under free discharge conditions. Oil pressure is obtained from a 300 psi water-motor operated oil pump located in the chamber or alternatively from the oil tank, 300 ft above in the service building. Opening or closing is accomplished by operation of valves on the panel board that is mounted near the oil tank in the valve chamber (Fig. 4). However, operating instructions require that when the operator leaves the valve chamber the valves be left in such a manner that the butterfly valve may be closed by opening valve No. 5 (Fig. 6) at the service house. This is a precaution to permit closing of the butterfly valve in the event some unforeseen occurrence flooded the valve chamber. The oil line to the service building tank is in an NX (3 in.) drill hole that connects to the tunnel.

Howell-Bunger Valve.—The Howell-Bunger valve was selected as a low cost, rugged and well balanced valve having a favorable cone shaped discharge jet for dissipation in the underground chamber. The 48-in. valve is hand operated, which is economical and is satisfactory for the infrequent operation that is required at this site and for that size of valve. The large air requirement for this type of valve is readily provided through the access shaft. The characteristics of the fixed-dispersion cone valve are well presented elsewhere[4,5].

Discharge Valve Chamber.—The discharge valve chamber is similar to that at Fontana[4] and Wishon[6]. Those installations, each having one 84-in. H-B valves, operate successfully. Courtright, with its small 48-in. H-B valve, is scaled down in size and is as illustrated in Fig. 3. A thick and well anchored steel liner is used where the jet impinges on the chamber. The wall separating the dry valve chamber and the discharge chamber and the concrete lining of the discharge chamber are arbitrarily thick and over-reinforced out of respect for the large amount of energy dissipated in the chamber. The air vents (Figs. 3 and 4) project 6 ft above the floor to protect the operator from high velocity air. As seen in Fig. 4, a tarpaulin has closed the air vent to reduce the natural

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4 "Characteristics of Fixed Dispersion Cone Valves," by Rex A. Elder and Gale B. Dougherty, Transactions, ASCE, Vol. 118, 1953, p. 907.

5 "Outlet Structures for Fixed-Dispersion Cone Valves," by Maurice L. Dickinson, Stanley M. Barnes, and Robert S. Milmoe, Jr., Proceedings, ASCE, Vol. 84, No. HY 4, August, 1958.

6 "Haas Hydroelectric Power Project," by J. Barry Cooke, Transactions, ASCE, Vol. 124, 1959, p. 989.

and cold draft while the outlet works is not in operation. A ladder is installed in one air vent to the floor of the discharge chamber (Fig. 5) to provide an additional access when the H-B valve is not operating.

Bypass Valve.—One regulating outlet assembly, consisting of a discharge valve and a shutoff valve, was adopted for the conditions at Courtright rather than two parallel regulating assemblies. As a reserve that may never be used, a pair of 24-in. gate valves (Fig. 3) were installed as an emergency outlet. The discharge capacity of the gate valves is such that they would drain the reser-

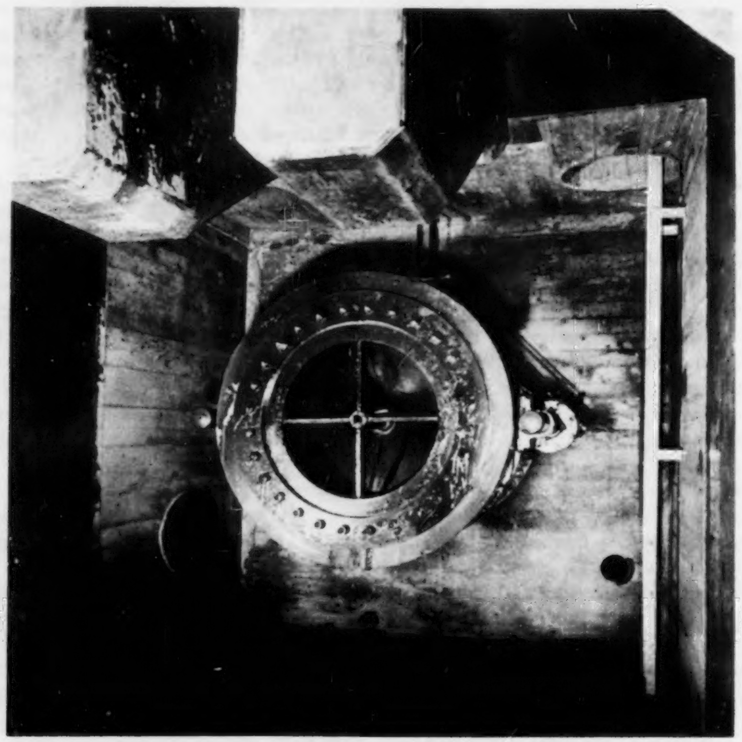

FIG. 5.—HOWELL-BUNGER VALVE AND DISCHARGE CHAMBER - COURTRIGHT DAM

voir during a summer season and permit access to the shutoff butterfly valves before the next runoff season.

Discharge Tunnel.—The discharge tunnel is a 10 ft horseshoe shaped unlined tunnel on a 2% grade. Its capacity as a flow tunnel is 1,050 cfs. It is essential that the discharge tunnel for the aerated H-B valve be a flow tunnel and that it should be impossible to release larger flows. The H-B valve with the design capacity of 800 cfs at 100 ft head will discharge 1,050 cfs at 170 ft head. When or before the reservoir level reaches the 170 ft height, operating instructions require that machined stop-collars be bolted on the two operating arms of the

H-B valve. These collars limit the flow to 800 cfs at 170 ft reservoir level and 1,050 cfs at the full 300 ft reservoir level. The maximum outlet capacity varies between 800 and 1,050 cfs. The granite is massive and believed entirely suitable to withstand the turbulent flow continuously without lining.

River Gage.—Downstream from the dam and spillway it is necessary to measure stream flow as obtained from outlet works release, spill, and dam leakage. At a narrow site of exposed bedrock in the streambed, located 950 ft downstream from the axis of the dam, a concrete weir 9 ft high and a recording float well were installed. A gate was provided for sluicing and a triangular steel edged V-notch weir, 3 ft deep, was provided to measure flows up to 30 cfs

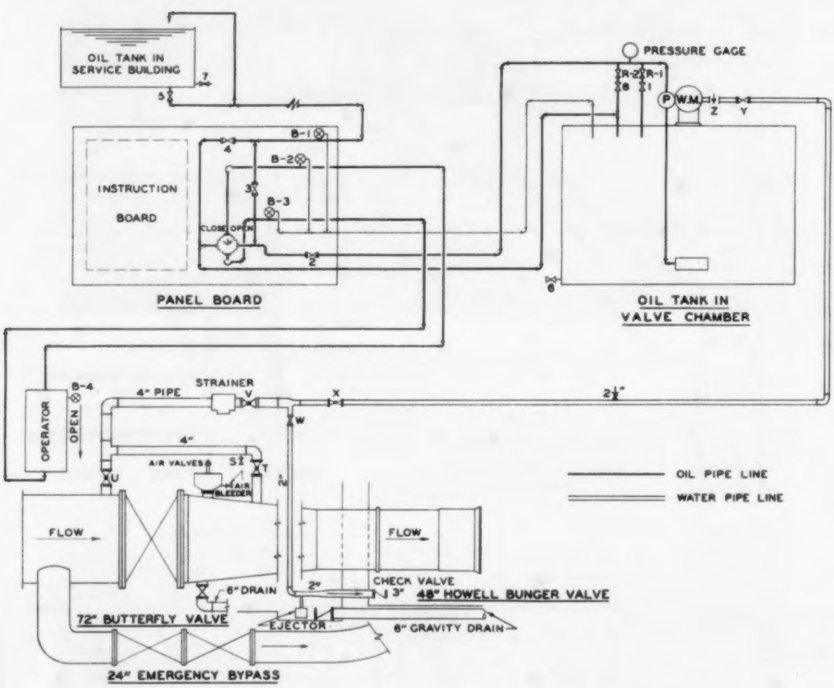

FIG. 6.—SCHEMATIC ARRANGEMENT OF CONTROL SYSTEM - COURTRIGHT OUTLET

accurately. Frequent gaging of this and other V-notch weirs has shown remarkably close agreement with the calibration as computed by the formula,

$$Q = (2.52)\,H^{2.47} \qquad (1)$$

in which H is the head. Further downstream a stream gage cable was installed at a suitable gaging location to provide calibration for higher flows over the full crest length of the weir.

Reservoir Level Indicator.—It is necessary to know the water level of Courtright Reservoir to 0.01 ft each day in order to allocate irrigation water on

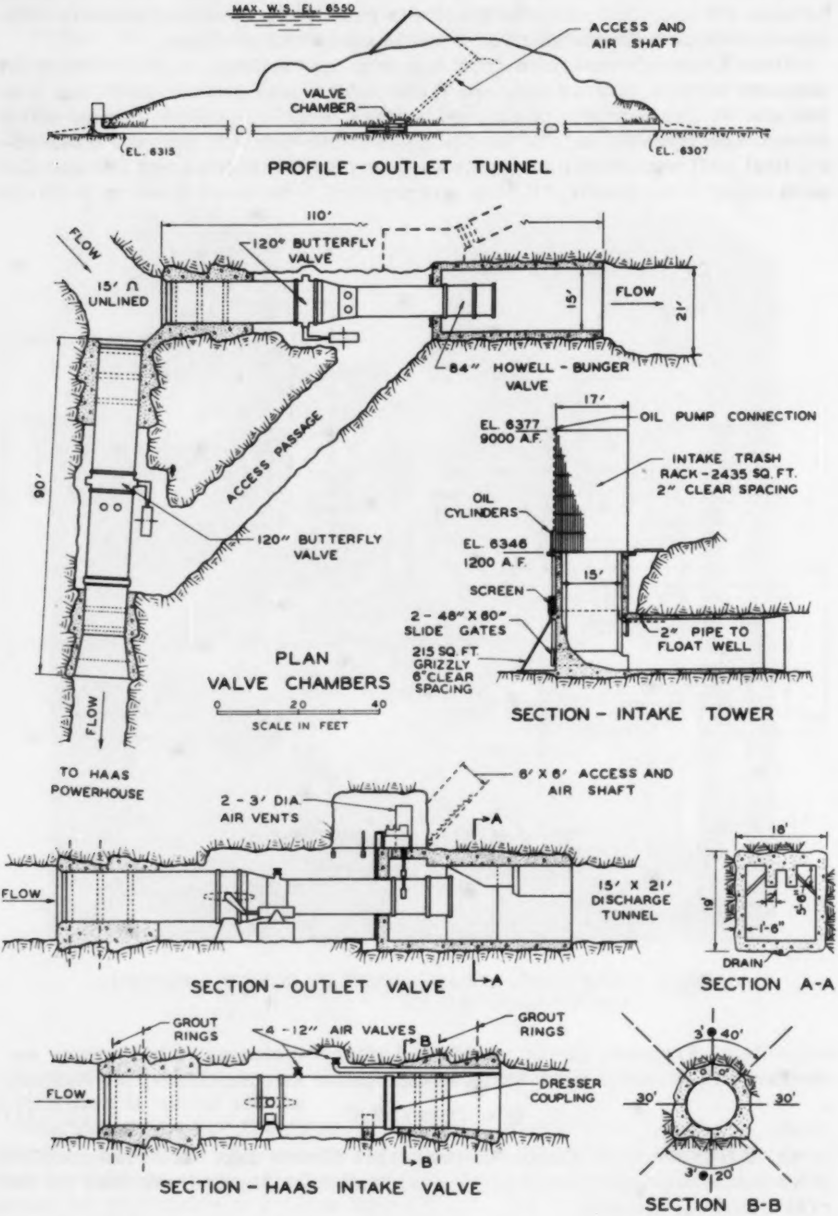

FIG. 7.—WISHON DAM OUTLET WORKS HAAS INTAKE WORKS

FIG. 8.—VIEW OF COMPLETED COURTRIGHT DAM

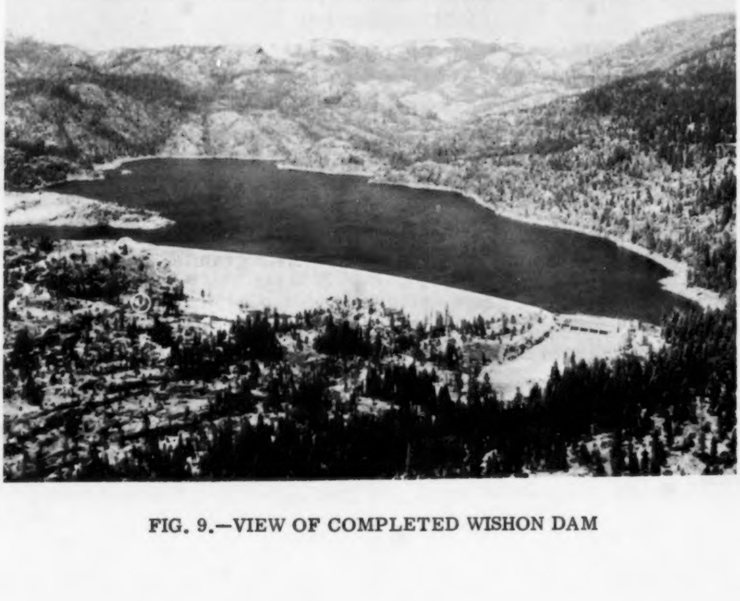

FIG. 9.—VIEW OF COMPLETED WISHON DAM

a basis of daily natural flow. The reservoir site is subject to severe freezing and heavy snow, and is inaccessible for about five months each winter. A 4 in. drill hole 300 ft deep, from service house to tunnel, is used as a float well for a water surface detector. The bottom of the well is connected to a screened opening outside the outlet tower by a 2-in. galvanized pipe in the tunnel. A blowdown valve is provided in the valve chamber to keep the 2-in. line clean. The detector has a 2-3/8 in.-diameter-by-14-in. float. The float is attached to a small-gage stainless steel wire that is controlled by a balanced lever arm mechanism. The wire is let in or out by a battery operated motor, actuated by mercoid switches on the lever arm. This mechanism, requiring neither a large float well nor a counterweight well, makes a low cost installation and has proven to be very reliable during its first year of service. It was planned to grout and re-drill the 4-in. hole to give a well that would not corrode, leak, or ravel. However, the cores were so good that it was considered unnecessary to drill and re-grout.

The detector actuates both a recorder and a telemark radio which transmits the water level to the Haas valve house. Transmission beyond that point is by cable and by power line carrier equipment. To insure reliability, two battery operated 1/2 watt transmitter units are set up to send a reading alternately at noon and midnight of each day.

Communications.—Courtright is an unattended dam and has no power or telephone circuits to it. Since it is desirable to communicate with operating or maintenance personnel when they are there, a battery powered portable radio is provided in the service house for communication with base radio stations.

OUTLET WORKS - WISHON DAM

The Wishon outlet works has been described previously.[6] Fig. 7 shows the layout of this site. The outlet tower, use of unlined tunnel and the outlet works are all similar to Courtright. Wishon outlet is combined with the underground Haas intake, which makes the arrangement particularly economical.

CONCLUSION

The dams and outlet works described herein are now (1960) completed and in service. Figs. 8 and 9 show the completed Courtright and Wishon Dams. On each figure the three surface features of the outlet works are circled: (a) service house, (b) portal of access and air-vent shaft, and (c) discharge portal of outlet tunnel.

The outlet works, as used at Courtright and at Wishon, were especially economical because of the use made of the excellent granite. This type of outlet requires good rock conditions.

ACKNOWLEDGMENTS

Planning and engineering of the Company's hydroelectric projects is under the direction of John F. Bonner, F. ASCE, Vice President in charge of Engineering, and J. D. Worthington, Chief Civil Engineer, Pacific Gas and Electric Company.

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SOME ASPECTS OF SURFACE WATER WAVE SCALE EFFECTS

By Gerritt Abraham[1]

SYNOPSIS

A study of the scale effects due to surface tension for model studies of water gravity waves generated by moving pressure disturbance is presented. The region of the wave system, generated by the moving pressure disturbance in which scale effects occur, has been determined by explaining the physical meaning of the asymptotes occurring in the fish-line problem.[2]

INTRODUCTION

Model studies of the water gravity waves generated by a local disturbance of pressure, advancing with constant velocity over the water surface have been performed at the University of California, Berkeley. One study was of a moving circular low pressure area, and the second study was of a moving annular air jet. The first study was made in order to gain insight on how the water (ocean) surface is affected by the pressure distribution of a hurricane, or gusts of low pressure in a wind blowing over the water surface. The latter investigation has been done to find out if ground effect machines, that is those aircraft that fly so low over the water surface that they gain a large part of their lift by maintaining an air cushion between the plane itself and the water surface, will generate waves that will endanger the stability of such planes. The investigation of the influence of the surface tension on the model results described herein

Note.—Discussion open until June 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 1, January, 1961.

[1] Hydr. Lab., Delft, Netherlands.

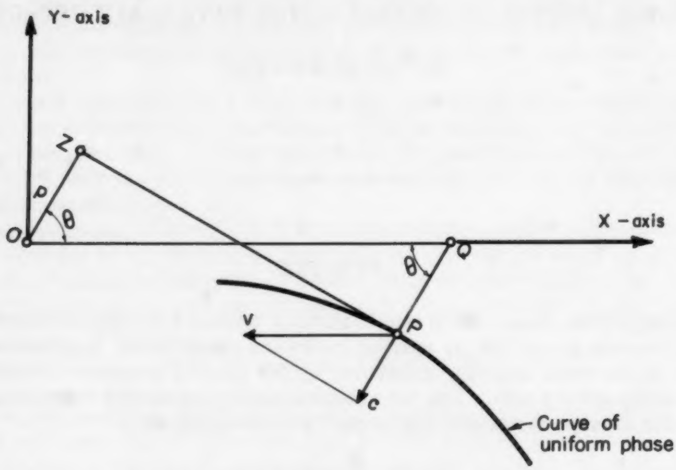
[2] "Hydrodynamics," by H. Lamb, Sixth Ed., Dover Publications, articles 256, 256B, 272, 1945.

was a part of these studies. The results of it could also be of interest for model studies of the wave resistance of ships.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

DETERMINATION OF SCALE EFFECTS

In order to determine the scale effects it is necessary to find the region of the wave pattern that is generated by the pressure disturbance, where neither



O: position of pressure point at time t = O
Q: position of pressure point at time −t

OQ: path of pressure point
PZ: tangent to curve of uniform phase
QP: normal to curve of uniform phase

v: velocity of pressure point (vt = OQ)
c: celerity of wave with wave length λ
λ: dominating wave length at point P

FIG. 1.—NOTATION

the shape of the wave pattern nor the wave height are influenced by surface tension. To do so we have to know what the wave pattern will look like when it is influenced by the surface tension and when it is not. For the present case the required theory has been given by H. Lamb.[2] A summary of his theoretical results follows. For the notation refer to Fig. 1.

Lamb's Theory (1945).—The dominating wave length at any point of a wave pattern has to satisfy:

$$v \cos \theta = c \qquad (1)$$

Eq. 1 means that the curves of uniform phase shift with the same speed as the pressure disturbance in the same direction.

The wave pattern not influenced by surface tension due to a point-disturbance of pressure advancing over infinitely deep water is given by:

$$p = a_1 \cos^2 \theta \qquad (2)$$

in which a_1 is a constant length that increases by $\frac{2\pi v^2}{g}$ in passing from one curve of uniform phase to the next with the same phase, and p is the distance from the position of the pressure point to the tangent in point P to curve of uniform phase. In the rectangular coordinate system the equivalent equations are

$$x = \frac{1}{4} a_1 (5 \cos \theta - \cos 3 \theta) \qquad (3)$$

and

$$y = - \frac{1}{4} a_1 (\sin \theta + \sin 3 \theta) \qquad (4)$$

The wave pattern consists of two wave systems[3] (Fig. 2), transverse waves, for which θ varies between 0° and 35°, and diverging waves, for which θ varies between 35° and 90°.

The wave pattern not influenced by surface tension due to a point-disturbance of pressure advancing over water of finite depth d is given by

$$\frac{p}{a_2} \tanh \frac{a_2}{p} = \frac{v^2}{g\,d} \cos^2 \theta \qquad (5)$$

where a_2 is a constant length that increases by a factor of $2\pi d$ in passing from one curve of uniform phase to the next with the same phase. The wave patterns described by Eq. 5 are to be seen in Fig. 2.

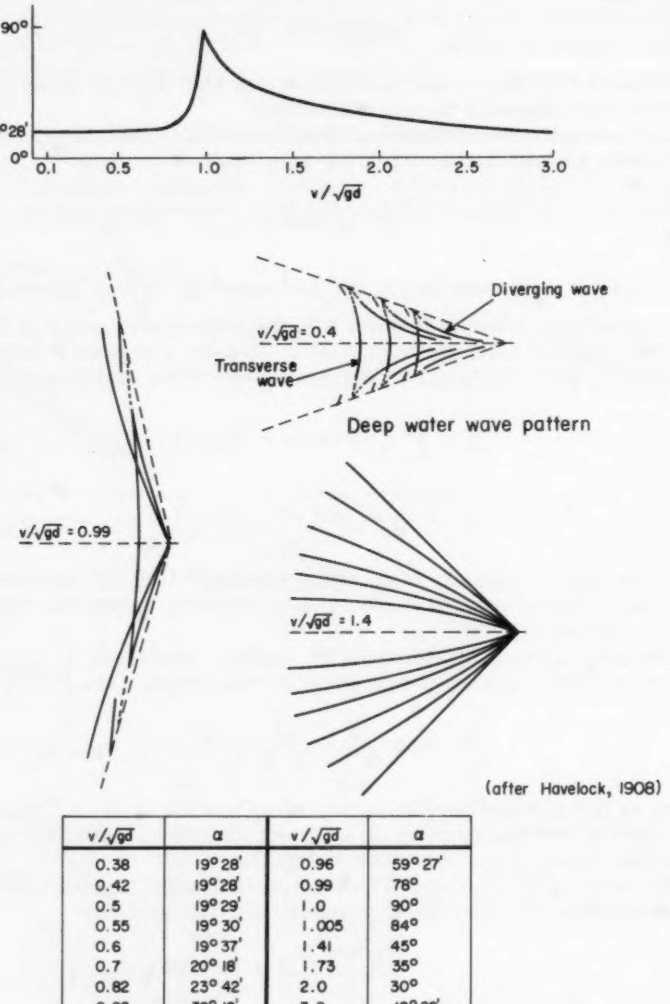
The wave pattern influenced by surface tension due to a point-disturbance of pressure advancing over water of infinite depth is given by

$$\frac{p}{a_3 \cos^2 \alpha_o} + \frac{a_3 \cos^2 \alpha_o}{p} = 2 \frac{\cos^2 \theta}{\cos^2 \alpha_o} \qquad (6a)$$

or

$$\frac{p}{a_3} = \cos^2 \theta \pm \sqrt{\cos^4 \theta - \cos^4 \alpha_o} \qquad (6b)$$

[3] "The Propagation of Groups of Waves in Dispersive Media, with Application to Waves on Water Produced by a Travelling Disturbance," by T. H. Havelock, Proceedings, Royal Soc., London, Series A, Vol. 81, 1908.



$v/\sqrt{gd}$	α	$v/\sqrt{gd}$	α
0.38	19° 28'	0.96	59° 27'
0.42	19° 28'	0.99	78°
0.5	19° 29'	1.0	90°
0.55	19° 30'	1.005	84°
0.6	19° 37'	1.41	45°
0.7	20° 18'	1.73	35°
0.82	23° 42'	2.0	30°
0.92	39° 19'	3.0	19° 28'

v: velocity of disturbance

d: water depth

α: half angle of aperture of wedge within which the wave pattern is contained

FIG. 2.—WAVE PATTERNS CAUSED BY POINT DISTURBANCE OF PRESSURE ADVANCING OVER WATER OF FINITE DEPTH (INFLUENCE OF SURFACE TENSION NEGLECTED)

in which a_3 is a constant length that increases by a factor of $\frac{\pi v^2}{g}$ in passing from one curve of uniform phase to the next with the same phase:

$$\cos \alpha_o = \frac{c_{min}}{v} \qquad (7)$$

and c_{min} is the minimum celerity of capillary waves. The wave patterns described by Eq. 6 are to be seen in Fig. 3.

The four straight lines for which $\alpha = \pm \alpha_o$ are asymptotes of the curves of uniform phase according to Eq. 6. Their equation is

$$\sin \phi = \frac{c_{min}}{v} \qquad (8)$$

in which ϕ is the angle between the axis of the wave pattern, this is the path covered by the disturbance, and the asymptote. The physical meaning of the asymptotes is closely related to the scale effects as will be shown now, first for deep water, and afterwards for shallow water.

Scale Effects for Deep Water.

Modification of the Form of the Wave Pattern due to Surface Tension Remote from the Disturbance.—The first term of Eq. 6 (a) is due to gravity and the second term is due to surface tension. When

$$\frac{p}{a_3 \cos^2 \alpha_o} >> \frac{a_3 \cos^2 \alpha_o}{p} \qquad (9)$$

the influence of surface tension on the form of the wave pattern is negligible and we get, from Eq. 6 (a)

$$\frac{p}{a_3 \cos^2 \alpha_o} = \frac{2 \cos^2 \theta}{\cos^2 \alpha_o} \qquad (10)$$

which is basically the same as Eq. 2.

Let us consider a wave pattern that is not influenced by surface tension and let us describe it by Eq. 10. Next let us consider a wave pattern influenced by surface tension. Its form is given by Eq. 6 (a). From these two equations it follows that the difference in p (for constant θ), Δp, due to surface tension is given by

$$\frac{p + \Delta p}{p} = 1 + \frac{a_3^2 \cos^4 \alpha_o}{p^2} \qquad (11)$$

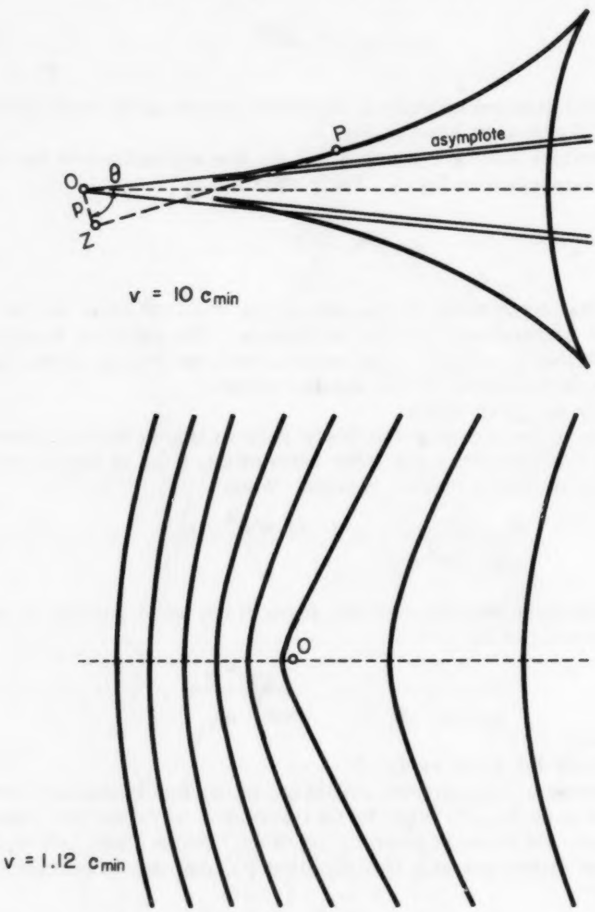
or

$$\frac{\Delta p}{p} = \frac{\cos^4 \alpha_o}{4 \cos^2 \theta} = \frac{c^4_{min}}{4 v^4 \cos^4 \theta} \qquad (12)$$

according to Eqs. 7 and 10.

It is obvious that the influence of the surface tension depends on the ratio v/c_{min} because when the speed of the disturbance is not much greater than the

minimum speed of capillary waves, it is possible that the two wave lengths that satisfy Eq. 1 are of the same order of magnitude. In this case the contribution of surface tension to the resultant wave pattern may not be neglected.



O: position of pressure point
ZP: tangent to curve of uniform phase
v: velocity of pressure point
c_{min}: minimum celerity of capillary waves

FIG. 3.—WAVE PATTERNS CAUSED BY A POINT DISTURBANCE OF PRESSURE ADVANCING OVER WATER OF INFINITE DEPTH (INFLUENCE OF SURFACE TENSION CONSIDERED; AFTER LAMB, 1945)

Eq. 12 enables us to compute the influence of the surface tension on the form of the wave pattern as a function of θ and $\frac{c_{min}}{v}$, in which θ refers to a wave pattern that is not modified by the surface tension.

From Eqs. 3 and 4 it follows that the points in the deep-water wave pattern where θ has a certain value θ_o are situated on the lines given by the equation

$$\psi_o = \text{arc tan} \frac{\sin \theta_o + \sin 3\,\theta_o}{5 \cos \theta_o - \cos 3\,\theta_o} = \text{arc tan} \frac{y_o}{x_o} \quad \ldots\ldots\ldots (13)$$

in which ψ_o is the angle between the lines where $\theta = \theta_o$ and the axis of the wave pattern. This equation holds for wave patterns not influenced by surface tension.

Suppose that in using Eq. 12 $\frac{\Delta p}{p}$ exceeds a certain desired value when $\theta > \theta_o$. When $\theta_o > 35°$ the part of the diverging wave pattern ($35° < \theta < 90°$) that is contained within the wedge with an angle of twice the corresponding value of ψ_o

TABLE 1

	$\Delta \bar{P}/\bar{p}$ = 1%			$\Delta \bar{P}/\bar{p}$ = 2%			$\Delta \bar{P}/\bar{p}$ = 5%			
c_{min}/v	θ_o	y_o/x_o	ψ_o	θ_o	y_o/x_o	ψ_o	θ_o	y_o/x_o	ψ_o	ϕ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	-	-	-	-	-	-	-	-	-	90°
1/2	-	-	-	20°	0.29	16°	41°	0.34	19°	30°
1/3	42°	0.34	20°	51°	0.30	16-1/2°	60°	0.25	14°	20°
1/3.2	46°	0.325	19°	54°	0.29	16°	62°	0.23	13°	19° 28'
1/5	64°	0.22	12°	68°	0.19	11°	72-1/2°	0.15	8-1/2°	11°
1/10	77°	0.11	6°	79°	0.10	6°	82-1/2°	0.07	4°	6°
1/20	83-1/2°	0.06	3°	84-1/2°	0.05	3°	86°	0.03	2°	3°

shows a larger deviation than the desired value, because of surface tension. As will be shown later, when $\theta_o < 35°$, the whole diverging wave pattern and the part of the transverse wave pattern ($o < \theta < 35°$) outside of the wedge with angle of twice the corresponding value of ψ_o shows a deviation larger than the desired value.

Thus Eqs. 12 and 13 make it possible to indicate the region of the wave pattern that is influenced to a certain degree by surface tension.

Some numerical values are shown in Table 1. The values of y_o/x_o occurring in this table have been calculated by means of Fig. 4, a plot of the functions ($\sin \theta + \sin 3\,\theta$) and ($5 \cos \theta - \cos 3\,\theta$). By comparing the values of ψ_o with the corresponding value of ϕ, the angle between the axis and the asymptotes, we see the following: (1) As long as $\phi > 19° 28'$ or $v < 3.2\ c_{min}$ the whole diverging wave system is influenced by the surface tension (2) As long as $\phi < 19° 28'$ or $v > 3.2\ c_{min}$ the portion of the diverging wave system within the asymptotes is influenced by surface tension, while the part outside of these asymptotes is barely influenced by surface tension. As a consequence, we may draw the conclusion that the area in which the form of the deep-water wave pattern is influenced by the surface tension is contained within the asymptotes.

Modification of the Height of the Crests of the Wave Pattern by Surface Tension Remote from the Distrubance.—The pressure normal to the water surface

due to surface tension (σ) is

$$N = \frac{\sigma}{R_1} + \frac{\sigma}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which N is the normal pressure, and R_1 and R_2 are the principal radii of

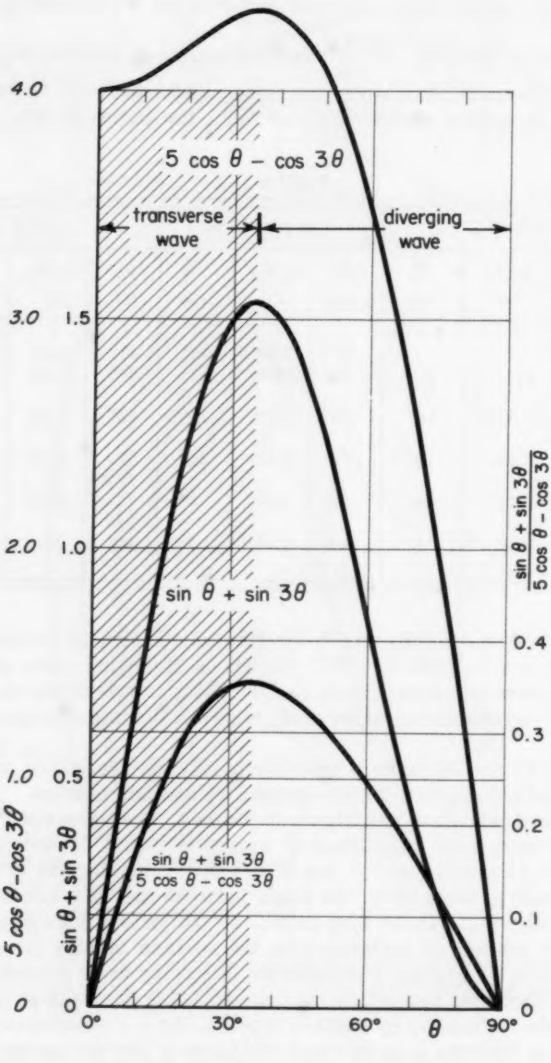


FIG. 4.—5 cos θ - cos 3θ, sin θ + sin 3θ AND $\frac{\sin\theta + \sin 3\theta}{5\cos\theta - \cos 3\theta}$ AS FUNCTIONS OF θ

curvature of the water surface. If we assume that $R_2 = \infty$, Eq. 14 reduces to

$$N = \frac{\sigma}{R} \quad \text{. . . . . (15)}$$

When the radius of curvature of the water surface is small, the normal pressure due to surface tension is large.

The curvature of a water surface in which waves occur is inversely proportional to the wave length; hence, the influence of the surface tension increases when the wave length gets small. In this case the effect of the surface tension is to make the water surface smooth.

Fig. 2 shows that the smallest wave length in the deep-water pattern occurs in the diverging wave system near the axis of the wave pattern. If any influence of the surface tension is noticed, it will be in this region.

Let us now consider a wave with length λ, and height H, in water infinitely deep. The shape of the wave surface is approximately

$$h = H \sin \frac{x}{\lambda}\ 2\pi \quad \text{. . . . . (16)}$$

The radius of curvature of the water surface is given by

$$R = - \frac{\left[1 + \left(\frac{2\pi}{\lambda}\ H \cos \frac{x}{\lambda}\ 2\pi\right)^2\right]^{3/2}}{\frac{4\pi^2}{\lambda^2}\ H \sin \frac{x}{\lambda}\ 2\pi} \quad \text{. . . . . (17)}$$

and

$$\left|\frac{1}{R}\right| \leq \left|\frac{4\pi^2}{\lambda^2}\ H \sin \frac{x}{\lambda}\ 2\pi\right| \quad \text{. . . . . (18)}$$

The average value of $\frac{1}{R}$ with respect to the wave length is given by

$$\left|\frac{1}{R}\right| \leq \left|\frac{4\pi^2}{\lambda^2}\ H\ \frac{1}{\pi}\int_0^{\pi} \sin x\, dx\right| = \frac{8\pi H}{\lambda^2} \quad \text{. . . . . . (19)}$$

The average value of the normal pressure with respect to the wave length, $\overline{N}$, is given by

$$\overline{N} \leq \frac{8\pi H}{\lambda^2}\ \sigma \quad \text{. . . . . (20)}$$

The average pressure in the plane of the undisturbed water surface due to gravity, $\overline{P}$, is given by

$$\overline{P} = \frac{1}{\pi}\int_0^{\pi} h\, \rho\, g\, dx = \frac{1}{\pi}\int_0^{\pi} H\, \rho\, g \sin x\, dx = \frac{2}{\pi}\ H\, \rho\, g \quad \text{. . . . . (21)}$$

An approximation of the ratio of the influence of the surface tension to the influence of gravity is

$$\frac{\overline{N}}{\overline{P}} \leq \frac{4\,\pi^2}{\lambda^2 \rho g}\,\sigma \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

According to Eq. 1

$$c^2 = \frac{g\,\lambda}{2\,\pi} = \frac{v^2}{{c_{min}}^2}\,{c_{min}}^2\,\cos^2\theta \quad \ldots\ldots\ldots\ldots \quad (23)$$

as long as the wave pattern is not modified by the surface tension. Substituting Eq. 23 into Eq. 22 gives

$$\frac{\overline{N}}{\overline{P}} \leq \frac{{c_{min}}^4}{v^4}\;\frac{\sigma g}{{c_{min}}^4\,\rho\,\cos^4\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

Eqs. 24 and 13 enable us to compute the influence of the surface tension on the wave height as a function of the ratio c_{min}/v and the location in the wave

TABLE 2

	$\overline{N}/\overline{p}$ = 1% if			$\overline{N}/\overline{p}$ = 2-1/2% if			$\overline{N}/\overline{p}$ = 5% if			
c_{min}/v	θ_o	y_o/x_o	ψ_o	θ_o	y_o/x_o	ψ_o	θ_o	y_o/x_o	ψ_o	ϕ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	-	-	-	-	-	-	-	-	-	90°
1/2	-	-	-	24°	0.32	18°	41°	0.34	20°	30°
1/3	41°	0.34	20°	53°	0.29	16-1/2°	60°	0.26	14°	20°
1/3.2	46°	0.325	19°	56°	0.28	15-1/2°	62°	0.23	13°	19° 28'
1/5	64°	0.22	12°	69°	0.18	10-1/2°	72°	0.15	8-1/2°	11°
1/10	77°	0.11	6°	80°	0.09	5°	82°	0.07	4°	6°
1/20	84°	0.06	3°	85°	0.04	2°	85°	0.03	2°	3°

pattern, assuming that the form of the wave pattern is not influenced by the surface tension. Some numerical results are given in Table 2. The computations are based on the assumptions: ρ = 1,000 kg per cu m, σ = 0.073 N per m, g = 9.81 m per sec^2, and c_{min} = 0.23 m per sec. By comparing the values for Ψ_o with the corresponding value of ϕ, the angle between the axis and the asymptotes, we see the following:

(1) As long as $\phi > 19° 28'$ or $v < 3.2\ c_{min}$, the whole diverging wave system is influenced by the surface tension;

(2) As long as $\phi < 19° 28'$ or $v > 3.2\ c_{min}$, the part of the diverging wave system within the asymptotes is influenced by surface tension while the part outside of them is barely influenced by surface tension.

As a consequence we may draw the conclusion that the region where the wave height in the deep-water wave pattern is influenced by the surface tension is contained within the asymptotes.

Because we could make a similar conclusion regarding the form of the wave pattern we may conclude that the region with scale effects is contained within the asymptotes. This explains the physical meaning of the asymptotes. When $v < 3.2\ c_{min}$ the whole deep-water wave pattern is contained within the asymptotes and there is no area free from scale effects.

Scale Effects for Shallow Water.—The wave length in shallow water corresponding to a certain wave celerity is larger than the wave length in deep-water corresponding to the same celerity. Therefore, the influence of surface tension in shallow-water wave pattern is, in general, smaller than the influence in the deep-water wave pattern. Hence the deep-water case will be the determinating case for scale effects.

The influence is the same only in that region of the shallow-water wave pattern where the wave pattern is the same as the deep-water wave pattern. We will now determine for which part of the two sets of wave patterns this applies.

The shallow-water wave pattern, without considering surface tension, is given by Eq. 5. This reduces to

$$\frac{p}{a_2} = \frac{v^2}{g\,d} \cos^2 \theta \quad \text{(25)}$$

when

$$\frac{p}{a_2} < \frac{1}{3} \quad \text{(26)}$$

Eq. 25 is the same equation as Eq. 2, which applies to the deep-water wave pattern except for the constants. From Eqs. 25 and 26 we may draw the conclusion that the shallow-water pattern is the same as the deep-water wave pattern in the area where

$$\frac{v^2}{g\,d} \cos^2 \theta < \frac{1}{3} \quad \text{(27)}$$

Eq. 27 enables us to compute the value of θ_{cr}, that is the smallest value of θ for which the water depth does not influence the wave pattern. We then calculate the value ψ_{cr} (which is the angle between the axis of the wave pattern and the line on which the points of the curves of uniform phase are situated for which $\theta = \theta_{cr}$ in the deep-water wave pattern) by means of Fig. 4 and Eq. 13. When $\theta_{cr} \geq 35°$ or $\psi_{cr} \leq 19° 28'$, 2 ψ_{cr} is the angle of aperture of the wedge within which the form of the diverging waves is not influenced by the water depth. As long as $\psi_{cr} > \phi$, the asymptotes will also occur in the shallow-water wave pattern, without being modified by the water depth. In that case the scale effects are the same for the shallow-water wave pattern as for the deep-water wave pattern. Table 3 shows ψ_{cr} as a function of $v/\sqrt{gd}$.

Conclusions.

Deep Water Remote from the Disturbance.—1. The region having scale effects is contained within the asymptotes. Outside of the asymptotes scale effects do not occur. The asymptotes are given by Eq. 8.

2. When $v < 3.2\ c_{min}$, then $\phi > 19° 28'$, and there is no area without scale effects because the whole wave pattern is contained within the asymptotes.

Shallow Water Remote from the Disturbance.—3. There is a certain area in the shallow-water wave pattern where the diverging wave system is the same as in the deep-water wave pattern. This area is contained within a wedge with half angle ψ_{cr}. The relationship between ψ_{cr} and $v/\sqrt{gd}$ is shown in Table 3. When

$$\psi_{cr} > \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

conclusion 1 also holds for the shallow-water wave pattern.

EXPERIMENTAL VERIFICATION

The theoretical data regarding the scale effects have been checked by comparison of measured values with the theoretical values of the angle ϕ between the axis of the wave pattern and the asymptotes as given by Eq. 8. This was done using a series of experiments in the ripple tank at the University of California. A description of the ripple tank is given by C. M. Snyder, et al.[4] The ripple tank includes an apparatus for taking pictures of the wave pattern. From the pictures (Fig. 5) the angle ϕ could be determined. In addition, during experiments on a larger scale, the angle ϕ was measured.

TABLE 3

$v/\sqrt{gd}$ (1)	θ_{cr} (2)	y_{cr}/x_{cr} (3)	ψ_{cr} (4)
0.7	35°	0.355	19° 28'
0.8	44°	0.33	18°
0.9	50°	0.305	17°
1.0	55°	0.28	15-1/2°
1.1	59°	0.255	14-1/2°
1.2	61°	0.24	13-1/2°
1.3	64°	0.22	12-1/2°
1.4	66°	0.20	11-1/2°
1.5	67-1/2°	0.19	11

Fig. 6 shows the measured angle ϕ plotted as a function of the velocity v. Since all the data shown in this figure satisfy Eq. 28, the theoretical predictions are in agreement with Eq. 8. The model results confirm the theory, which has been used to derive the scale effects.

Characteristic wave patterns are presented in Fig. 5. These pictures were taken in the ripple tank. For Fig. 5 (a), $v = 1.3\ c_{min}$. Here only transverse waves exist but the angle between the asymptotes becomes more acute. For Fig. 5 (c), $v = 3.8\ c_{min}$. Here both transverse and diverging waves exist according to the theory. They do not show up however. Only the asymptotes can be seen clearly.

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4 "Laboratory Facilities for Studying Water Gravity Wave Phenomena," by C. M. Snyder, R. L. Wiegel, and K. J. Bermel, Proceedings, Sixth Conf. on Coastal Engrg., Council on Wave Research, Engrg. Foundation, Berkeley, Calif., 1958, pp. 231-251.

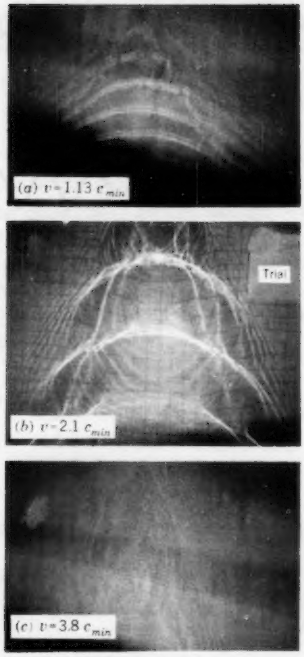

FIG. 5.—WAVE PATTERNS INFLUENCED BY SURFACE TENSION

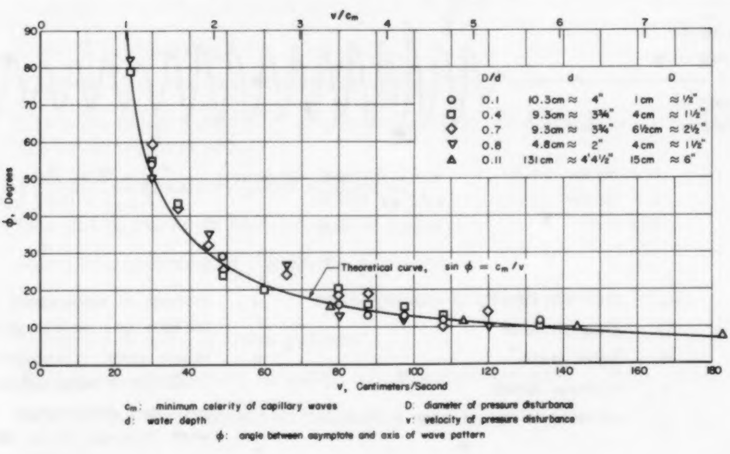

FIG. 6.—ANGLE OF ASYMPTOTE AS A FUNCTION OF VELOCITY OF PRESSURE DISTURBANCE

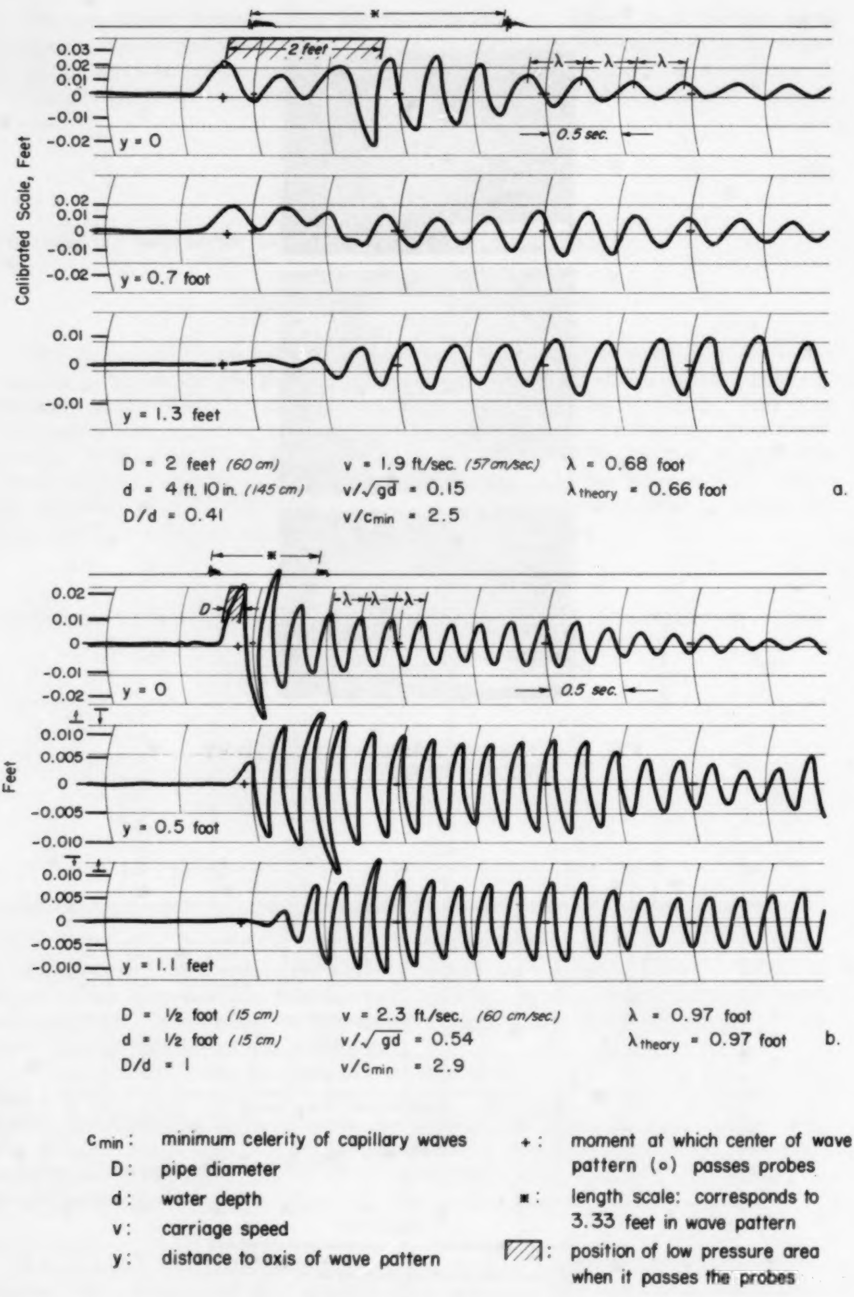

FIG. 7.—DEEP WATER WAVE PATTERN INFLUENCED BY SURFACE TENSION

Figs. 3 and 5 show a similarity between the theoretical and the observed wave patterns. Fig. 7 shows the deep-water wave pattern when $v < 3.2\, c_{min}$. According to the theory only transverse waves occur for this case. Their wave length along the axis of the wave pattern can be calculated by Eq. 1, substituting $\theta = 0°$. The observed wave length along the axis compares well with the theoretical prediction.

ACKNOWLEDGMENTS

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The author wishes to express his appreciation to J. W. Johnson and R. L. Wiegel for facilitating this investigation.

APPENDIX. NOTATION

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c	Celerity of wave with wave length λ
c_{min}	Minimum celerity of capillary waves
D	Diameter of pressure disturbance
d	Water depth
g	Acceleration of gravity
H	Height of wave
N	Normal pressure
O	Position of pressure point at time $t = 0$
OQ	Path of pressure point
PZ	Tangent to curve of uniform phase
p	Distance from position of pressure point to the tangent in point P to the curve of uniform phase
Q	Position of pressure point at time $-t$
QP	Normal to curve of uniform phase
R	Radii of curvature of the water surface
v	Velocity
y	Distance to axis of wave pattern
α	Half angle of aperture of wedge within which the wave pattern is contained
λ	Dominating wave length at point P
ρ	Density

σ Surface tension

ϕ Angle between asymptote and axis of wave pattern

ψ Angle between the lines where $\theta = \theta_0$ and the axis of the surface tension

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PROTOTYPE MEASUREMENTS OF THE COLUMBIA RIVER ESTUARY

By John B. Lockett,[1] F. ASCE, and Harold A. Kidby,[2] F. ASCE

SYNOPSIS

Modern methods of measuring flow characteristics of the extremely complex Columbia River entrance system define the environmental conditions affecting the problem of shoaling in the entrance area and reveal phenomena which appear to have important significance relating to this problem.

INTRODUCTION

The Columbia River estuary (Fig. 1), lying along the boundary between the States of Oregon and Washington, connects the inland empire of the Columbia River basin with the Pacific Ocean and provides an essential water route for the expanding ocean-going commerce of the Pacific Northwest.

For nearly a century, the Corps of Engineers has been engaged in the task of meeting the ever-increasing needs of deep-draft navigation for a safe and dependable channel at the entrance to Columbia River. This task has involved the construction of several types of estuary improvements, each conceived to serve a specific purpose leading to the achievement of the collective goal of providing adequate channel dimensions in the most economically feasible, engineering manner.

Note.—Discussion open until June 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 1, January, 1961.

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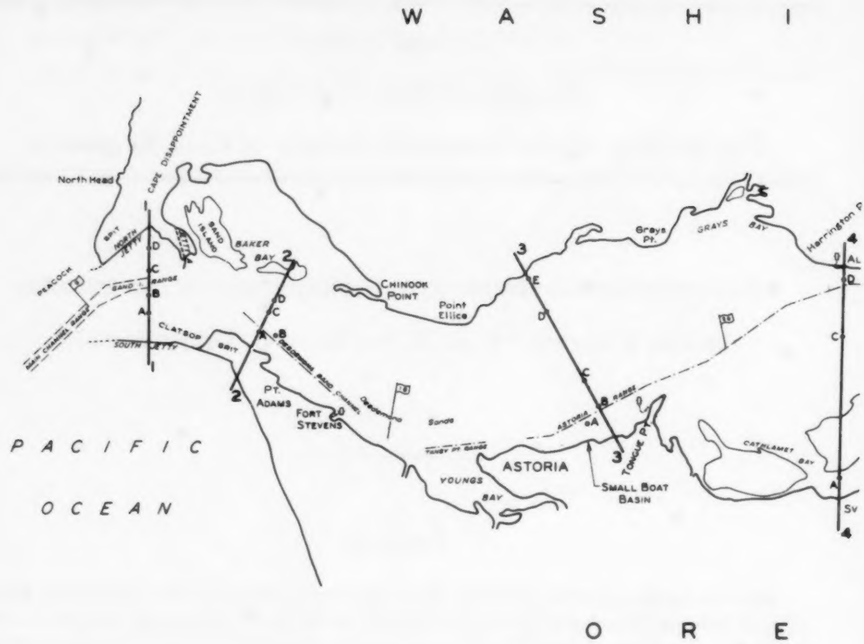
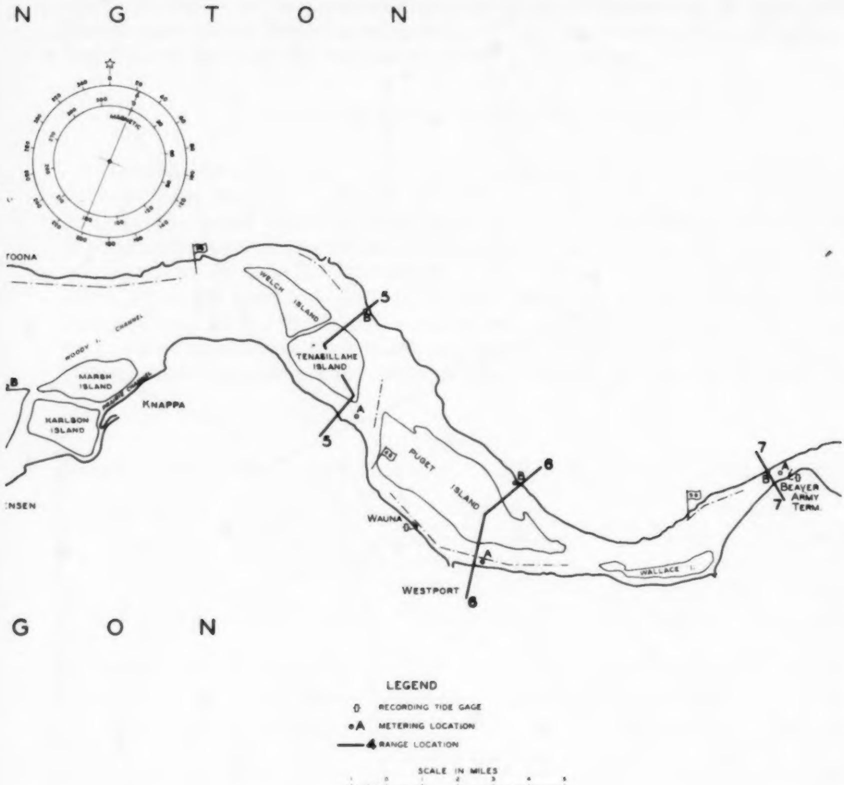


FIG. 1.—COLUMBIA

Prior to the adoption of the initial permanent navigation project for the Columbia River entrance in 1882, early improvements consisted of sporadic dredging in combination with construction of temporary training structures. It was soon realized that such limited work could not be effective in controlling or even influencing to any appreciable degree the forces of nature prevailing in the entrance and that works of more permanent character would be required. Accordingly, construction of a South Jetty, extending northwesterly 4-1/2 mi from its base at Point Adams (Fig. 2), was initiated in 1885 and completed in 1895. Although shoal areas marking the birth of Clatsop Spit began to form along the north side of the jetty during its construction, a general improvement in navigation conditions over the bar was evident during this period which culminated in the achievement of bar depths of 31 ft. Shortly afterward, however, channel conditions deteriorated rapidly until, in 1902, depths of only 22 ft were available. This situation led to the adoption by Congress in 1905 of a modified project designed to secure dependable depths of 40 ft over a width of one-half



RIVER ESTUARY

mile by extension of the South Jetty, construction of a North Jetty, and dredging. By 1914, the year after the South Jetty had been extended to a total length of 6.6 mi, entrance depths had returned to 30 ft and, with completion of the North Jetty in 1917, depths of 40 ft were available through a narrow bar channel. Thenceforth, depths and usable widths throughout the channel further improved and, in 1927, minimum depths of 47 ft were available for at least full project width.

As previously mentioned, shoals began to form along the north side of the South Jetty concurrently with the original construction of that structure. These shoals subsequently encompassed areas south of the jetty and consolidated to form, in 1902, a hook-shaped land mass, called Clatsop Spit, connecting the mid-point of the jetty with Point Adams. In the years that followed shoal areas north of the jetty continued to enlarge, particularly toward the north and west, until by 1931 entrance depths had been reduced to about 43 ft. Rehabilitation of both jetties and construction of Jetty "A" during the period 1931 to 1939 were

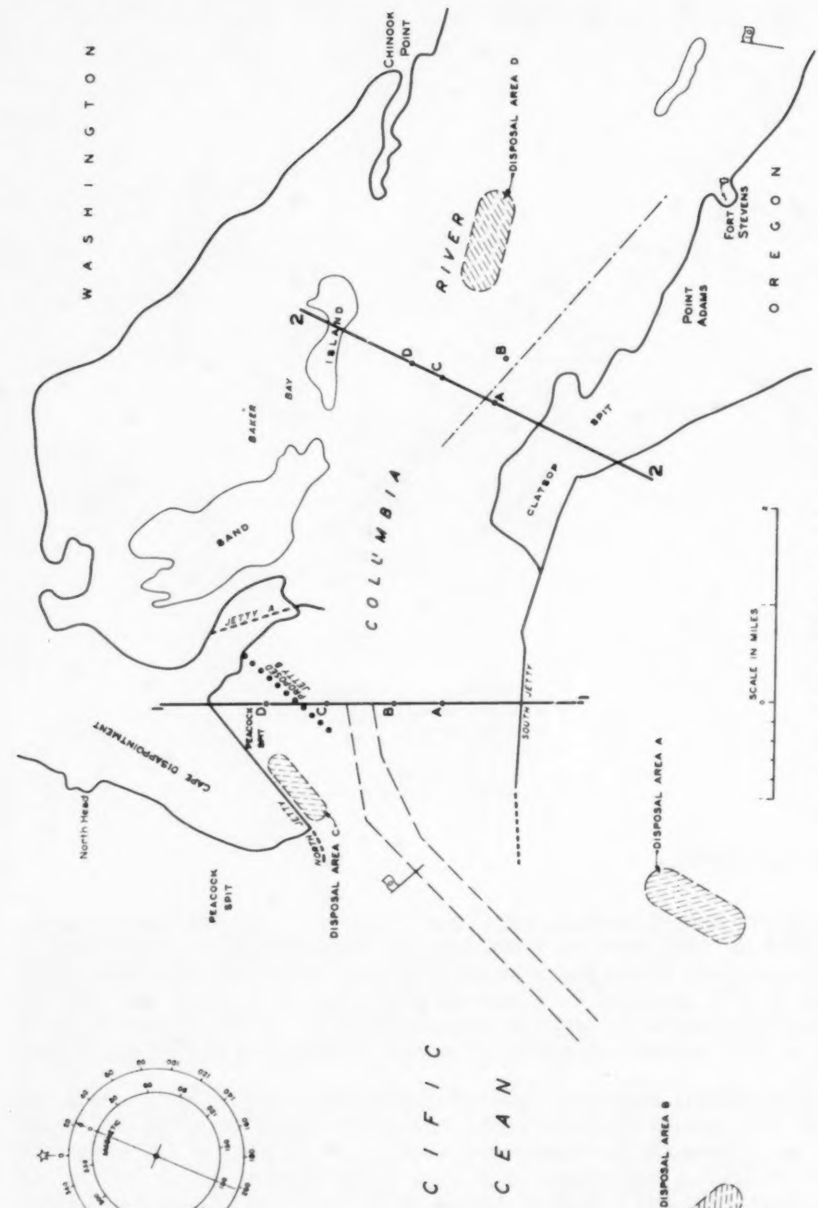


FIG. 2.—LOCATION OF RANGES 1 AND 2

accompanied by further westward growth of these shoals and, by 1952, notwithstanding an active dredging program, an inner bar in extension of Clatsop Spit had formed between the outer ends of the main jetties.

EXISTING 48-FT ENTRANCE PROJECT

To meet the needs of modern ocean navigation, Congress, in 1954, modified the entrance project to provide for minimum depths of 48 ft throughout the half-mile channel width to be secured initially by dredging and subsequently, if experience so warranted, by construction of spur Jetty "B" along the north shore, shown on Fig. 2. Dredges were successful in 1956, the initial year of dredging on the modified project, in securing by the end of the season in October depths of 48 ft over the full channel width, in spite of heavy amounts of erratic shoaling and scouring which accompanied the dredging operations. Extremely heavy shoaling of the newly dredged channel during the following winter

TABLE 1

Calendar year (1)	Quantity dredged, in 1000 cubic yards (2)	Shoaling		Scouring	
		Amount, in 1000 cubic yards (3)	Percentage of Column (2) (4)	Amount, 1000 cubic yards (5)	Percentage of Column (2) (6)
(a) 1956 and 1957					
1956	14,436	8,492	59	6,263	43
1957	3,909	10,255	262	4,535	116
Total	18,345	18,747	102	10,798	59
(b) 1958 and 1959					
1958	2,603	3,602	138	2,893	111
1959	2,289	3,771	164	2,034	89
Total	4,892	7,373	151	4,927	101

months, together with lack of sufficient dredging plant, resulted in the decision to concentrate the 1957 dredging effort to the securing of project depths throughout a channel of 1,500-ft width. Dredging in 1957 was also accompanied by erratic shoaling and scouring over the 1,500-ft channel and was similarly followed by proportionately heavy shoaling during the ensuing winter season. Approximately 75% of the material dredged in 1956 and 1957 was disposed in an offshore area approximately one mile south of the extreme end of the South Jetty (Area A), with the balance disposed in deep water along the entrance range (Area B); adjacent to the inside of the North Jetty (Area C); and upriver in the vicinity of Chinook Point (Area D). Quantities dredged during these years and the amounts of apparent shoaling and scouring noted within the authorized channel by frequent condition surveys are given in Table 1(a).

Beginning in 1958, disposal practices were modified so as to concentrate disposal of dredged materials in deeper water offshore (Area B) and to eliminate disposal of these materials south of the South Jetty and near the North Jetty. Although in 1958 and 1959 shoaling and scouring generally followed the

previously noted pattern, the magnitude of such occurrences was reduced considerably from that experienced in previous years, as shown in Table 1(b).

While dredges have been able by the end of each dredging season to secure project depths within authorized or reduced entrance channel widths, shoaling developing during the winter non-dredging months has reduced available depths to the point where each spring it is necessary to initiate maintenance dredging operations in order to secure a usable entrance channel even though of reduced width during the remainder of each year. Study of available dredging records shows that, as a rule, the entrance channel has evidenced a trend to shoal during the month of June and throughout the months of September through February. Scouring in the channel appears to predominate during the months of March, April, July, and August, but no predominant trend to either shoal or scour has been noted during the month of May. These trends, although not yet fully understood, have been evident from the beginning of dredging on the 48-ft project and, apparently, have not been influenced by the change in disposal practices initiated in 1958. The magnitude of shoaling and scouring has, as previously mentioned, been altered somewhat since that date.

PROTOTYPE MEASUREMENT PROGRAM

The Columbia River entrance, due to the severity of storms and accompanying high seas, is one of the most difficult and hazardous river entrances in the world to successfully navigate. Storm waves as high as 40 ft have occasionally prevailed in the entrance area for brief periods during the winter months, and even moderate storms make it necessary for incoming vessels to await offshore and outbound vessels to delay departure for days at a time. A more significant hazard to navigation, however, is created throughout the year by the treacherous Clatsop Spit shoals which, extending northward and westward of the South Jetty, encroach upon the entrance channel at the most critical point where its alignment changes, just inside the outer ends of the two main jetties. These shoals and the Peacock Spit shoals lying along the Washington shore are reportedly responsible for some 2,000 shipwrecks that have occurred at or near the Columbia River entrance in the past. Although the hazards presented by the Peacock Spit shoals have steadily diminished through the years with the gradual movement of the navigation channel to the north, the Clatsop Spit shoals continue to threaten and endanger the entrance channel and show no signs of diminishing in influence. In fact, these latter shoals encroach upon the entrance channel year after year even though they are temporarily reduced by dredging each summer.

In view of difficulties experienced in the maintenance of the previous 40-ft entrance project against the continual encroachment of Clatsop Spit shoals on the adopted channel alignment, the problem at the Columbia River entrance was reapproached in 1956 in the light of expanding knowledge of tidal hydraulic phenomena concurrently with prosecution of the newly authorized 48-ft entrance project.[3] In making this new approach, the advice and recommendations of the Corps of Engineers Committee on Tidal Hydraulics were sought as a guide for future study. After careful consideration of all known aspects of the problem, including the history of past improvements and findings of past studies, the Committee recommended that a general investigation of the entrance area be

3 "Interim Consideration of the Columbia River Entrance," by John B. Lockett, Proceedings, ASCE, Vol. 85, No. HY1, January, 1959.

launched. As the initial step of this investigation, a program of prototype measurements of current velocity and direction, as well as salinity, would be undertaken to broadly define the density current regime established by environmental river, ocean, and hydrographic conditions within the estuary. With the effects of density currents thus defined, it was the view of the Committee that any additional structural measures, designed to reduce shoaling in the entrance, could be properly evaluated in a hydraulic model study.

In order to achieve the above goal, it would be necessary for the program to provide data sufficient to meet the design and verification needs of a model study. To this end, the proposed program would consist of three cycles of measurements to obtain data for conditions of low, normal, and high river discharge, each cycle to include observations of current direction and velocity for a continuous full tidal period of about 25 hr at 19 stations located on six ranges across the lower 52 mi of the estuary. At each station, current velocity and direction measurements would be taken at 30-min intervals at the surface and near the bottom and at the intervening quarter points of depth. Simultaneous observations would be made initially at one station on each range for a continuous period of about 25 hr to establish the relationship between ranges which would be followed by simultaneous observations at all stations on each range (generally one range per day) until all observations have been obtained. Salinities would be observed at hourly intervals concurrently with the observations of current velocities and directions. It was also suggested that an attempt be made to obtain a few measurements of current velocities, current directions, and salinities on a range between the jetties. It was recognized that it would, no doubt, be necessary to modify details of the proposed program relating to the location of the measurement ranges to fit local field conditions.

The program of prototype measurements as undertaken in 1959 closely followed the recommended program with the exception that measurements were taken at 23 stations along seven ranges as shown on Fig. 1, and included observations of current velocity and direction, salinity and temperature at different levels throughout the estuary, all observations being taken at 30-min intervals. This program, due to its magnitude, the complexities involved in instrumentation, administration and operation, not to mention the trying conditions of weather and heavy seas, represented a monumental task. However, in spite of these difficulties, reliable measurements were secured which will serve to provide the basis for factual analysis of the forces controlling the regimen of the estuary area.

PROGRAM INSTRUMENTATION

To satisfy the major instrumentation requirements of the program, consideration was given to the use of several types of directional current velocity meters. Each had characteristics limiting the scope of the measurement program. A versatile, sturdy current meter was desired that could measure velocities from a small fraction of 1 fps to over 12 fps. The Price current meter met these velocity requirements but was not being used with an acceptable direction indicator.

The direction-velocity problem was discussed with members of the Waterways Experiment Station at Vicksburg, Mississippi, and the United States Geological Survey Research Laboratory at Columbus, Ohio. The Waterways Experiment Station designed a cylindrical container about 8-in. diam and 7-3/4 in. high that was fastened to a 1/2-in. diam, rubber coated, 4-conductor line

used for suspension of a magnesyn compass transmitter and a Price meter. The magnesyn compass was mounted inside the cylinder and the Price meter was suspended on a hanger bar below the cylinder. Since the cylinder and bar moved as one unit, any horizontal change of direction of the Price meter was indicated on a compass dial mounted on a control box located above water. Direction readings to the nearest five degrees could be obtained by this method.

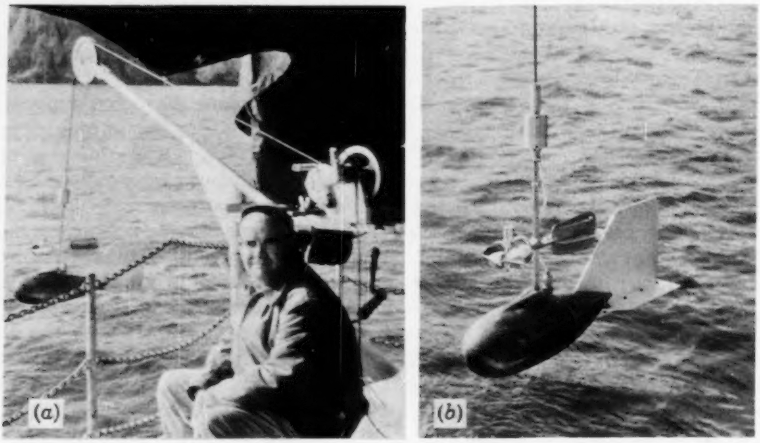


FIG. 3.—VELOCITY - AZIMUTH-DEPTH ASSEMBLY

FIG. 4.—SALINITY - TEMPERATURE ASSEMBLY

Similar equipment was developed by the United States Geological Survey, but mounted in a different manner (Fig. 3). That agency had been testing one instrument consisting of a transducer for a Raytheon Fathometer mounted in the bottom of a 140-lb brass weight suspended on the bottom of a Price meter

hanger bar in place of the lead weight normally used. It was decided to mount the magnesyn compass transmitter in the same weight as the transducer, thereby giving current velocity, direction, and distance between the bottom of the weight and the riverbed. A seven-conductor, pancake swivel was mounted between the current meter hanger bar, and the seven-conductor, 1/4-in. diam, suspension cable.

Both of these velocity-direction assemblies were tested around dredge disposal areas in the Columbia River estuary and out in the adjacent ocean areas during May and June, 1958. It was soon determined that the drag on the 1/2-in. suspension cable and cylinder of the WES instrument caused by high velocities was too great. With current velocities of 6 to 8 fps and with up to 50 ft of cable in the water, the drag was so large that the angle of the cable from vertical, where it left the supporting crane, was as much as 58°. This trouble appeared to stem from the use of a 100-lb weight instead of one much heavier, and the use of 1/2-in. conductor cable instead of one with smaller diameter. Furthermore the mounting of the magnesyn compass transmitter in a non-streamlined vertical cylinder, instead of in a streamlined "fish," which would also serve as the weight, exposed additional surface to the high velocity current. Subsequent models, with the compass transmitter mounted in the weight and the meter mounted ahead of the weight, have much better operating characteristics.

No major objections were found with the USGS velocity-azimuth-depth assembly (VADA) as initially designed.[4] Some difficulty was encountered with waterproofing the swivel and connector to the weight. Salt water entered those parts causing weak signals after several hours of operation. Corrosion of copper spring contacts in the swivel necessitated their replacement with phosphor-bronze contacts when the original contacts broke. On the basis of these comparative tests, six sets of the USGS instrument, complete with reels, were purchased for the measurement program. Minor revisions were made to the swivel in an attempt to seal out the water. Also, the fins of the weight were anodized to prevent a thick film from forming.

Consideration was given to taking water samples at each station at the five elevations and determining the salinity by titration later in the laboratory. Five readings were to be taken on each of five boats every 1/2 hr for each of seven 25-hr periods for the three different stages of upland discharge. If all of the readings were taken, there would have been 26,775 samples to be collected, transported, stored, and eventually titrated. In the interest of reducing the workload, it was felt that a less exact method of measuring salinity would suffice for the needs of the data collection and analysis program. A search for information concerning various conductivity cells and remote-reading resistance indicators revealed that there were several instruments that could be used. Specifications were prepared for purchase of needed conductivity cells and resistance indicators. Salinity was expected to vary from fresh river water to the salt water of the ocean, thus a scale indicating salinity in parts per thousand from 0 to 40 was specified.

As salinity varies with the temperature of the water samples, it was necessary to obtain concurrent measurements of water temperature. Temperature cells and remote reading wheatstone-balanced bridges were used for that purpose.

The conductivity and temperature cells were mounted on a bracket, shown in Fig. 4, above a 140-lb brass weight identical in shape to the VADA weight

[4] "New Instruments Developed by the U.S.G.S. for the Measurement of Tidal Flow," by Edgar C. Barron, Presented at the July 1959 ASCE Hydr. Conf. in Fort Collins, Colo.

but solid inside. Short leads from the cells were connected to leads from the suspension cable inside a short stainless steel hanger that was clamped to the lower end of the suspension cable. The soldered connections were then embedded in a hard plastic filler that waterproofed the connection. Seven-conductor suspension cable identical to that provided for VADA was used. A standard USGS reel was modified to receive the 1/4-in. diam cable and lay it one thickness on the drum, the same as for the VADA unit. A smaller diameter secondary reel or spool was mounted on the right end of the salinity-temperature reel and wound with 70 ft of 4-conductor wire so that the wire on the spool would unwind while the 1/4-in. cable on the reel was winding up. Both the salinity and temperature bridges were connected to the 70 ft of wire on the spool.

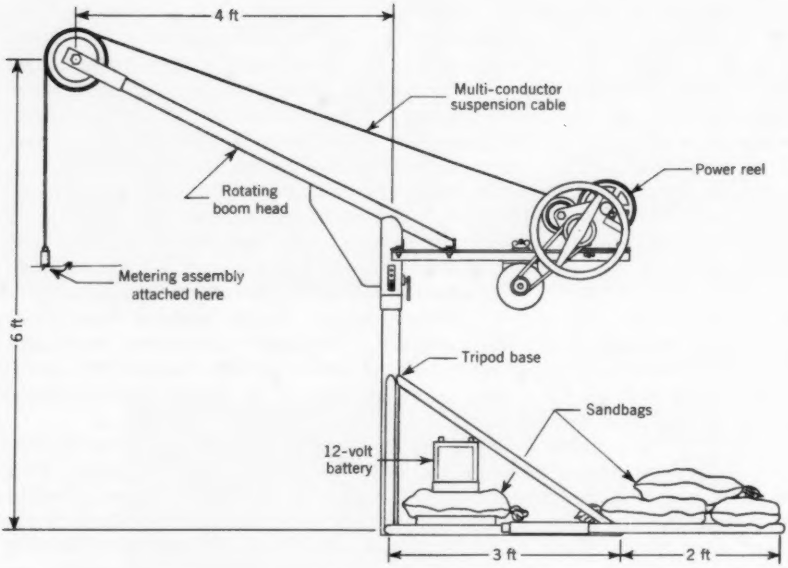


FIG. 5.—PILLAR CRANE

Such an arrangement was found necessary in order to have solid connections between the cells and bridges. None of the manufacturers of the cells and bridges would guarantee results if a spring-loaded slip connection were used as in the VADA units to connect the bridges to the cells.

As the metering and salinity-temperature assemblies were to be used over the side of small boats, a crane was needed that could raise the bottom of the weights 30 in. above the deck and far enough away from the side of the vessel so that the danger of striking the instruments against the boat hull would be a minimum. Pipe was used for the main structural members of the pillar crane shown in Fig. 5. A rectangular base supported the upright pipe that held the rotating boom head. Either reel could be bolted to the metal base and operated by a 12-v electric motor mounted underneath the reel base. An 8-in.-diam

sheave was mounted on the upper end of the boom head so that the supporting cable for the metering assembly would be 4 ft 4 in. forward from the axis of the vertical support for the boom. Either U-bolts fastened directly to the tripod base or filled sandbags on extensions to the base were used to counterweight the pillar crane to prevent it from overturning.

Other equipment required to make the measurements included six 12-v batteries for each boat and one 30-amp gasoline-engine powered battery charger. A cable angle-indicator was provided to measure the angle of the cable from vertical when the instruments were used in high velocities and the angle was appreciable. Flashlights, as well as electric head lamps and spare batteries, were provided for each boat. Spare parts for the Price meters were also available. One complete set of VADA and salinity-temperature assemblies with electric motors and pillar cranes was kept on the control boat for emergency use.

Automatic tide gage recorders were maintained at Fort Stevens boat basin, Tongue Point Naval Base, Wauna, Beaver Army Terminal and Columbia River Packers Association Dock at Altoona in order to keep a record of tide stage and variation during the data measureing periods.

PROGRAM ADMINISTRATION

Administration tasks associated with the program included the procurement of necessary equipment, rental of vessels, training of equipment operators, and coordination of program details with fishery and navigation interests. Assemblies and pillar cranes previously described were obtained by contract with local manufacturers. Small items, including repair parts and flashing lights for buoys, as well as battery chargers, were obtained by direct purchase. Storage batteries were rented for each of the three measurement periods. All equipment was assembled at the Government Moorings in Portland prior to movement by truck to the small-boat basin at Astoria which served as field headquarters for the program. Specifications were prepared[5] to meet the particular requirements of the control boat, the master metering boat and four metering boats and these vessels were hired through normal contractual procedures on an hourly rental basis. Several bids were rejected for lack of deck space or adequate cabin space for setting up recording and auxiliary equipment.

Professional engineers and engineer technicians from the Engineering Division of Portland District were selected as equipment operators and were indoctrinated by means of a special training program. Each operator was furnished a 20-page brochure describing the purpose of the prototype measurement program and related administrative and technical details, prior to a general orientation session to insure a thorough understanding by all of their specific duties. This was followed by a 4-hr session in which each operator was taught how to assemble, operate, and maintain each item of measuring equipment. It later became obvious that at least 10 hr to 12 hr should have been devoted to that purpose.

Discussions were held with the United States Coast Guard to determine the type of buoy required to mark the location in the estuary of each measurement station, the availability of such buoys, and needed appurtenant equipment. As the result of these discussions it was necessary to purchase by contract additional anchor chain to meet the requirements of the program. At the request

5 Specifications Accompanying Invitation No. CIVENG-35-026-59-142, U. S. Army Engr. Dist., Portland, Ore., February 13, 1959.

of the Columbia River Fisherman's Protective Association, arrangements were made to forego the use of buoys at Stations C and D on Range 2 to eliminate interference with fishermen's nets on established drifts and to remove and replace buoys at Stations D and E on Range 3 during the August fish runs. Some minor shifting of station locations in the back channels was made at the suggestion of tug and barge operators to avoid accidental removal of buoys by log tows. The Columbia River Bar Pilots Association and Columbia River Pilots

TABLE 2.—ANCHOR DATA

Range and station	Approximate depth,[a] in feet	Date of survey	Type of bouy (unlighted)	Anchor weight, in tons	Chain	
					Size, in inches	Shots required[b]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1-A	19	Aug 1957	First class can	9	1 1/2	3
1-B	48	Aug 1957	First class can	9	1 1/2	3
1-C	36	Aug 1957	First class can	9	1 1/2	3
1-D	26	Aug 1957	First class can	9	1 1/2	3
2-A	35	Aug 1957	First class can	6	1 1/2	1
2-B	39	Jun 1958	First class can	9	1 1/2	1
2-C	54	Aug 1957	No buoy			
2-D	50	Aug 1957	No buoy			
3-A	40	Jun 1958	First class can	3.2	1 1/2	1
3-B	33	Aug 1957	No buoy			
3-C	20	Aug 1957	First class can	2.1	1 1/2	1
3-D	37	Aug 1957	First class can	2.1	1 1/2	1
3-E	58	Aug 1957	First class can	2.1	1 1/2	1
4-A	50	Sep 1957	First class can	2.1	1 1/4	1
4-B	22	Sep 1957	First calss can	2.1	1 1/4	1
4-C	20	Sep 1957	First class can	2.1	1 1/4	1
4-D	30	Sep 1957	First class can	3.2	1 1/4	1
5-A	27	May 1950	First class can	2.1	1 1/4	1
5-B	55	Sep 1957	First class can	2.1	1 1/4	1
6-A	30	Dec 1958	First class can	2.1	1 1/4	1
6-B	46	Dec 1957	First class can	2.1	1 1/4	1
7-A	45	Nov 1956	First class can	2.1	1 1/4	1
7-B	50	Dec 1957	First class can	2.1	1 1/4	1

[a] All depths referenced from mean lower low water.
[b] One shot of chain equals 15 fathoms, or 90 ft.

Association were notified of proposed activities and the Coast Guard was furnished the latitude and longitude of each buoy for publishing in its notice to mariners well in advance of the placement of buoys.

PROGRAM OPERATION

Field operations, with the attendant requirement of extreme accuracy and timing of observations, occasionally in the face of adverse weather and wave conditions, represented the most difficult phase of the measurement program. Initial operations involved the placement of buoys by the Coast Guard early in

April, 1959, to mark the locations of measurement stations. First class can buoys were selected for this purpose and were painted international orange with a broad white band within which the station and range number was conspicuously painted in orange. Amber-flashing, waterproof, dry-cell-powered lights were bolted to a 90° ell welded to the top of each buoy and concrete sinkers ranging up to 9 tons in weight were provided to anchor the buoys in position. Approximate depth, size of anchor, and length and type of anchor chain used for each buoy are indicated in Table 2.

Four equipment operators were assigned to each of the six vessels providing two 2-man operator crews for alternate 12-hr shifts throughout each 8-day measurement cycle. One operator was designated as party chief for each boat and was made responsible for administrative control and proper functioning of the team.

TABLE 3.—DAILY DISTRIBUTION OF METERING BOATS

25-hr metering period	CATANA		MORNING STAR		ROSIE		MY BOAT		MARY K	
	Station	Distance, in nautical miles	Station	Distance, in nautical miles	Station	Distance, in nautical miles	Station	Distance, in nautical miles	Station	Distance, in nautical miles
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Boat basin		Boat basin		Boat basin		Boat basin		Boat basin	
1	3-A	1.0	3-C	1.7	3-B	1.2	3-D	5.8	3-E	6.0
2	3-A	0	1-A	12.9	1-B	12.6	1-C	9.9	1-D	10.0
3	3-A	0	2-A	3.3	2-B	3.0	2-C	3.1	2-D	3.4
4	3-A	0	4-A	16.1	4-B	15.8	4-C	16.1	4-D	16.6
5	3-A	0	5-B	12.6	5-A	9.0	6-A	18.9	6-B	15.9
6	3-A	0	5-B	0	6-A	6.7	7-A	7.7	7-B	7.1
7	3-A	0	4-D	10.1	2-C	24.6	5-B	15.1	7-B	0
	Boat basin	1.0	Boat basin	7.9	Boat basin	9.3	Boat basin	17.1	Boat basin	32.7
Total miles		2.0		64.6		82.2		94.3		91.7

The control boat, the SUJAN, in addition to representing an immediate source of supply of spare equipment and the means for supervision and control of operations, was also used to ferry operator crews to and from shore bases established at Fort Stevens, Astoria, Svenson, Knappa, and Westport for the purpose of reducing the time required for changing shifts.

Two operators, having experience the previous year in operating the VADA unit, arranged and set up the measuring equipment in each metering boat. In general, one VADA unit and one salinity-temperature unit were mounted on pillar cranes on opposite sides of each vessel in such a manner to insure that the equipment would not strike the sides when the vessel was tossed about by waves. Control of the amount of cable in the water was effected by means of a productimeter counter attached to the shaft of each reel. At the higher river velocities, the equipment did not hang vertically, causing the reel counter to indicate erroneous depths. When this occurred, the cable-angle indicator was used to measure the angle of the cable from the vertical and an appropriate depth correction value was applied. All vessels were equipped for radio communication and special crystals with frequency of 2,326 kc were provided.

Station A, Range 3, the master metering station, was occupied by the metering boat, CATANA, continuously throughout each 8-day measurement cycle. The four other metering boats, the MORNING STAR, ROSIE, MARY K, and MY BOAT, were phased among the remaining 22 stations to obtain combinations of 25-hr measurements at each station as shown in Table 3.

All boats, personnel, and equipment were assembled at the Astoria small-boat basin at 8:00 a.m. on the first day of the first measurement cycle. Equipment was placed aboard, assembled, and tested before each boat was dispatched to its initial station. The control boat contacted each boat by radio as soon as it was underway, and later the exact starting hour of all measurements was determined. Thereafter radio communications were limited to emergency and essential messages, with adequate coverage being provided by a 5-min listening period beginning at 15 min before and after each hour. For the first measurement cycle all boats occupied stations on Range 3 to reduce the movement of the control vessel, thereby facilitating the checking of operations, repairing of equipment, and assisting the operator-crews to become proficient with the measurement routine. Considering the difficulties that arose from lack of experience during this initial phase, it is believed that the program would have been severely hampered if the metering boats had been spread over two or more ranges at that time. At the completion of the initial 25-hr period, equipment was secured, and the boats moved to the next assigned station in accordance with the schedule shown in Table 3.

Data obtained consisted of observations every 1/2 hr of current velocity and direction, salinity and temperature at five different levels at each station. These levels were located 3 ft below the water surface, 2 ft above the bottom, and at the intervening 1/4, 1/2, and 3/4 depth levels. Observations were recorded on a specially prepared form (Fig. 6).

The initial cycle of measurements was undertaken from May 5 through May 13, 1959, a period of normal river flow, when the discharge ranged from 365,000 to 404,000 cfs in the lower estuary. The tidal pattern at Tongue Point during that period and subsequent measurement cycles is shown in Fig. 7. Successful times of observation at each station during the initial cycle are indicated by the bar graph of Fig. 8. Records were kept of missed observations which were subsequently analyzed to determine whether caused by equipment failure or other reasons with the result that minor modifications were made to the equipment where needed to improve its operation.

The second cycle of measurements was accomplished between June 16 and June 24, 1959, a period of high river flow, when the discharged ranged between 532,000 and 577,000 cfs. The bar graph shown in Fig. 9 indicates considerable improvement in the number of successful observations obtained.

The final cycle of measurements was made between September 15 and September 23, 1959, a period of low river flow, when the discharge ranged between 153,000 and 214,000 cfs. Fig. 10 shows that the number of successful operations during this cycle was almost as great as during the preceding cycle.

ANALYSIS OF DATA

Observations taken in the Columbia River Prototype Measurement Program during 1959, which aggregate approximately 25,920 measurements, are now being analyzed by Portland District. The analysis study of that office is oriented toward determining from current observations the flow predominance

at the different stations and depth levels indicated during each of the three conditions of upland discharge. These determinations, confirmed by concurrent salinity and temperature observations, will broadly define the pattern of currents within the estuary and thus provide a basis for evaluation of the effects

CURRENT VELOCITY, DIRECTION, DEPTH, SALINITY, AND TEMPERATURE DATA

RANGE: 3 STATION: A DATE: 1 MAY 1959 SURVEYBOAT: CATANA

OBSERVERS: TERVOOREN CURRENT METER NO: 4

CARLSON CURRENT DIRECTION ASSEMBLY NO: 5055

SALINITY CELL NO: 604574

WIND: NONE TEMPERATURE CELL NO: 604580

WEATHER CONDITION: CLEAR, COOL

REMARKS: EBB

HOURLY OBSERVATIONS

Time start traverse: 0955 Sounding:[1] 36 ft.

End traverse: 1010 Depth: 37 ft.

Sounding (ft.)	Depth (ft.)	Revolutions		Time (sec.)	Velocity (fps)	Current direction[2]	Measured salinity (PPT)	Temperature		Corrected salinity (PPT)
		Single	Penta					Water °F	Air °F	
33	3	50		40	2.78	265	3.1	58.2	72	
26.8	9.2	50		40	2.78	265	5	59.1	72	
17.5	18.5	40		50	1.79	250	4	59.2	72	
8.3	27.7	10		40	0.58	260	3	58.4	72	
1.0	35	10		50	0.47	260	3	57.7	72	

HALF-HOURLY OBSERVATIONS

Time start traverse: 1025 Sounding:[1] 33 ft.

End traverse: 1041 Depth: 34 ft.

VADA ∠	Sounding (ft.)	Depth (ft.)	Revolutions		Time (sec.)	Velocity (fps)	Current direction[2]	Measured salinity (PPT)	Temperature		Corrected salinity (PPT)	SAL. TEM ∠
			Single	Penta					Water °F	Air °F		
18	30	3.0	200	40	40	11.12	265	2	59.3	75		20
22	24.5	8.5	200	40	40	11.12	260	3	59.0	75		23
24	16	17.0	150	30	40	8.34	260	3	58.6	75		25
21	7.5	25.5	100	20	50	4.45	260	3	58.1	75		24
16	1	32.0	50	10	50	2.23	265	3	57.8	74		18

1. Sounding taken with transducer 1 ft. below meter with axis of meter at water surface.
2. Magnetic azimuth - degree

FIG. 6.—SAMPLE RECORD FORM

of the currents on shoaling at the estuary entrance. In addition, the District Engineer is undertaking studies of past dredging of the entrance channel and the lower estuary area, changes in areas of scouring and shoaling, amounts of scouring and shoaling occurring within the channel below the 50-ft depth level,

and new areas of shoaling which may have developed since attainment of modified project depths. The results of these latter studies, together with the office analysis of observed data, will be presented for review by the Committee on Tidal Hydraulics and other echelons of the Corps of Engineers in the fall of 1960, and it is anticipated that decision will be reached shortly thereafter regarding the course of further investigation which may include a hydraulic model study.

Due to the present status of the office analysis of observed data which is in progress, it will not be possible to present at this time a complete and fully evaluated analysis of all the data obtained. It will, nevertheless, be possible to consider some of the indications revealed by the portion of the analysis which has been completed to date, particularly that relating to observations taken on Ranges 1 and 2. In doing so, it should be recognized that observations taken

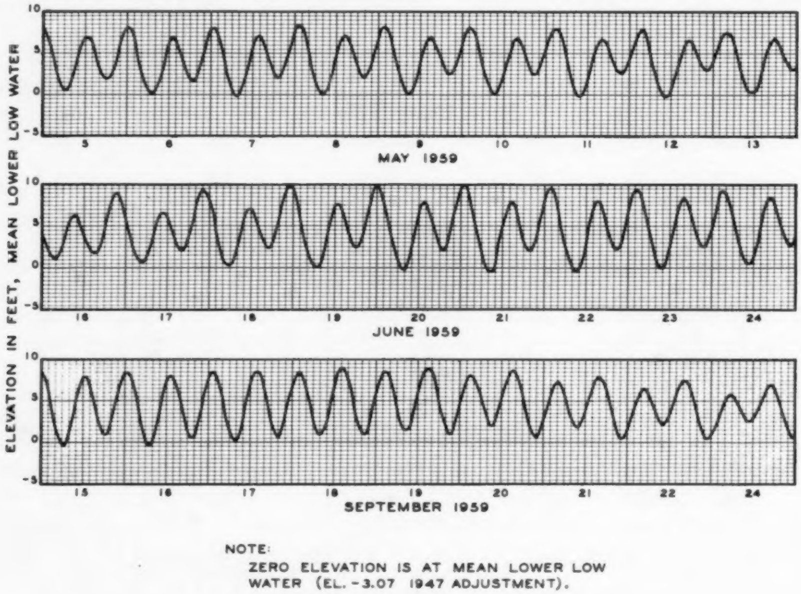


FIG. 7.—TONGUE POINT TIDAL PATTERNS

along these particular ranges deflect the action of forces prevailing at these locations only and, in order to arrive at sound conclusions relating to the nature of these forces throughout the estuary, it will be necessary to evaluate the observations along Ranges 1 and 2 with those taken along the other ranges throughout the estuary and available information on the nature of other related estuarine phenomena. The preliminary examination of observations on these two ranges discussed below is made with that qualification.

In analyzing observations of current direction and velocity, plots, similar to that shown on Fig. 11, were prepared indicating for each station the changes in current direction between flood and ebb and the velocity noted at each depth level throughout each observation cycle of approximately 25 hr duration. From these plots it was possible, by computing the subtended areas above and below the zero velocity line, representing the volumes of flood and ebb flows, to determine the predominant direction of flow and the degree of predominance of

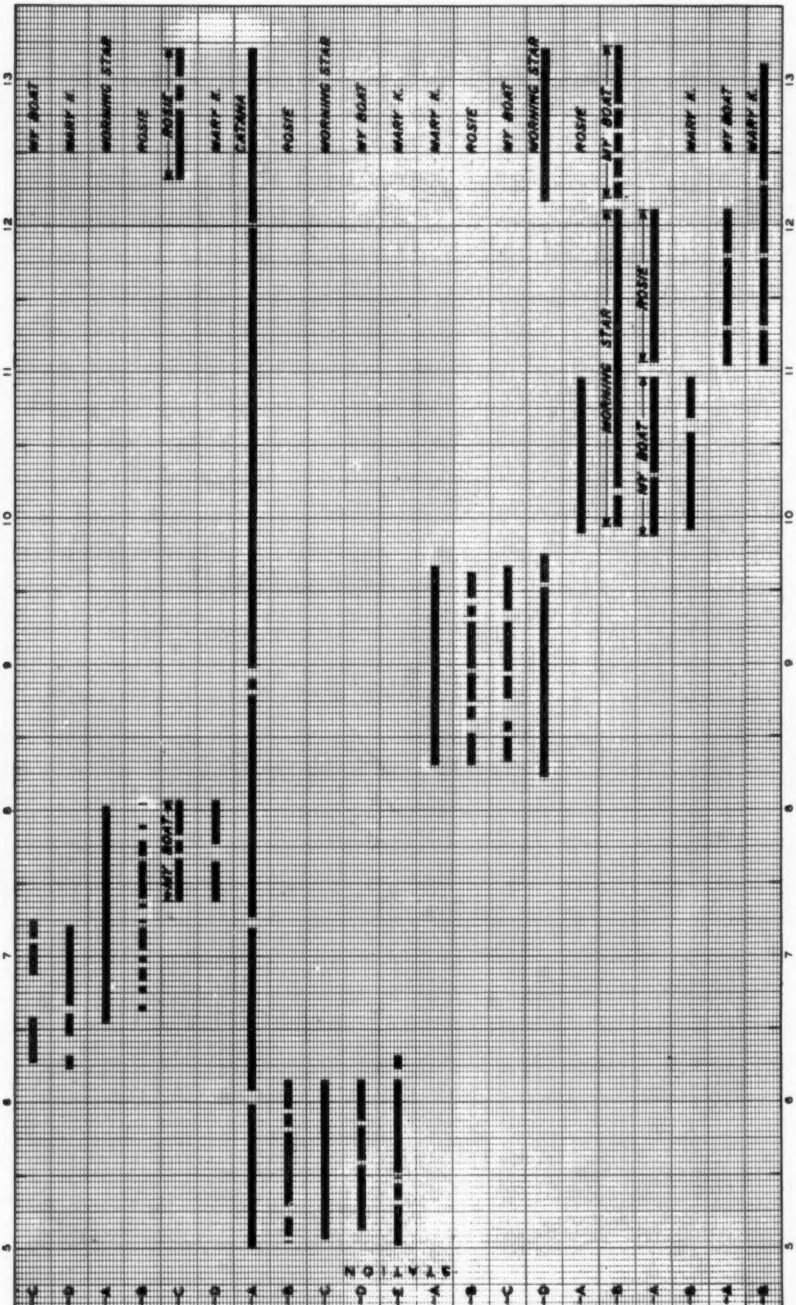


FIG. 8.—READINGS TAKEN FROM MAY 5 TO 13, 1959

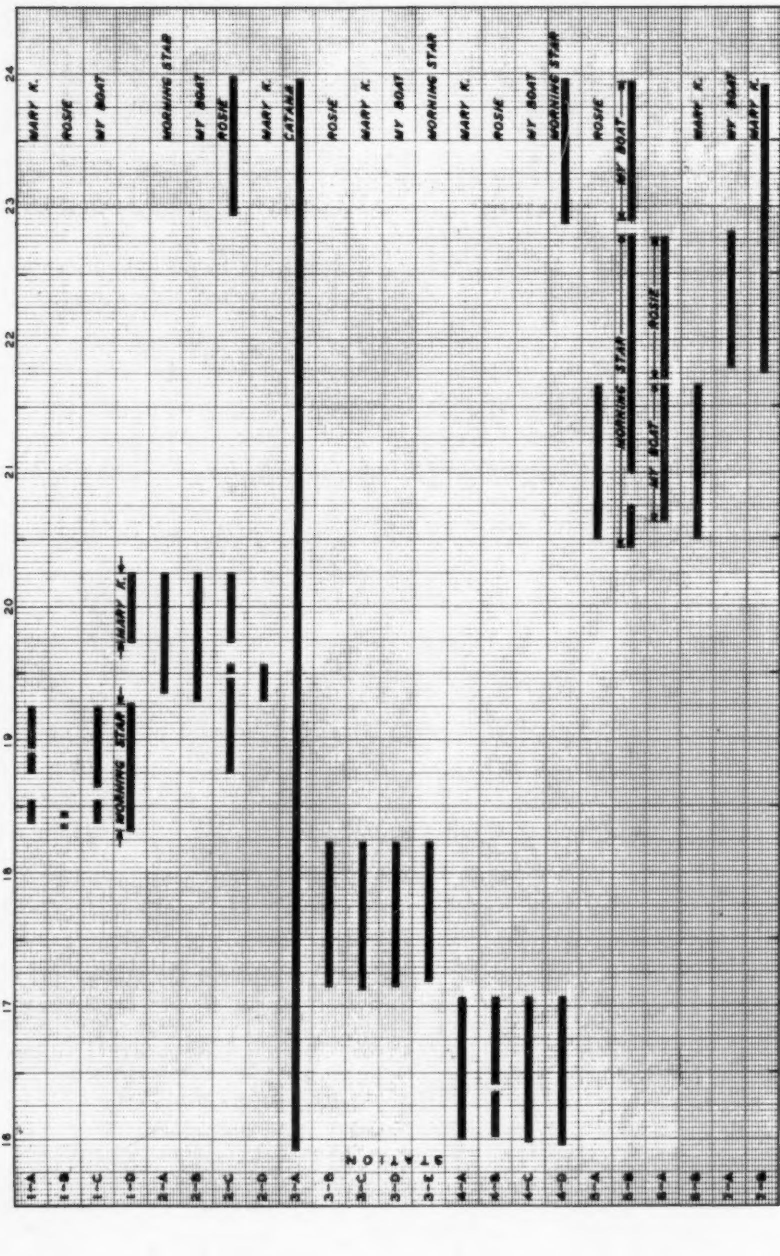


FIG. 9.—READINGS TAKEN FROM JUNE 16 TO 24, 1959

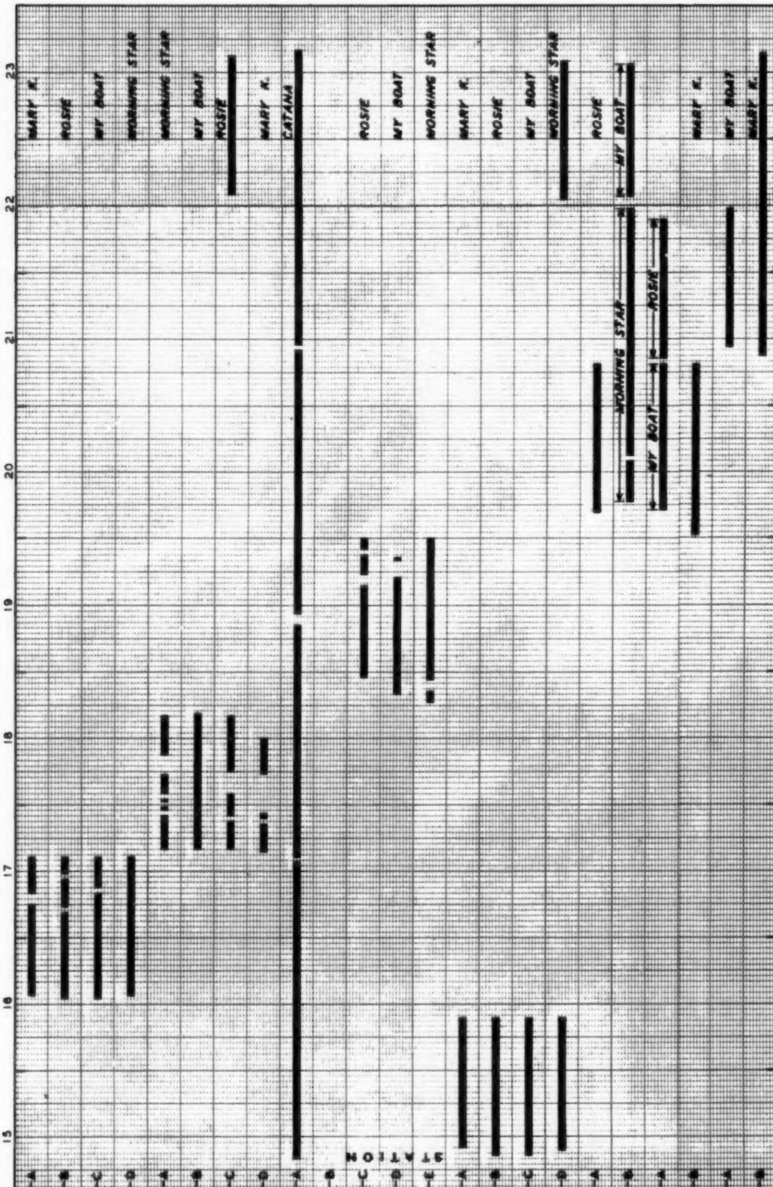


FIG. 10.—READINGS TAKEN FROM SEPTEMBER 15 TO 23, 1959

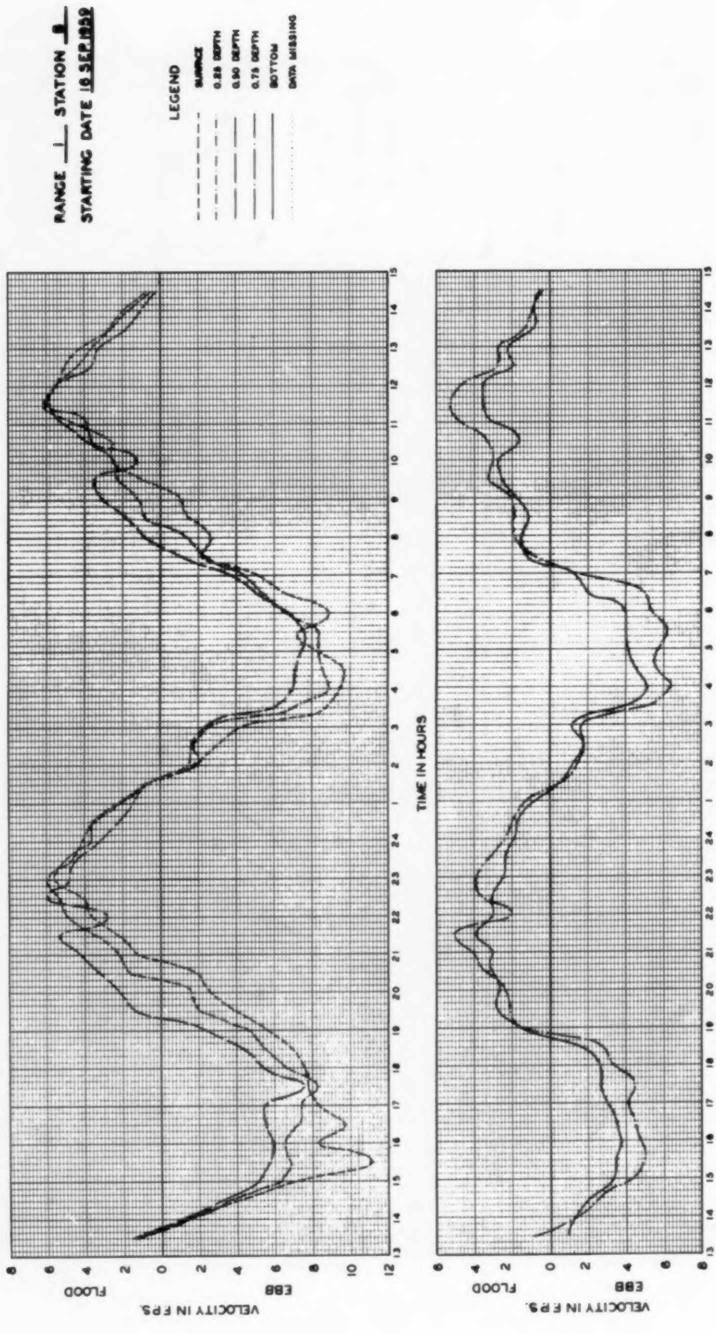


FIG. 11.—VELOCITY - TIME-DIRECTION CURVE (STATION 1-B)

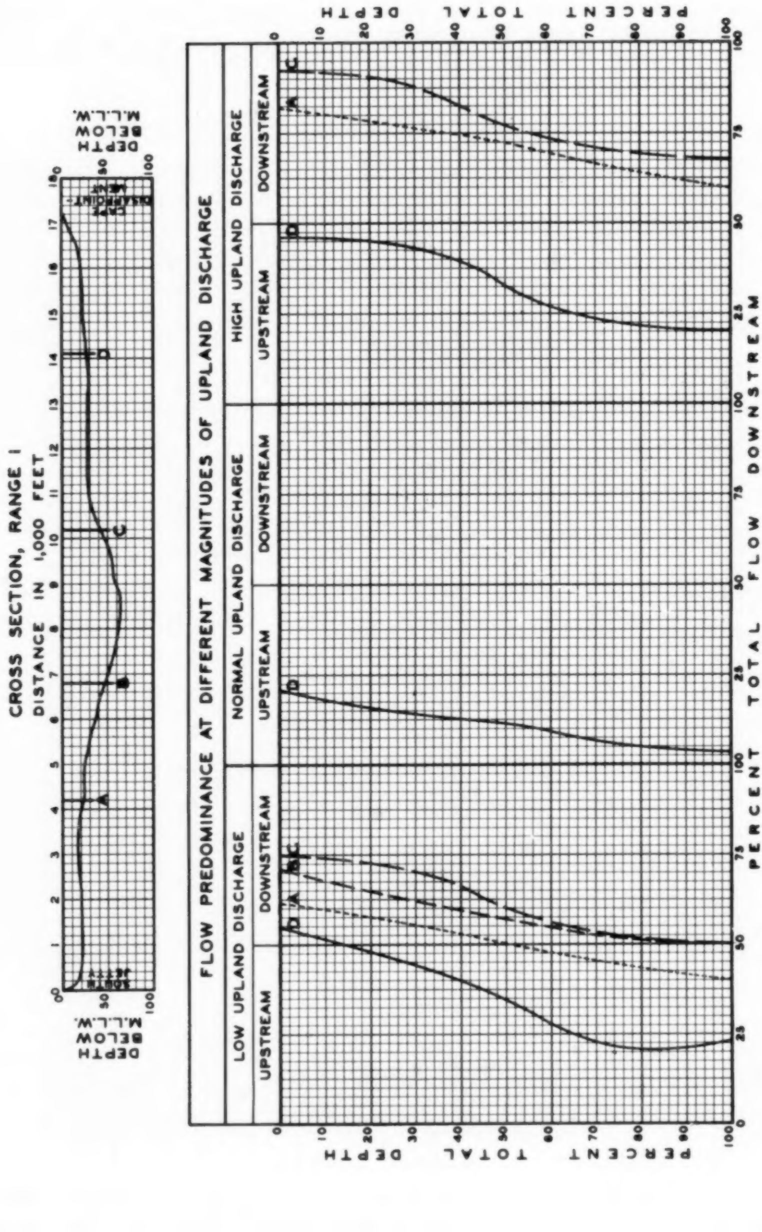


FIG. 12.—FLOW PREDOMINANCE - RANGE 1

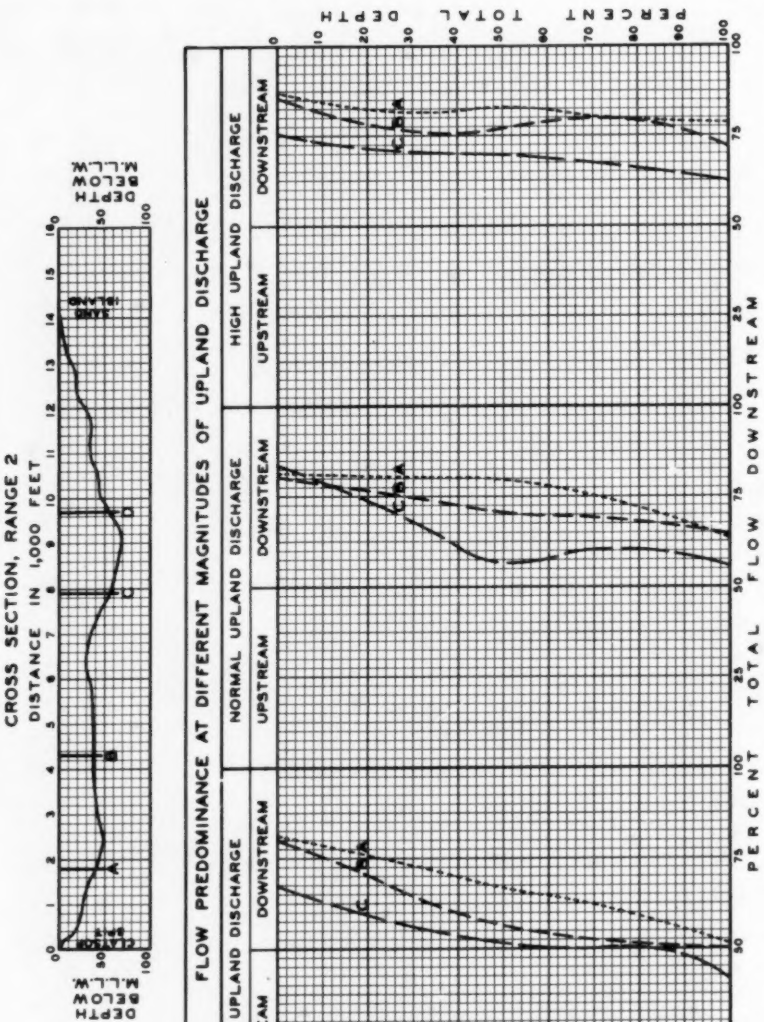


FIG. 13.—FLOW PREDOMINANCE - RANGE 2

such flow at those levels. Fig. 12 shows changes in flow predominance noted at all stations and levels during each river discharge cycle of measurement on Range 1 and Fig. 13 shows similar information obtained from observations taken on Range 2. Data from which these curves were derived are contained in Tables 4 and 5, respectively.

RANGE 1 OBSERVATIONS

Range 1, located between the jaws of the jetties and approximately 1-1/4 mi above their seaward ends, marked the seaward geographical limit of the measurement program. Due to its proximity to the sea and its generally exposed position, vessels taking observations along that range were subject to the vagaries of unfavorable waves, wind, and weather and, occasionally, failure of equipment was experienced. In general, observations on this range were,

TABLE 4.—FLOW PREDOMINANCE - RANGE 1

Station and Depth	Percentage Flow Downstream at Different Magnitudes of Upland Discharge		
	High[a]	Normal[b]	Low[c]
(1)	(2)	(3)	(4)
Station A			
Surface	81.5	No Observations Taken	61.6
1/4 depth	77.2		56.9
1/2 depth	72.5		50.2
3/4 depth	65.1		44.7
Bottom	59.8		-
Station B			
Surface	No Observations Taken	No Observations Taken	71.0
1/4 depth			63.9
1/2 depth			57.0
3/4 depth			51.7
Bottom			50.1
Station C			
Surface	92.4	89.1[d]	74.7
1/4 depth	89.4	75.7[d]	72.0
1/2 depth	77.2	57.7[d]	59.5
3/4 depth	70.0	44.3[d]	52.4
Bottom	67.9	40.3[d]	50.1
Station D			
Surface	46.1	20.9	54.8
1/4 depth	44.5	15.0	46.4
1/2 depth	32.8	11.1	35.2
3/4 depth	22.5	6.0	21.8
Bottom	20.3	3.6	22.8

[a] Upland discharge at entrance on June 18, 1959 estimated at about 577,000 cfs.
[b] Upland discharge at entrance on May 6, 1959 estimated at about 378,000 cfs.
[c] Upland discharge at entrance on September 16, 1959 estimated at about 153,000 cfs.
[d] For one-half tidal cycle period of 12.42 hr.

except for the low river discharge cycle of measurements, incomplete at one or more stations. At the high river discharge cycle, equipment failure prevented full observations at Station B and, during the normal river discharge cycle, rough sea conditions and stormy weather prevented the taking of complete observations at Stations A, B, and C.

At Station A, which lies 4,200 ft north of the South Jetty and where the total depth averages about 25 ft, flows at all depths during the high river discharge cycle were predominantly downstream in direction, although somewhat less in downstream predominance along the bottom than at the surface. During the low river discharge cycle, surface flows were predominantly downstream although less predominant in that direction than during the high river discharge cycle. However, below mid depth, flows were slightly predominant in the upstream direction, reaching the greatest upstream predominance at the lowest depths.

TABLE 5.—FLOW PREDOMINANCE -RANGE 2

Station and Depth	Percentage Flow Downstream at Different Magnitudes of Upland Discharge		
	High[a]	Normal[b]	Low[c]
(1)	(2)	(3)	(4)
Station A			
Surface	87.1	81.1	81.1
1/4 depth	81.3	80.3	74.3
1/2 depth	83.0	79.8	66.7
3/4 depth	79.9	73.9	60.7
Bottom	78.5	63.4	51.2
Station B			
Surface	85.3	80.3	77.4
1/4 depth	77.1	75.5	67.1
1/2 depth	77.1	70.5	56.6
3/4 depth	79.9	68.5	52.3
Bottom	72.4	64.6	50.2
Station C			
Surface	75.6	82.9[d]	67.1
1/4 depth	71.0	70.7[d]	57.9
1/2 depth	69.8	56.7[d]	51.9
3/4 depth	66.8	60.5[d]	50.7
Bottom	62.1	55.9[d]	41.3

[a] Upland discharge at entrance on June 19, 1959 estimated at about 570,000 cfs.
[b] For stations A + B, upland discharge at entrance on May 7, 1950 estimated at about 404,000 cfs.
[c] Upland discharge at entrance on September 17, 1959 estimated at about 163,000cfs.
[d] Upland discharge at entrance on May 12, 1959 estimated at about 365,000 cfs.

During the low river discharge cycle, surface flows at Station B, where the total depth is about 50 ft, had a greater downstream predominance than those at Station A, which predominance was gradually lessened in the deeper levels until it vanished entirely along the bottom, where flows were neither downstream nor upstream in predominance.

Flows at all depths at Station C, where the total depth is about 44 ft, during the high river discharge cycle were predominantly downstream although considerably less predominant in that direction along the bottom than at the surface, where more than 90% of the total flow was downstream. However, during lower river stages, notwithstanding a significant downstream predominance at the surface, flows at the bottom, like those at Station B, demonstrated neither a downstream nor upstream predominance.

At Station D, located in the cul-du-sac between the North Jetty and Jetty "A", where the total depth was about 27 ft, flows at all depth levels were predominantly upstream in direction at all different river discharge measurement cycles, except at the surface during the low flow cycle where a slight downstream predominance was noted. This was to be expected since the station was located in an area that was known to have a clockwise eddy for most tides and flows.

RANGE 2 OBSERVATIONS

Observations at three of the four stations on Range 2 were, in general, fairly complete throughout each different river discharge measurement cycle. At Station D, stormy weather caused the loss of equipment during the low river discharge cycle and during the normal and high river cycles, the measuring boat anchors failed to hold. As a consequence no complete observations were taken at Station D during any measurement cycle.

At Station A, which lies along the left side of the navigation channel some 1,800 ft north and east of the low water line of Clatsop Spit and where the total depth is about 42 ft, flows at all depths during the normal and high upland discharge periods were found to be predominantly downstream in direction although along the bottom the downstream predominance of these flows was less than at the upper depth levels. This pattern of downstream predominance generally prevailed but to a lesser extent during the low upland discharge cycle, except along the bottom where flows had a very slight downstream predominance.

Flows at Station B, lying just off the right side of the navigation channel where the total depth is about 43 ft, evidenced a similar predominance to those of Station A during the high river discharge cycle. During the normal river discharge cycle, flows at Station B, although similar to those at Station A at the surface and bottom, revealed a somewhat less downstream predominance at median depths. At the low river discharge cycle, flows above the bottom were predominantly downstream, although to a lesser degree than at Station A, but along the bottom neither an upstream nor downstream predominance was evidenced.

At Station C, which lies along the left side of a natural deep arm of the estuary some 6,000 ft south and west of Sand Island, where the total depth is about 54 ft, flows at all depths and at all stages of upland discharge revealed lesser evidence of downstream predominance than was noted at Station B. During the high upland discharge cycle, flows were predominantly downstream at all depths although of lesser predominance than indicated at Stations A and B. This trend was noted throughout median depth levels during the normal upland discharge

period. During the low upland discharge period, flows below the 3/4 depth level were predominantly upstream in direction.

PRELIMINARY EVALUATION OF OBSERVATIONS ON RANGES 1 AND 2

Study of the observations taken at Station D on Range 1 suggests the possible existance of a predominant upstream flow at all depth levels and at all stages of upland discharge in this general vicinity. Although probably the result of an eddy condition created in this cul-du-sac area, the existance of such an unusual flow is evidence of very complex current action in this area which may be related to the general regimen of the adjacent estuary bottom.

If observations on Range 1 are further considered without regard to those obtained on upstream ranges, it might be logical to expect evidences of greater upstream predominance as depths become greater throughout the cross-section. This would indicate that bottom flows at Stations B and C, with average depths of 50 ft and 44 ft, respectively, should show greater upstream predominance than bottom flows at either Station A (depth about 25 ft) or Station D (depth about 27 ft). This, however, is apparently not the case as bottom flows at both Stations A and D demonstrate a more marked upstream predominance than flows at either Stations B or C. The reason for this variance from the normally expected condition is not clear unless, under the influence of the eddy condition just mentioned, the strength of density currents at this range varies not only in the vertical but also in the horizontal, thus evidencing a tendency to be more pronounced at Stations A and D. It also appears possible that an eddy condition similar to that presumed along the opposite shore may prevail near the westerly limits of Clatsop Spit.

Examination of flow predominance for Range 2 reveals a general tendency for the downstream predominance of flows at all levels and at all river stages to progressively diminish beginning at Station A and proceeding across the range to Station C. This suggests that the northern portion of this range lies within a region of more active density current influence, which region could constitute the primary route of advance and retreat of the saline wedge phenomena.

Considering the relative degrees of bottom flow predominance noted along Ranges 1 and 2, it appears doubtful that at stages of low upland discharge the flow predominance pattern permits material moving down the river to be permanently discharged into the sea. In fact, bottom upstream velocities observed during the low river discharge cycle appear to be more than competent to move sand from the sea into the entrance area. At normal and high cycles of upland discharge, bottom flows, except at Station D on Range 1, are predominently downstream in direction and are competent to move bottom materials to the sea during times which those stages prevail.

The degrees of bottom flow predominance noted at any station during the different measurement cycles generally reflect the changing competency of currents, as influenced by corresponding magnitudes of upland discharge, to move shoal materials in either a downstream or upstream direction. For example, at Station C on Range 2, when the upland discharge was 570,000 cfs, the percent of bottom flow downstream was computed to be 62.1; when this discharge was reduced to 365,000 cfs, the percentage was reduced to 55.9; and finally, when the discharge was 163,000 cfs, the percentage was further reduced to 41.3, indicating at this discharge a majority 58.7% of the total bottom flow

was in an upstream direction. From interpolation of these data it may be assumed, for river discharges greater than 280,000 cfs, bottom flows have varying degrees of downstream predominance and for river discharges less than 280,000 cfs bottom flows have varying degrees of upstream predominance. Reference to the duration curve of the Columbia River at The Dalles, with due allowance for inflow between that point and the river entrance, shows that river flows greater than 280,000 cfs prevail for only about 25% of the time. Accordingly, it would appear that in order to assure a net permanent downstream movement of bottom materials throughout the year, the net competency of bottom ebb flows during times of discharge greater than 280,000 cfs would have to be more than three times the net competency of bottom flood flows during the remainder of the time. Since, however, observed maximum bottom ebb velocities are only slightly greater than maximum bottom flood velocities, 3.8 as compared to 3.6 fps, prespectively, it appears resonable to conclude that there is no net downstream bottom movement of materials at this station. Similar examination of bottom flow predominance patterns at other stations on Ranges 1 and 2 raises doubt that the inner bar portion of the estuary is capable of maintaining project dimensions at the present time without yearly removal of shoal materials or without further structural improvement to render annual dredging unnecessary. Past history, of improvement of the Columbia River entrance appears to confirm this conclusion.

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HYDRAULIC COMPUTATIONS FROM LIMITED INFORMATION

By Cecil K. Oakes,[1] F. ASCE

SYNOPSIS

A shortage of data and information on a project area often makes it necessary to employ methods designed to make the most of what are available in climatological, hydrological, and physical data in studies and investigations for drainage and flood control and the design of hydraulic facilities.

The writer describes methods, together with sample results, recommended for use in certain types of hydrologic and hydraulics computations where lack of time and/or funds will not permit obtaining additional basic data and information. Methods are discussed for computing peak discharge rates and discharge hydrographs from watersheds; discharge rates over embankments; low flows or runoff from watersheds; and stage-discharge relationships. Some of the improbabilities in computed results by the methods discussed are mentioned and where feasible, at least two methods are recommended as a check on the reliability of these results.

INTRODUCTION

The lack of sufficient hydrologic information on a project area when studies and investigations for drainage and flood control and the design of hydraulic facilities are required places the engineer in a predicament. This lack of data frequently stems from the urgency for an answer demanded by his clients. The

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nature of the problem is such that sufficient time is not available to collect needed hydrologic data. In most cases a long period of waiting for extreme climatological conditions to occur would be required.

The use of methods which are described in this paper are recommended where factual data are lacking. Louisiana is looking to its water resources and have in progress a vigorous and comprehensive program of data collection. A state-wide network of continuous record gaging stations supplemented by high and low-flow partial record stations provides a wealth of surface water data and information.

Comprehensive studies of Louisiana rainfall, including intensity, duration, and frequency data is available and in print. Flood-frequency relationships have been developed on an areal basis for most of the state which provide flood data for watersheds above 25 sq miles. Studies are underway and in the next few years it is expected that the water supply characteristics of Louisiana streams will be published which will provide low flow data for most of the areas in the state.

In the past, a shortage of data has necessitated the use of synthetic methods. There is yet in need of flood information on runoff from small watersheds. This paper deals with methods to be used in those areas where these deficiencies exist.

With a limited amount of climatological, hydrological, and physical data covering the project area, the accuracy of the answers obtained will be largely a measure of the engineer's judgment and diligence in searching for and using all available data and information, including the transposition of data from other watersheds. In many cases, he can at best hope to come up with computed answers that are reasonably close to what will result from the actual occurrence of rainfall or flood flows on which the computations are based.

The computation of discharge hydrographs, peak discharge rates for various storms, and for recurrence frequencies and low or minimum runoff from a given watershed; the discharge over obstructions; and the construction of stage-discharge and stage-slope-discharge curves are discussed herein.

DISCHARGE HYDROGRAPHS AND PEAK RATES

In computing discharge hydrographs for storm runoff from an area, the details with which the computations should be made will be dictated by the nature of the structures involved. Instantaneous peak rates will be required for drainage channels, culverts, and bridges. Where peak inflow rates are dampened by storage in areas that serve as reservoirs, even for short time periods, it is often more essential to determine the volume of runoff for time periods of certain length. From this hydrograph and a stage-volume (capacity) curve of the reservoir area, a flood routing computation is then made to determine the instantaneous peak outflow from the area.

For small watersheds, having periods of concentration (the time required for all the watershed to contribute runoff to the point in question, design point in this paper) of a few hours the rational formula ($Q = C\ i\ A$) is satisfactory for determining instantaneous peak discharge rates. For larger watersheds some other method should be used. A unit hydrograph or a distribution graph can be more reliably employed to make this determination. For storms having recurrence intervals up to about 50 yr, these peak rates can be determined for most

areas in Louisiana in excess of about 25 sq miles from a 1952 publication.[2] This determination, made by the U. S. Geological Survey, was based on the annual instantaneous peak discharge rates of record over the state. In many cases the existence of hydrographs having considerable variation in peak time will cause considerable doubt as to whether the computed instantaneous peak discharge rates are reasonably close to what can be expected from the watershed under the assumed conditions of rainfall and runoff. In such cases two methods of approach should be used. If the resulting answers are considerably different, more research and refinement of the assumptions and computations may be required. By use of a distribution graph, transposed from another area of similar size and characteristics, and an actual storm equal to one of a given recurrence frequency, the writer, in computing these peak discharge rates from watersheds in Louisiana, has obtained results that were within 5% of the values obtained by the U. S. Geological Survey.[2] This strengthens the reliability of using this method.

SYNTHETIC HYDROGRAPH

For small drainage areas ranging up to a few square miles, a synthetic triangular shaped unit hydrograph can be satisfactorily employed in computing runoff hydrographs for larger storms. This hydrograph has its apex at the time of concentration, which is determined from nearby gaging station records or from the engineer's experience and observations, and the base is equal to 2.0 to 2.5 times this time of concentration. The volume under the hydrograph is of course 1.0 in. of runoff from the watershed. The peak discharge is twice the average discharge for the period of time representing the base of the triangle. Table 1 and Fig. 1 are examples of this method. Fig. 1 shows a computed runoff hydrograph using synthetic unit graph for the March 13, 1947 storm. The drainage area was 6,700 acres.

MAXIMUM DISCHARGE RATES

All available stream flow data which are applicable for the area under study should be used to establish the relationship between drainage area and maximum peak discharges. These data plotted on Log-Log co-ordinate paper will often permit a curve for maximum discharge rates that will serve as a guide as to the limit for which facilities should be designed in the majority of cases. The envelope curve on Fig. 2 embraces the maximum peak rates that have been observed in Louisiana. No attempt has been made to draw a curve representing the maximum rates to be expected. Extension of this curve for drainage areas beyond approximately 1,000 sq miles is not recommended without additional peak discharge data from major floods. Until additional flood information is obtained, the transposition of data from other areas is necessary.

CONCENTRATION TIME

The determination of the time of concentration is due careful consideration in any method employed. The writer has found that an error ranging up to about

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2 "Floods in Louisiana, Magnitude, and Frequency," U. S. Geol. Survey, in cooperation with Louisiana Dept. of Highways, Baton Rouge, La., 1952.

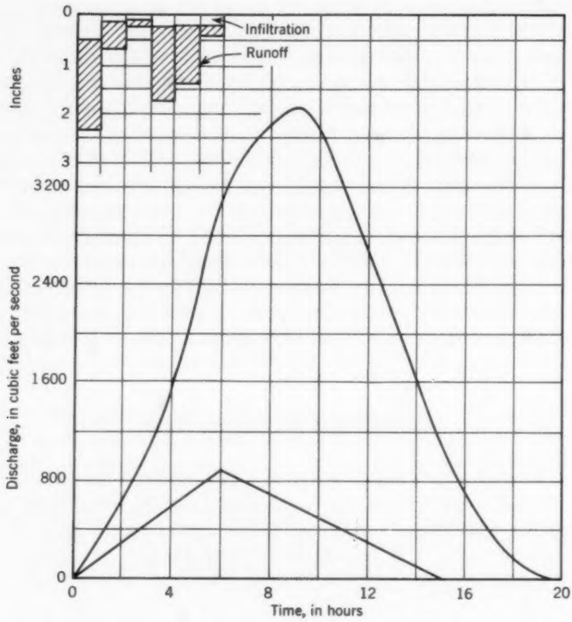

FIG. 1.—COMPUTED RUNOFF HYDROGRAPH USING SYNTHETIC UNIT GRAPH

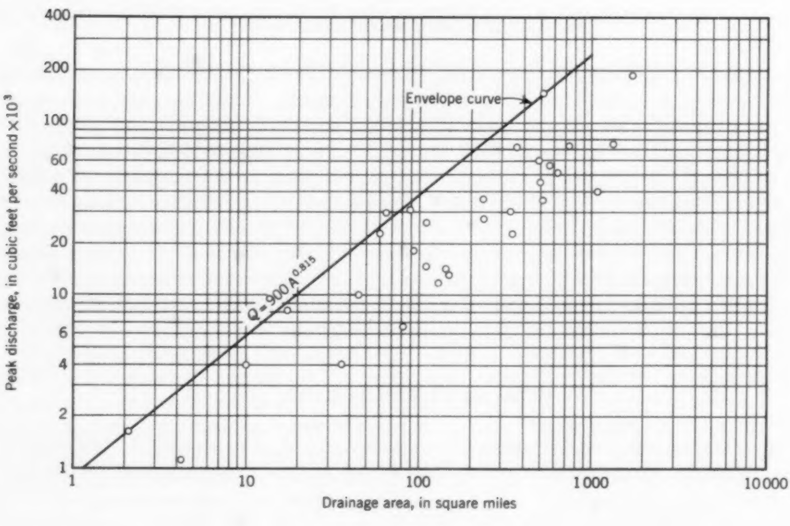

FIG. 2.—FLOOD DISCHARGES IN RELATION TO DRAINAGE AREA IN LOUISIANA

50% is relatively common if too much reliance is placed on computing this time period by dividing distances, scaled on a map, from the remotest limits of the watershed to the design point, by assumed velocities in the channels and over the land areas. The periods computed by this method are often applicable after improvements have been made to tributary channels and on the watershed.

Available stage and stream flow hydrographs from gaging station records should be plotted to determine periods of concentration for various watersheds in the general area of a study. In cases with several hydrographs from one watershed, different peak time periods will often occur. This is due to the time and/or areal distribution of the rainfall provided no major changes have been made on the watershed. The rainfall records embracing the area should be studied to determine this distribution. The hydrograph having the shortest peak

TABLE 1.—COMPUTATION OF RUNOFF HYDROGRAPH BY USE OF SYNTHETIC TRIANGULAR UNITGRAPH

Time, in hours	Unit-graph (cfs)	Runoff, in inches						Total (cfs)
		1.80	0.50	0.10	1.50	1.20	0.20	
0	0	0	...	...	...	...	...	0
1	149	268	0	...	...	...	...	270
2	298	536	75	0	...	...	...	610
3	448	806	149	15	0	...	...	970
4	597	1075	224	30	224	0	...	1555
5	745	1340	299	45	447	179	0	2310
6	895	1610	373	60	672	358	30	3105
7	795	1430	448	75	896	538	60	3445
8	696	1250	398	90	1120	717	90	3665
9	596	1075	348	80	1340	895	119	3855
10	497	895	298	70	1190	1075	149	3675
11	398	715	249	60	1045	955	179	3205
12	298	536	199	50	895	836	159	2675
13	199	358	149	40	745	716	139	2145
14	100	180	100	30	597	596	119	1620
15	0	0	50	20	447	478	99	1095
16	...	...	0	10	298	358	80	745
17	...	...	...	0	150	239	60	450
18	...	...	...	...	0	120	40	160
19	...	...	...	...	...	0	20	20
20	...	...	...	...	...	...	0	0

[a] Fig. 1. [b] Drainage area 6,700 acres.

time when the entire watershed is contributing represents the most critical one for the area. Its use for computing a unit hydrograph or a distribution graph will result in the maximum peak discharge rates computed for larger rainfalls.

In determining the runoff, from any given storm, the antecedent rainfall must of course be considered. At best, one can hope to arrive at a reasonably reliable amount of runoff and the resultant hydrograph from any given storm over the area. Although he may be in error by a sizable quantity for any specific storm, the probability of a storm occurring at any time that will produce this computed runoff should not be overlooked. For this reason, it is not advisable to adhere too unyieldingly to a storm frequency criterian (one that is often arbitrary) in the design of drainage, flood control, and other facilities. Computed

peak discharge rates should be compared with those obtained from the envelope curve such as is shown on Fig. 2. The design storm may be greater than any for which discharge measurements have been made and should result in computed peak rates that fall outside this curve. Remembering that 1 in. of runoff per acre per hour is practically 1 cfs, when these computed rates expressed as cfs per acre from the watershed approach or exceed the average maximum rainfall over the area, expressed in inches per hour for the period equal to the time of concentration, the computations should be reviewed very critically for errors in assumptions. Such phenomenon could only result from 100% runoff and this is unlikely.

COMPUTING DISCHARGES OVER OBSTRUCTIONS

In flooded areas, it is often necessary to compute discharge rates over obstruction such as road embankments. If a weir formula, $(Q = C\ L\ H^n)$ is used there is often considerable doubt as to the proper C value to be used, expecially where submerged flow conditions have obtained. Where head and tailwater stages are ascertainable from gage records, highwater marks, or computed flow lines (water surface profiles), the writer often treats the stage differential across the embankment as the velocity head when the submergence is in excess of about 0.7. The area used is the product of the submerged length and the average depth of the tailwater above the crest of the embankment. As a check on the results by this method and in computing discharge where submergence is less than 0.7, the values of c and n by D. L. Yarnell and F. A. Nagler are used.[3]

ESTIMATING LOW FLOW YIELDS

It is often necessary to estimate the runoff from a watershed during periods of extreme low rainfall and there are few or no low flow measurements available. The duration of the thought period may vary from a few months to several years.

The most reliable means of estimating the yield is by comparison with nearby streams where records are available. This comparison is strengthened when a few discharge measurements are available on the problem stream.

Low-flow yields cannot be correlated with rainfall in the same manner as storm runoff. Low-flow is a relatively small part of residual runoff relating to sub-surface characteristics. However, in the absence of other data, this method must be employed in some instances.

Usually more information is available on the rainfall embracing an area than on the runoff. In determining the probable runoff for prolonged drought periods, the daily rainfall for the stations embracing the watershed should be tabulated. Infiltration is estimated on the basis of the daily amounts and the time distribution of the rainfall. Where data are available this method should be checked on comparable areas having discharge records available. The writer has found

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[3] "Flow of Flood Water over Railway and Highway Embankments," by David L. Yarnell and Floyd A. Nagler, Public Roads, April, 1930.

this method to be more reliable than simply using a percentage of the rainfall as the amount of runoff.

DISCHARGE RATING CURVES

It is recognized that the method of synthesizing rating curves by means of the slope conveyance method, as is employed by the U. S. Geological Survey, where time and funds permit is highly preferable. The approach described is to be used where time is too limited to obtain the necessary factors for a slope conveyance study.

Quite frequently it is necessary to determine the stage-discharge relationship at a point on a stream where no measurements have been made. In such cases, one approach is to obtain a field cross-section of the channel and the flood plain at the point in question, and with this information, together with a stage-velocity curve, the discharge for various stages is computed. This will require a stage-velocity relationship determined from a gaging station at nearby points on this stream or one having flow characteristics similar to the one in question. By plotting the stage above the bottom of the channel as the ordinate, against the velocity as the abscissa, for several such channels, a good idea can often be obtained as to the magnitude and slope of the line through these points. Where the channel and flood plain flows are funneled through bridges, the area and velocity through the bridges are used. As many points as desired on the stage-discharge curve are then determined by the product of the cross-section area at a given stage and the velocity at that stage from the stage-velocity curve. The velocity curve shown on Fig. 3 for the Tickfaw River at Holden, is representative of many of the major streams in southeast Louisiana having similar characteristics at the main bridge sections. The drainage area for Fig. 3 was 242 sq miles.

For stages above bank-full, the discharge rating curve will in most cases bend more rapidly to the right as the stage increases depending on the amount of overbank or flood plain flow. This flow should be computed separately from the channel flow and the two then added. The average slope of the flood plain can often be determined from topographic maps with a reasonable degree of accuracy. This slope and the average depth of flow, used as the hydraulic radius, together with the estimated roughness factor "n" will permit a computation of the velocity for various stages, provided there is no point of control such as road embankments, bridges, or constrictions, further downstream that will result in backwater effect at the point in question.

In cases where backwater effect occurs at the problem or design point, backwater computations for various discharge rates will be necessary. The starting point for these computations will either be at any effective control point, just mentioned, or at some other point where the stage-discharge relationship can be determined with reasonable accuracy from existing data. Where it is not feasible to establish reasonably reliable stages for starting these backwater computations, a starting point must be selected that is a sufficient distance downstream so that the effect of any reasonable error in the starting elevation will become negligible by the time the computations have progressed upstream to the point in question. The flatter the gradient downstream from this design point the further the starting point will be for the backwater computation in order to eliminate the effects of any error in the starting elevation. Where the

slope can be obtained from topographic maps and using an estimated roughness factor (n) for Manning's formula, a reasonably reliable rating curve can be computed. Experience in making backwater computations is indispensable to the individual in tempering his judgment as to how far downstream from the design point to begin these backwater computations.

STAGE-SLOPE DISCHARGE RELATIONSHIP

In channels where backwater results from a source other than the stream on which computations are being made, there is no pattern of relationship between the stage and discharge. That is to say the stream will not "rate." In

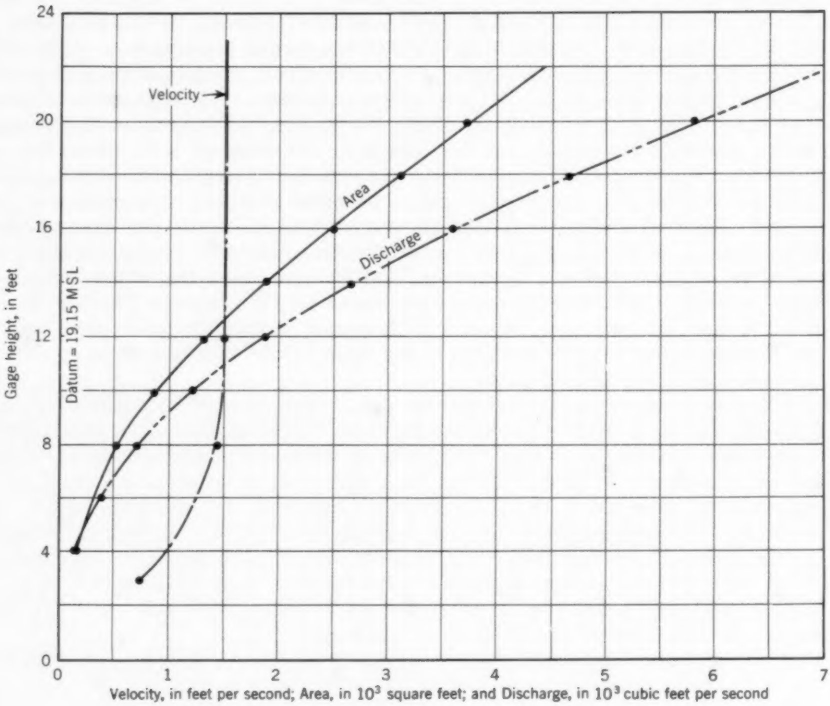

FIG. 3.—STAGE VERSUS VELOCITY, AREA, AND DISCHARGE

such cases a few discharge measurements together with the stages at each end of the channel reach in question can be used to compute the discharges for other combinations of stages without having the hydraulic elements of the channel reach. Considerable time and expense are often necessary to obtain these data from a field survey, especially on the larger streams.

Within certain undetermined limits, the discharge will vary as the square root of the slope of the hydraulic gradient. Or $Q = K\,(S)^{\frac{1}{2}}$. The constant, K, embraces the mean cross-sectional area, the hydraulic radius, and the coef-

ficient of roughness for the channel reach. The value of K will vary for different stages. The length of the reach remaining constant, the difference in stages, H, at the two ends of the reach may be substituted for S in the formula. For each set of gage readings, when discharge measurements were made, the value of K is computed. Other discharge rates are then computed by varying the stages at each end of the project reach. The average stage for the reach

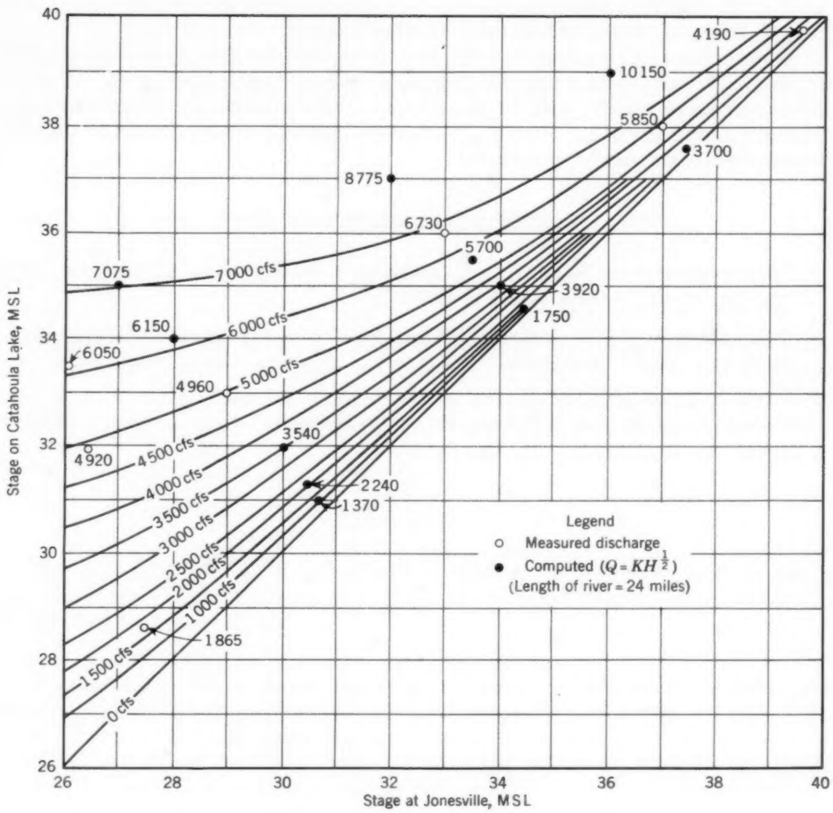

FIG. 4.—STAGE-SLOPE DISCHARGE DIAGRAM

and the value of K remain constant for all computations based on that one discharge measurement. By plotting these data on graph paper as shown in Fig. 4 (for Little River, La.), lines of equal discharge are drawn.

CONCLUSIONS

The most difficult task in the design of a drainage system is the determination of peak discharge rates for design purposes. In many of the hydrologic and hydraulics computations the best that can be expected is to arrive at results

that can reasonably be expected to occur about as often and of the magnitude as the computations indicate. For one thing the relationship between rainfall, as recorded on a daily basis, and peak discharge rates from small watersheds will vary over a considerable range. In such cases there is no intelligible relationship between the two insofar as can be ascertained from available data. The probability of a storm at any time that is greatly in excess of the design storm must not be overlooked.

The relationship between stages and discharges is subject to the influence of channel and flood plain changes such as channel meandering, sloughing of the banks, siltation, fallen trees, etc., as time passes.

In the majority of cases one can test computed results by applying coefficients and other constants used in formulae that are both too small and too large. If the computed results with these extreme values straddle the results that have been computed as "being correct," then more reliance can be placed on the computed results for design purposes.

ADDITIONAL REFERENCES

1. "Streamflow from Rainfall by Unit-Graph Method," by L. K. Sherman, Engineering News Record, April 7, 1932.

2. "Louisiana Rainfall, Intensity-Duration-Frequency Data and Depth-Area-Duration Data." Louisiana Dept. of Pub. Works, Baton Rouge, July, 1952.

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Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

DEVELOPMENT OF A VARIABLE RELUCTANCE VELOCITY METER

By Iury L. Maytin[1]

SYNOPSIS

Presented herein is a new method of measuring fluid flow based on the principle of variable reluctance. The device used by the author has been called a Variable Reluctance Velocity Meter. The basic theory of operation and the outstanding characteristics of this miniature instrument are discussed.

INTRODUCTION

The measurement of fluid velocities has always presented a challenge to the engineer. The following are some of the conditions that may affect the accuracy of measurements:

1. The limited range of a particular velocity meter.
2. The difficulty of visual observation through a fluid due to turbidity and spurious light reflections caused by turbulence and wave action.
3. Transported sediments that may clog tiny orifices.
4. Non-linearity of flow patterns through different regions of depth.
5. The human error associated with the use of apparatus requiring counting, timing, positioning, and interpretation or mathematical translation.

The Variable Reluctance Velocity Meter (hereafter referred to as the VRVM) was designed primarily as a miniature free-wheeling propeller meter possessing the following qualities:

1. The ability to give reproduceable results with a degree of accuracy consistent with that required by a particular test.

Note.—Discussion open until June 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 87, No. HY 1, January, 1961.

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2. A stable long-term operational characteristic.
3. Independence from the physical properties of fluids.
4. Physical dimensions suited to the requirements imposed by different tests.
5. Possession of a wide velocity range.
6. Simplicity of operation and maintenance.
7. Provisions for the automatic recording of velocity data.

DESCRIPTION

The VRVM is basically a propeller-type meter pickup assembly incorporating an electromagnetic circuit whose output is fed into an integrator. Fig. 1

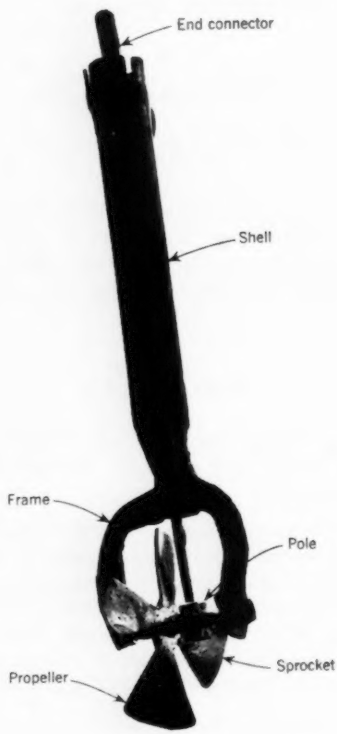

FIG. 1.—VRVM PICKUP UNIT

shows a close-up of the pickup unit. The overall length of the prototype model is 4 in.

A propeller is mounted within a frame and is free to rotate. The frame is joined to a mu-metal shell containing a coil of fine wire, a permanent magnet, and a steel pole piece. This pole piece is axially located along the longitudinal plane of the coil and its upper end is magnetized by contact with the permanent magnet. Fig. 2 is a schematic showing the VRVM pickup unit details.

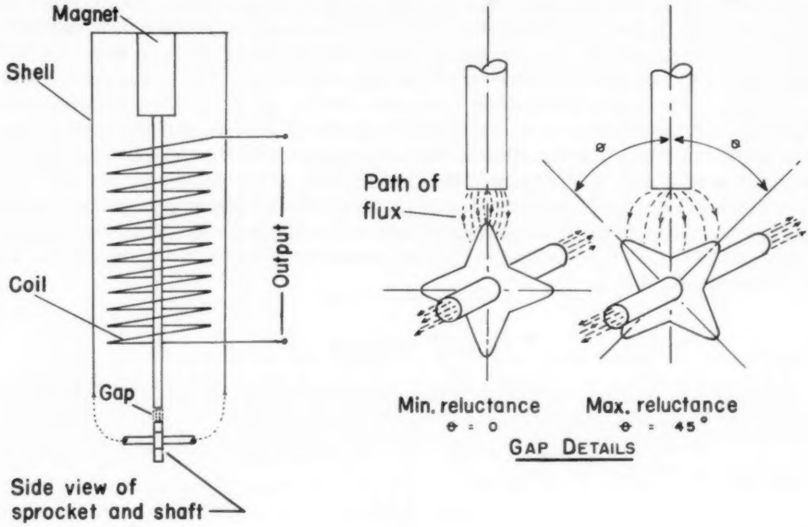

FIG. 2.—VRVM DETAILS

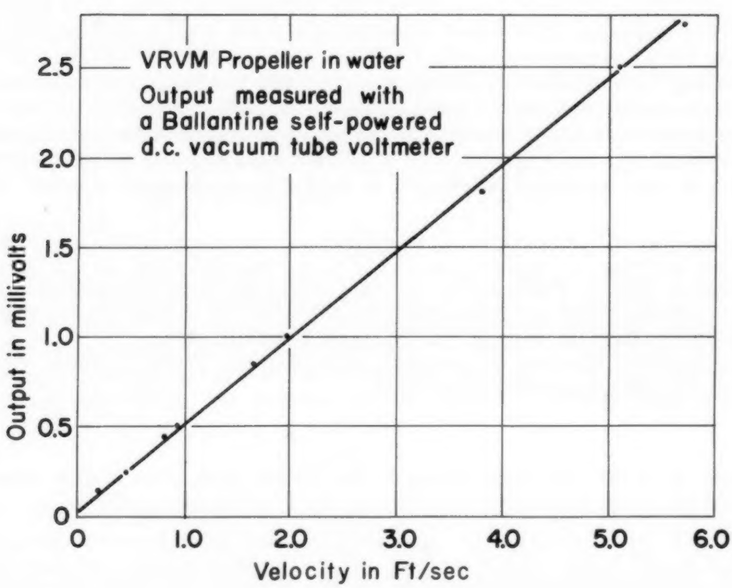

FIG. 3.—CALIBRATION OF PICKUP OUTPUT VERSUS VELOCITY

A steel sprocket is attached to the propeller shaft in such a way as to be centered directly beneath the lower end of the pole piece without making contact to it. The path of the magnetic flux radiating from the pole reaches the steel sprocket through the intervening air gap. Rotation of the sprocket results in a change in air gap dimension. The gap and its reluctance to the magnetic flux are minimum when any one tooth is directly in line with the axis of the pole piece. The gap and reluctance are at a maximum when any two teeth are centered evenly below the pole piece. Reluctance of the magnetic circuit will cycle from maximum to minimum at a rate that is directly proportional to the speed of the propeller. The varying reluctance causes a voltage to be induced in the coil. This voltage is sinusoidal in character and of a frequency equal to the reluctance cycling.

BASIC THEORY

The operation of the VRVM is based entirely on electromagnetic induction. Faraday's Equation states that the electromotive force (volts) induced in a coil of N turns is a function of the rate of flux change. This statement may be written as follows:[2,3]

$$e = \frac{N\, d\theta}{dt} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which e = electromotive force, in volts; N = number of turns in the coil; θ = magnetic flux, in maxwells; and t = time, in seconds.

The facility with which a material can transmit magnetic lines of force, relative to the ease with which these lines of force may be transmitted in a vacuum, is called its permeability, (u). The permeability for a vacuum or for air is unity. In most steels it will range from 1,000 to 10,000. Some materials, such as permaloy, will have a permeability of 80,000 or more.

The reluctance (R) of a magnetic circuit is analogous to the resistance of an electrical circuit. It is a function of both physical dimension and of permeability of the material supplying a path for the magnetic circuit. Then

$$R = \frac{l}{u\, A} \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which l is the length of the material and A is its cross-sectional area. Because for all intent and purpose the reluctivity of air is unity, it is virtually impossible to have an open or discontinuous magnetic path.

The magnetic intensity (in oersteds) along the axis of the magnetic core is equal to the total magnetizing force, or

$$H\, l = F \quad \ldots\ldots\ldots\ldots \quad (3)$$

in which H is the unit field strength. The magnetizing force is also equal to the product of the flux and the reluctance. This can be expressed as

$$F = \theta\, R \quad \ldots\ldots\ldots\ldots \quad (4)$$

If F is constant in a given magnetic circuit, then any decrease in R must be accompanied by an equivalent increase in θ. Conversely, any increase in R

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[2] "Electrical Engineering," by E. E. Kimberly, 3rd edition, International Textbook Co., 1951.

[3] "Modern College Physics," by Harvey E. White, D. Van Nostrand Co., 1951.

must result in a proportional decrease in θ. A change in reluctance is also a function of a change in physical dimension. Thus

$$\Delta R = f\left(\frac{\Delta l}{A}\right) \qquad (5)$$

Eq. 4 shows that any change in the reluctance of a magnetic circuit will be accompanied by an inverse and proportional change in the circuit's magnetic flux if the total magnetizing force of this circuit remains constant. A change in linear dimension will cause this reluctance change (Eq. 5) and thus will satisfy the conditions needed for the generation of a voltage (Eq. 1).

APPLICATIONS

1. The meter is suited for laboratory or for field work. Its small size makes the instrument adaptable to the study of shallow draft flow conditions.
2. The meter is adaptable to automatic recording. This is an asset whenever a long-term velocity study is required, as in the correlation of biological response (fish, snail, algae) to flow conditions.
3. The meter's operation is not dependent on the fluid's electrical conductivity. It can be used with any gas or liquid.
4. The meter can be equipped with a very lightweight propeller to respond quickly to changes in velocity.
5. The response of the meter to fluctuating velocities can be minimized by the use of a heavy propeller possessing an inherent degree of self-integration.
6. The meter can be used in conjunction with an electronic counter to record average velocities.

The propeller used for the prototype design was 1.0 in. in diameter. A propeller of 0.25 in. in diameter and weighing 0.5 g was also tried. It proved extremely responsive to the most minute fluid velocity changes.

ASSOCIATED ELECTRONIC EQUIPMENT

The voltage generated by the meter is small (See Fig. 3) and has to be amplified before it is fed into a frequency meter of the type built by Hewlet-Packard, the HP 500 B. This commercial unit covers a frequency range of 3 cycles per sec to 100,000 cycles per sec. Frequency is read directly on a large meter mounted on the front panel. The HP 500 B or any equivalent type of frequency meter comprises the integrating section of the complete meter set-up and translates the rotation of the propeller into meaningful data.

CALIBRATION

The VRVM was calibrated by towing it through a circular towing tank. The pickup was mounted on a boom whose speed was fully controlled (Fig. 4). A tachometer was shaft-mounted to the electric drive motor to serve as a reference of boom speed. An accurate plot of boom peripheral speed versus tachometer readings was made. The VRVM pickup was then installed at the tip of the boom and aligned tangentially to its direction of travel. The output

signal was fed to a preamplifier and then to a Hewlet-Packard model 500 B frequency meter.

Figs. 5 and 6 show characteristics of the VRVM frequency response versus velocity in fresh water. Fig. 3 shows a plot of voltage output versus velocity for the pickup.

CONCLUSIONS

The propeller used with the VRVM was the determining factor in the establishment of the minimum velocity recordable. Fig. 5 shows a value of 0.15 fps as the lowest velocity at which the propeller registered a valid reading.

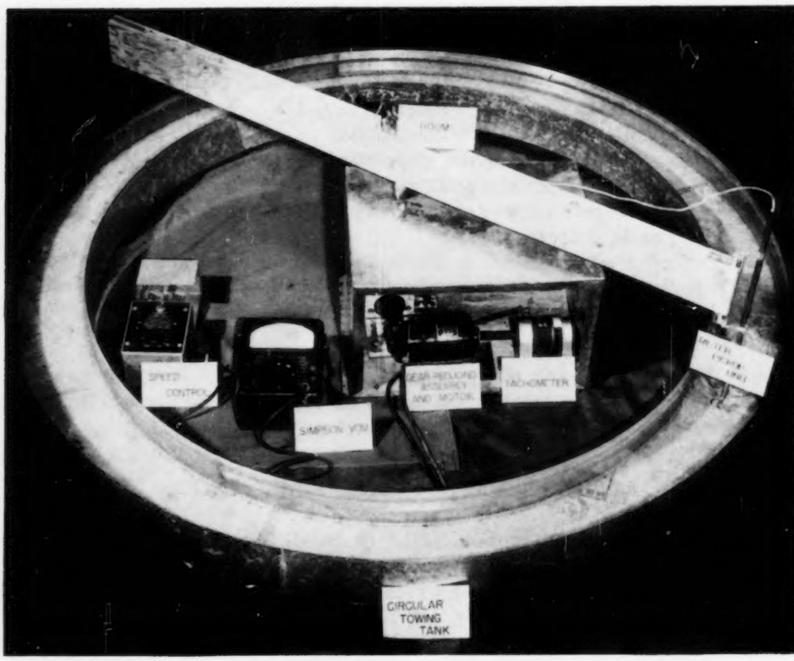

FIG. 4.—CALIBRATING APPARATUS

Fig. 6 shows an upper velocity limit of 5.4 fps. This is not the true limit but an arbitrary one that was imposed by the physical limitations of the calibration apparatus. The propeller was capable of much higher speeds although a more powerful boom motor and a larger calibrating tank would have been needed to record the corresponding velocities with consistent accuracy.

Random scattering of the points on the VRVM calibration curves discloses an overall accuracy of approximately 95% or better. The performance would most probably have been improved by more refined calibration equipment and techniques.

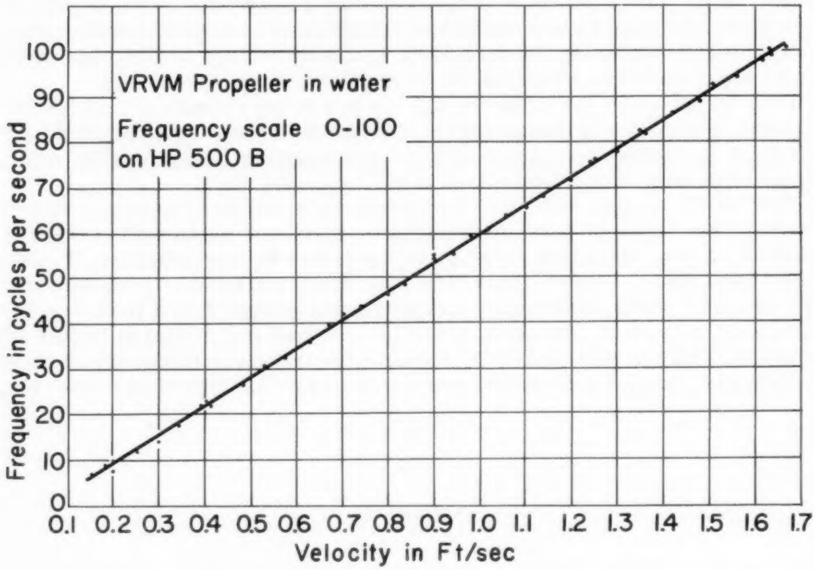

FIG. 5.—METER CALIBRATION CURVE

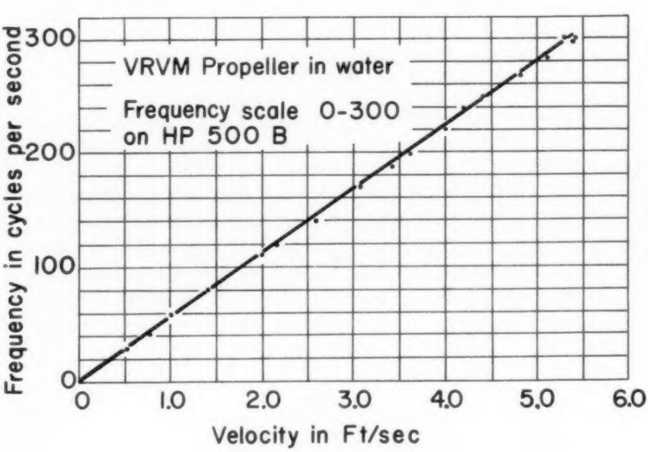

FIG. 6.—METER CALIBRATION CURVE

Frequencies are read on the large panel meter of the HP 500 B. A frequency range switch offers 9 ranges covering a spectrum from 5 to 100,000 cycles per sec. In actual operation, the minimum amount of conversion could be attained by replacing the frequency scales by velocity scales; this would enable the operator to get direct and continuous velocity indications.

Coil output is in the order of 0.2 mv at a water velocity of 0.15 fps and requires a high-gain preamplifier to satisfy the input sensitivity of the HP 500 B. It is desirable to design the coil for maximum output feasible within the physical dimensional limitations of the mu-metal shell.

The VRVM pickup unit and its associated electronic equipment is well suited to the type of service requiring a small and lightweight outfit. For example, if the electronic section of the meter is transistorized, it could easily be packed to remote areas. The inclusion of a miniature transistorized tape recorder would enable quick and permanent storage of data from velocity studies of all sorts. This data could be analyzed and studied at leisure by merely playing the tape back to the frequency meter whose output signal feeds an Esterline-Angus I ma. recorder or any other suitable type of pen recorder.

Journal of the
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HYDRAULICS OF SOUTHWEST PASS, MISSISSIPPI RIVER[a]

By Chester A. Peyronnin, Jr.[1]

SYNOPSIS

This paper summarizes and generalizes the results of an extensive data collection program at Southwest Pass of the Mississippi River. Various theories of shoaling in highly stratified estuaries are examined and the salt water wedge theory selected as the most probable cause of heavy shoaling in this pass.

INTRODUCTION

Southwest Pass is one of the major distributaries of the Mississippi River into the Gulf of Mexico. Because it discharges a large volume of fresh water into a saline body, it provides an excellent example of a highly stratified estuary and demonstrates the influence of density difference upon siltation. Recently the pass was the subject of an extensive data collection program by the United States Army Engineer District, New Orleans, in order to verify a model of the pass. This model had been built to determine the optimum configuration of a deeper channel through the pass. The purpose of this paper is to generalize the findings of that study as related to contemporary theories on siltation

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[a] Presented at the August 1958 Hydrs. Conf. in Atlanta, Ga.

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in such estuaries. The complete data collection program results have been published[2] in a two-volume publication. A historical account of the pass is also available.[3]

GENERAL

The Mississippi River drains approximately 1,245,000 sq miles of the United States and Canada. It is navigable to ocean-going commerce to the port of Baton Rouge, Louisiana, 247 miles from the Gulf of Mexico via Southwest Pass. New Orleans, one of the major maritime ports is 114 miles from the gulf by the same route. The river does not receive any tributary flow below Baton Rouge and so is a closed channel from there to the distributary system near

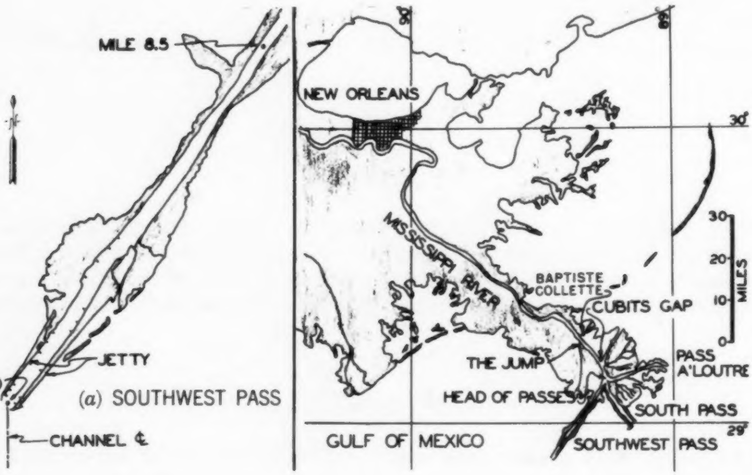

FIG. 1.—GENERAL MAP

the mouth. Joining at the Head of Passes are the three major distributaries, Pass a Loutre, Southwest Pass, and South Pass. Upstream from this junction are several minor distributaries: Baptiste Collette Bayou, the Jump (Grand Pass), and Cubits Gap (Main Pass and Octave Pass) (Fig. 1).

The river is building a natural delta by advancing in extensions resembling, in plan, a bird's foot and so named by geologists. These extensions are forming in relatively deep water and are depositing fine sand and silt over the pro delta muds off the coast. Because of the bars forming at the mouth of each pass, the natural depth of each is limited to approximately 9 ft. South Pass

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2 "Investigations and Data Collection for Model Study of Southwest Pass, Mississippi River," U. S. Army Engr. Dist., New Orleans, La.

3 "The Passes of the Mississippi River," by W. C. Cobb, Transactions, ASCE, Vol. CT, 1953, p. 1147.

has been improved to a depth of 30 ft by the construction of jetties and Southwest Pass has likewise been developed to 35 ft with a proposed depth of 42 ft. Extensive maintenance dredging is required to keep these passes open but a new configuration of the 42-ft channel will minimize this dredging.

FRESH WATER FLOW

Below New Orleans the Mississippi River has a capacity of approximately 1,250,000 cfs at project design flood conditions. Of this, approximately 40% or 500,000 cfs may be expected to pass out through Southwest Pass. At low-water conditions, the flow past New Orleans will be approximately 100,000 cfs, but has been as low as 50,000 cfs in an extreme case. At the normal low flow in the river, the tide in the gulf will materially affect the flow out of the pass and may cause a variation of several hundred percent within a tidal cycle. One measurement taken at low flow in the program, indicated a flow variation from 42,400 cfs to 128,000 cfs in the pass during a tidal cycle.

At high river flows, there will be less of an effect due to the tide, but it is always present to some extent. Flows out of the passes are generally related to the stage of the Mississippi River at Carrollton, La. (New Orleans). Because of the difficulty in correlating stage and flow quantity due to the tide and river variation, it is customary to refer to flows as being low, intermediate, or high depending upon the physical effects. At low flows, less than 2 ft on the Carrollton gage, there is a pronounced intrusion of salt water into the river. At high flows, above 12 ft, the bed material will begin to move and the sediment transport in the river will rise significantly. Flood stage is 17 ft on this gage. At intermediate flows, there are no adverse hydraulic activities.

The fresh water flows into the gulf in the form of a hypopycnal jet, with lighter water flowing over denser saline water at all stages of the river. This delineation between fresh water jet and the saline gulf water is clearly visible from the air and extends several miles into the gulf before the natural diffusion process diffuses the stream with the salt water. The expanse and extent of this jet varies with flow rate as indicated by Fig. 2. Fresh water salinity, expressed as chlorinity, is less than 50 ppm chloride ion.

GULF OF MEXICO FLOW

The Gulf of Mexico is a salt body that has a parent salinity, expressed as a chlorinity, of approximately 18,000 ppm chloride ion. The complete littoral flow in the gulf has not been adequately determined, but in the area adjacent to Southwest Pass may be generalized as westerly. The current is almost due west past South Pass. It is deflected southwesterly by Southwest Pass and then reverts to a westerly direction once clear of the river influence. Currents flowing upriver under the outflowing fresh water form a highly stratified wedge (Fig. 3). There is occasionally a counterclockwise flow just at the tip of the east jetty.

The tide at the mouth of the pass is diurnal and has a range of approximately 1.1 ft, but is affected by the wind to a great extent. The tide rises at approxi-

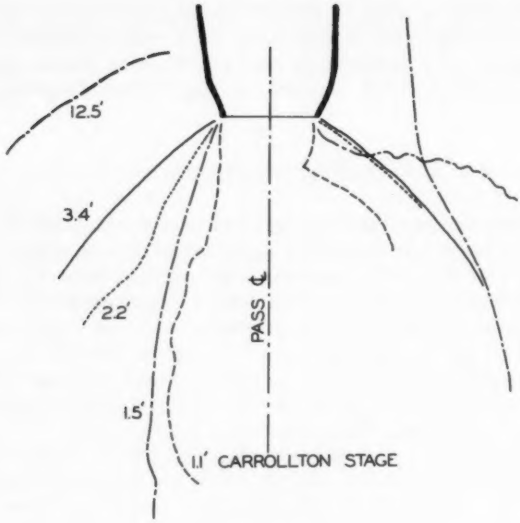

FIG. 2.—EFFLUENT JET

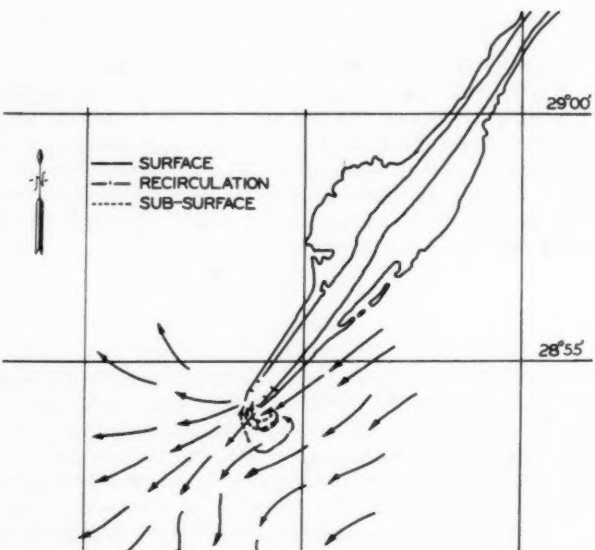

FIG. 3.—LITTORAL FLOW, HIGH STAGES

mately the same time on both sides of the pass and, therefore, there is no head differential transverse to the pass.

SALT WATER WEDGE

The saline water flowing under the fresh water is commonly referred to as a salt water wedge and the interface between it and the fresh water is defined as 5,000 ppm chloride ion. This definition is adequate for most purposes, but it is sometimes desirable to choose a higher value for study purposes. This line of demarcation does not always define the line of zero velocity, but generally flows at greater depths will be upstream and flows at lesser depth will be downstream. Strong density currents exist at the vicinity of the tip of this wedge and there are diffusion currents all along the interface.

At low water conditions in the river, this wedge will proceed upstream, reaching just above the latitude of New Orleans, as indicated in Fig. 4. At high stages this wedge is pushed back into the pass proper (Fig. 5, for 10,000 ppm Cl^-) and will reach the mouth of the pass at approximately a 12-ft stage. There is evidence to indicate that this wedge advances slowly for falling river stages, but retreats quickly for rising stages. During a tidal cycle the tip will adjust its position back and forth slightly so that it is never quite stagnant even for steady stages. During a tidal cycle there will be a variation of salt water flow up and fresh water flow down the river and a variation of the depth of the wedge interface and line of zero velocity.

Typical values of flow, depth of wedge and flow reversal line, and stage are shown for a low-flow condition in Fig. 6. At low flows, the water surface slope will actually reverse as the tide is rising as may be noted by comparing the stage at the East Jetty Station and the stage at mile 8.5, which is approximately 11.5 miles upstream. This reverse slope does not occur at medium and high flows. As a result of the tide, there will be a decrease in downstream flow and an increase in the upstream flow. This in turn raises the wedge and line of velocity reversal.

Fig. 7 illustrates a section across the mouth at mile 20 showing the isovels of upstream and downstream flow during one day. The transverse slope of the flow reversal and wedge interface does not always conform to the same slope and will slope in the opposite direction at other times.

SEDIMENT

Sediment particle sizes in the pass are rather small. In the data collection program, the maximum sample size found in the bed material was 1 mm (0 phi units) with 0.5 mm (1 phi unit) being the more general maximum size. Median sizes of bed material approximated 0.025 mm (5.22 phi units). Material carried in suspension was much finer with the maximum size being in the order of 0.1 mm (3.33 phi units). At low-water conditions, the bulk of the material in transport is carried in suspension. At high stages, approximately 12 ft on the Carrollton gage, the bed load will begin to move and will provide the greater percentage of material in transport.

SHOAL FORMATIONS

The shoaling problem in the pass is a very complex one. Shoaling is due to the unbalance of depositional and transport forces and these forces are continually in the transient state in this pass. In order to study the problem, it is

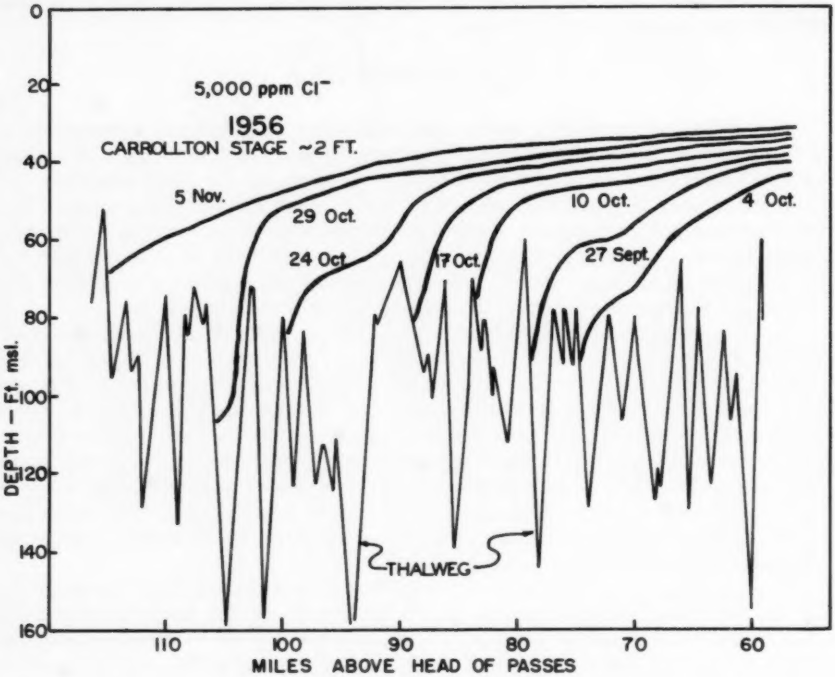

FIG. 4.—PROGRESSION OF SALT WEDGE

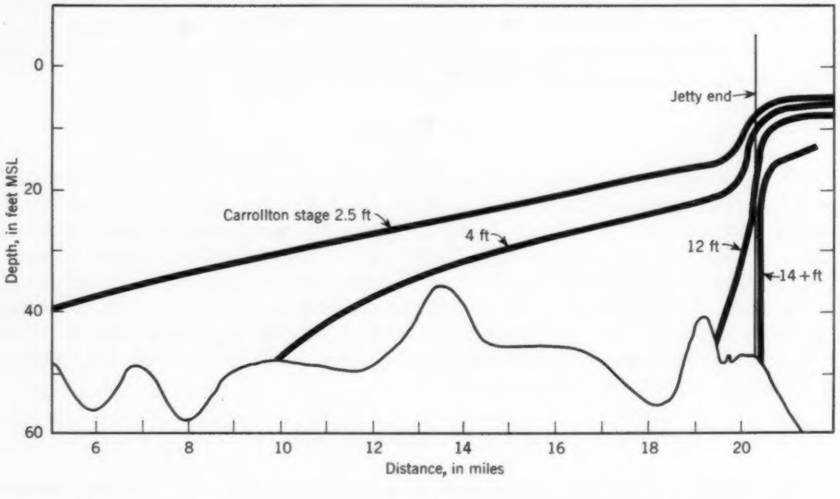

FIG. 5.—SALT WEDGE IN THE PASS

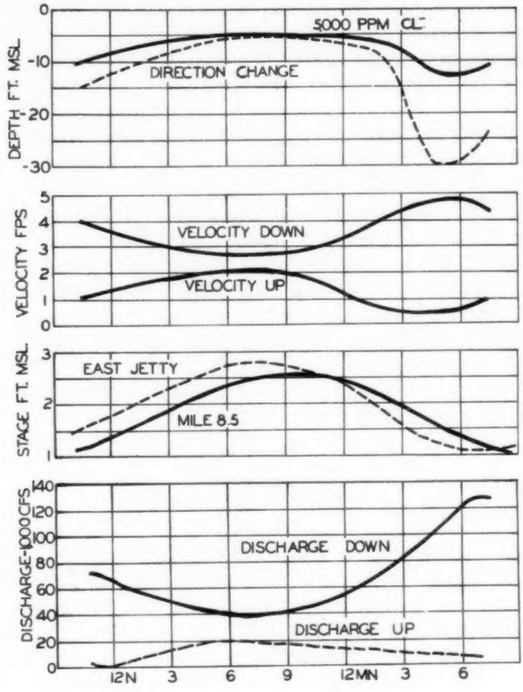

FIG. 6.—HYDRAULIC CYCLE, LOW STAGES

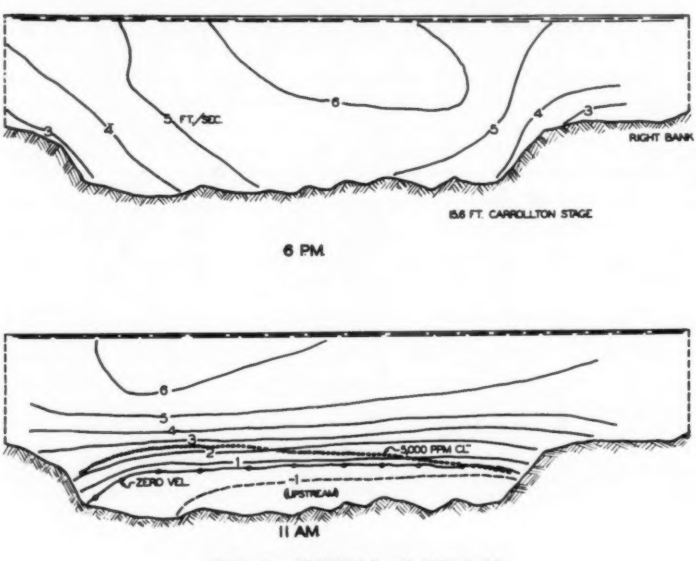

FIG. 7.—ISOVELS AT MILE 20

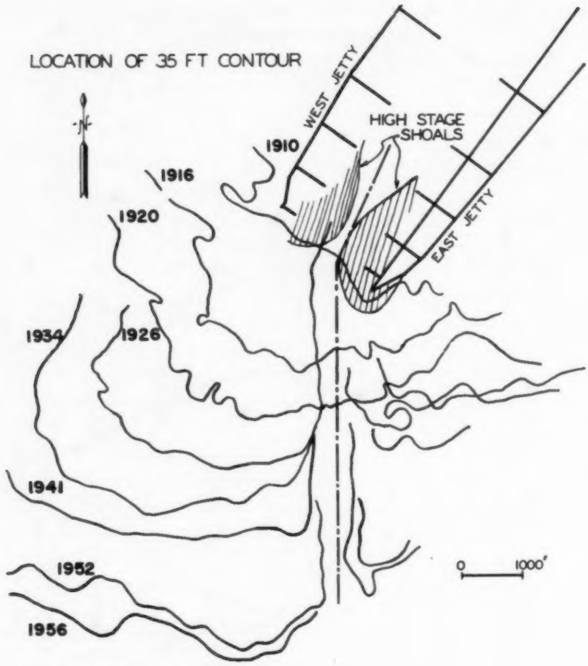

FIG. 8.—SHOALS AND CONTOUR ADVANCE

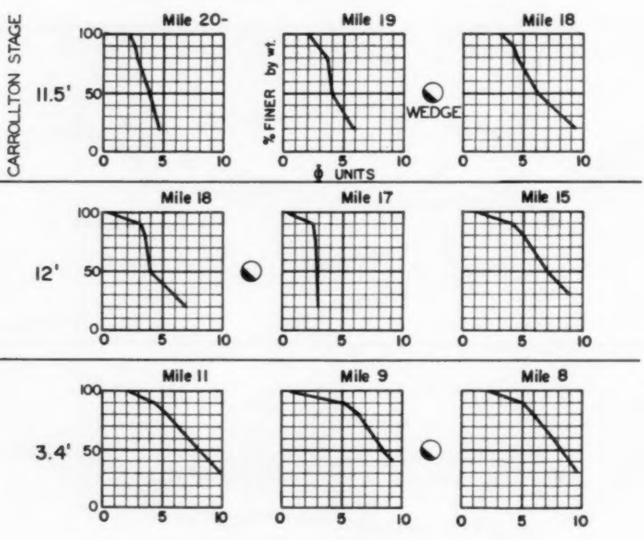

FIG. 9.—BED MATERIAL GRADATION

necessary to identify the major shoals and relate them to the hydraulic phenomena in process as they occur. The only method of locating shoals is by hydrographic surveys but this does not always provide a complete picture because some shoaling may possibly be obscured by rapid changes in river conditions, dredging activities, or incomplete records.

Fig. 8 shows the net advance of the 35-ft contour off the Southwest Pass. The bar channel, as indicated, has been utilized since 1921 when it was changed from a direct prolongation of the pass to a more easterly alignment. The predominant shoaling problem is that which occurs at high water stages. At these stages, large shoals will begin at the mouth of the jetty, close across the mouth and then progress upstream. Minor shoaling is apparent in the shallow area to the west of the channel and approximately 5,000 ft or approximately 3 mouth widths out from the mouth along the channel in a lunate bar formation.

At low water stages, the shoals form at the ends of the jetty and gradually extend inward to fill the channel. At medium stages there is a similar shoal formation at the mouth plus additional shoals 3,000 ft to 5,000 ft out into the gulf. These low and medium shoals are of minor importance from an economic maintenance standpoint.

THEORIES OF SHOALING

The most widely accepted theory on shoaling is that explained by E. A. Shultz and H. B. Simmons and concerns[4] the action of the salt water wedge as a barrier to bed load material. Basically this theory states that bed load material will deposit out when it reaches the tip of the wedge because the horizontal velocity will be essentially zero at this point and the viscous drag will cease moving the bed material. Lighter material will of course continue in suspension and be transported over the wedge and out to the gulf.

This theory seems to be well founded and the results of this investigation program illustrate this effect. Fig. 9 shows the gradation of material above and below the wedge for various stages. There is no heavy material so the analysis must be based on a percent composition basis. At the 11.5-ft stage, the wedge is positioned at approximately mile 18.5. Below the wedge the material is of a coarser percent gradation with median size of 4 phi units. Above the wedge there was a median size of only 6 phi units. At a 12-ft stage the wedge was at mile 17.5 and the material had a median size of 4 phi units below the 3 units above and further upstream. This indicated a larger percentage of heavier material adjacent to the wedge tip activity than upstream from it. At the low stage of 3.4 ft there was no difference in gradation at the wedge tip because of the fact that there is no bed movement and so there could be no significant change. The median sizes of 8.3 phi units above and below the tip indicate this.

It must be emphasized that the tip location was varying with tide in the gulf and river stage and was moving during the time required to take the samples above and below the area. Other plots of similar data indicate results similar to this example. Undoubtedly material carried over the wedge settles out and

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4 "Fresh Water-Salt Water Desnity Currents, A Major Cause of Siltation in Estuaries," by E. A. Shultz and H. B. Simmons, Tech. Bulletin 2, Beach Erosion Bd., April, 1952.

is transported back to the tip area by the upstream saline currents. This theory therefore adequately accounts for the heavy shoal formation at high stages.

Another theory for delta formation is given by C. C. Bates and J. C. Freeman, Jr. and is based[5] upon the emerging, less dense, hypopycnal jet dropping its load approximately 4 to 5 mouth widths out as it fans out over the denser saline waters. This would account for the formation of the lunate bar and undoubtedly explains a certain amount of shoaling. It does not account for the major deposits that occur within the channel and so, while a valid theory, applies mainly to material in suspension from natural streams and is not of prime importance in this dredged channel.

Another common theory that must be considered is that the shoaling is caused by the transition of the pass from a meandering stream with natural curves to a controlled, essentially straight section. This means that the bed changes from a bed with deep sections at the outer bend and shallow sections at the inner bend to that of a flat channel. The higher velocities in the bed curve tend to scour the bottom material, and the lower velocities, when the water reaches the flatter portion, cause deposition. There is a slight evidence of this type of action several miles up the pass where this bottom transition takes place, but this theory does not explain the heavy shoals at the mouth where the transition does not occur. This theory must therefore be dismissed as being relatively unimportant.

In any stream problem there must always be a discussion of the effect of the Coriolis acceleration theory. Actually this theory deals more with where the material is deposited rather than whether it is deposited or not. This theory states that because of the rotation of the earth, the acceleration of a body on it is with respect to a rotating axis and there is an additional accelerative force imposed upon it. In this northern hemisphere, this means that more material should be deposited on the west bank than on the east and that the delta should build more on that side. Historically this is true, but the effect of this acceleration is very small and is overshadowed by littoral forces that would cause the same effect. An examination of the complete distributary system of the Mississippi River reveals more evidence of deposition by the littoral effect and bifurcation due to sedimentation effect than due to this accelerative force. The Coriolis effect should theoretically deflect the effluent stream to the west with a radius of approximately 10 miles. Actually, the turning is much more rapid because of the momentum interchange between the stream and the gulf currents, which indicates the superiority of the littoral forces.

The minor shoals occurring at the eastern side of the pass can best be explained by the circulation phenomena which is of minor importance but which has not been considered. As the emerging jet leaves the mouth of the river, it is unconfined and will widen and decelerate. Because it is directed to the southwest, there is an attempted separation from the east jetty and a counter clockwise recirculation of currents back to this jetty. Sediment is therefore carried back where it drops into the saline underflow and is deposited at the end of the jetty. Examination of surface littoral currents indicates this re-

5 "Inter-Relations between Jet Behavior and Hydraulic Processes at Deltaic River Mouths and Tidal Inlets," by C. C. Bates and J. C. Freeman, Jr., Coastal Engineering, Vol. 3, 1952.

curvature and circulation about a point east of the east jetty. This is of minor importance but accounts for the minor shoals.

CONCLUSIONS

From the study of this pass, it is apparent that the salt water wedge theory is the most plausible explanation for the majority of the shoaling. Other theories have not been disproven by this study, but their contributions are shown to be of a minor nature.

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DISCUSSION

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DESIGN METHODS FOR FLOW IN ROUGH CONDUITS[a]

Closure by Henry M. Morris

HENRY M. MORRIS,[26] F. ASCE.—The writer is grateful to Messrs. Engel, Ackers, Bilonok, and Roberson for their thoughtful and helpful discussions of the paper. He is pleased that the concept of turbulence regimes as related to boundary roughness, suggested in this and an earlier paper,[12] is viewed favorably by each of the discussers. In the absence of comprehensive test results for specific verification purposes, their hesistance in accepting the particular equations and design curves for the respective flow regimes is understandable.

Before attempting to defend these equations, the writer wishes to affirm his high regard for the brilliant work of Colebrook and White[4] nearly 25 years ago. In pointing out inadequacies of the design procedure based on their studies, no criticism was intended of the individuals, or of their work as such. The same statement applies to the writer's comments on the equations of Manning, Hazen-Williams, and others. All of these have been, and are being, used successfully in design practice. The engineer should be familiar with the sphere of applicability of each of them. Because each is essentially an empirical formula, it may be legitimately applied for design purposes within the range of data for which it was derived.

In effect, use of an empirical formula in this manner is tantamount to design by actual test and is, therefore, highly accurate. But when the basic design data are uncertain, or when they are such as to require extrapolation, then use of such an empirical method becomes dangerous, unless applied judiciously by a competent designer.

This limitation is true of the Colebrook formula as well as the others, even though its range of applicability may be considerably greater. The most serious danger in its use is the widespread tendency to equate the equivalent sand diameter to the roughness height. This error is observed fairly frequently in hydraulic literature and is probably even more common in design practice.

As an illustration of the seriousness of a mistake of this type, one might consider the problem of estimating a friction factor for a 3-ft diam corrugated metal pipe, at a Reynolds Number of 500,000. If the roughness height of $\frac{1}{2}$ in. is assumed to be the equivalent sand roughness for use in the Colebrook formula, the latter will yield a friction factor of about 0.042. If the hyper-turbulence concept, with the corresponding curves, is used, the correct friction factor of about 0.070 is obtained. At fully normal turbulent flow, the friction factor for this pipe size approaches 0.090, more than double the value estimated from the Colebrook function using the corrugation height.

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[a] July, 1959, by Henry M. Morris.

[26] Prof. and Head, Dept. of Civ. Engrg., Virginia Poly. Inst., Blacksburg, Va.

[12] "Flow in Rough Conduits," by Henry M. Morris, Transactions, ASCE, Vol. 120, 1955, pp. 373-410.

[4] "Experiments with Fluid Friction in Roughened Pipes," by C. F. Colebrook and C. M. White, Proceedings, Royal Soc. of London, Vol. 161, 1937, p. 367.

Mr. Roberson has criticized the writer's exposition of the theory of hyper-turbulent flow, one of the main aspects of which is that it approaches fully normal turbulent flow at sufficiently high Reynolds Numbers, regardless of the wall roughness type. At this final condition, the spacing of the roughness elements becomes the sole variable determining the friction factor. The implication, questioned by Mr. Roberson, is that the wakes behind the elements are then geometrically similar for all types of elements.

The attainment of full geometric similarity of wakes may be uncertain in the absence of specific measurements, but a general approach toward similarity, whether caused by adjustment of separation points or by some other phenomenon, seems certain to take place as the inertial aspects of the flow increase and the wall zone of hyper-turbulence is compressed.

Even if the separation zones do not become fully similar, the "hyper-turbulence" generated in the wakes is gradually broken up into "normal turbulence" as the vortices are propelled away from the wall. At sufficiently high Reynolds Numbers, the thickness of the zone of hyper-turbulence approaches zero. Normal turbulence (with its implied statistically normal distribution of turbulence parameters) then prevails throughout the flow, with all "hyper-turbulent" vorticity distintegrating into "normal" vorticity immediately after generation.

Mr. Ackers and Mr. Bilonok understandably desire that the equations and curves proposed in this paper and the previous paper should be verified by more experimental data before adoption in design practice. The writer, in 1950, made such studies on all pertinent data that were then available, concluding that the equations were quite adequately substantiated. Publication of the analyses of these data, in either of the papers, was precluded by space limitations, but they are available[27] for any who wish to review them.

The writer is at present (1960) engaged in a similar study of other experimental data that have been obtained since that time. Although these analyses are not yet completed, preliminary results indicate that they likewise can be adequately referenced within the framework of the proposed concepts and formulas. A systematic experimental study devoted specifically to the elucidation of these questions is, of course, still needed, but the evidence already at hand indicates the proposed method to be more nearly rational and more widely applicable then any previously used.

Mr. Ackers questioned the writer's interpretation of the St. Anthony Falls tests on concrete culvert pipes[5] and the tests of Burke on large penstocks[8] as confirming the equation of semi-smooth turbulent flow. It appears, however, that he has misunderstood the implications of this equation. Contrary to the Colebrook-White function, the semi-smooth flow equation implies that the friction factor decreases indefinitely with increasing Reynolds Number (a fact strikingly confirmed by Burke's results) and depends otherwise only on roughness element form and dimensions, not on pipe radius (a fact supported by the concrete culvert pipe tests that were made on very smooth pipes, with the joints providing the "isolated-roughness" elements).

Mr. Engel asks further elucidation of the relation between the Prandtl equation and the writer's suggested equation for normal turbulent flow. The equa-

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[27] "A New Concept of Flow in Rough Conduits," by Henry M. Morris, thesis presented to the University of Minnesota, Minneapolis, Minnesota, 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[5] "Hydraulic Tests on Concrete Culvert Pipes," by L. G. Straub and Henry M. Morris, St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn., 1950.

[8] "High-Velocity Tests in a Penstock," by Maxwell F. Burke, Transactions, ASCE, Vol. 120, 1955, pp. 863-896.

tions are essentially the same except that the latter replaces the sand-grain diameter by the roughness spacing. The two dimensions are presumably the same for pipes densely coated with sand grains, so that the two equations are equivalent for this type of rough boundary.

For other types of roughness, producing hyper-turbulent and normal turbulent flow, however, the writer's contention is that the roughness spacing (rather than the roughness height) is the correct dimension to use. This permits the same equation to be used for sand-coated boundaries, for corrugated metal boundaries, and for other types of roughness yielding the "wake-interference" boundary phenomena.

The roughness height has, therefore, no direct effect on the turbulence structure when the condition of statistically normal turbulence is attained. The value of Reynolds Number at which this condition is reached, however, with the flow changing from "hyper-turbulent" to "normal turbulent," does depend on the geometric form of the boundary roughness elements.

The roughness height also has an indirect effect, in that it determines the effective diameter of the flow. For this type of flow, the regions between roughness elements are largely occupied by wakes, with the actual flow occurring above them. Computations of friction factor, therefore, should be based on the crest-to-crest diameter. If laboratory measurements are used to compute friction factor data and this is not done, the computed friction factors will be too high, in proportion to the fifth power of the diameter. That is,

$$f_c = f_w \left(\frac{D_c}{D_w}\right)^5 = f_w \left(1 - \frac{e}{r}\right)^5 \quad \dots\dots\dots\dots\dots \quad (19)$$

in which f_c is the friction factor as computed on the basis of the crest-to-crest diameter D_c, and f_w is that computed from the wall-to-wall diameter D_w.

This error is small in pipes of small relative roughness, but may be quite large if the relative roughness $\frac{e}{r}$, in Engel's notation is large. If the friction factors of the Mōbius curves are recomputed on this basis, it will be seen that the apparent influence of relative roughness, stressed by Mr. Engel as in conflict with the writer's equation, very largely vanishes. This is shown in Fig. 11.

The four Mōbius curves, shown by heavy lines in Engel's Fig. A, are here shown as dashed lines. The two solid-line curves represent the topmost and lowermost of these curves recomputed on the basis of the crest-to-crest diameter (the two intermediate curves are not shown because of the excessive crowding that would be necessary).

Therefore, instead of refuting the implications of the writer's equations, the Mōbius data appear to provide excellent further confirmation of them. It appears, too, that Mr. Engel was mistaken in identifying the flows represented by these curves as hyper-turbulent. They are more properly identified as semi-smooth turbulent flows, as indicated by the fact that the friction factor decreases with increasing values of $\frac{L}{e}$ $\left(\text{or } \frac{\lambda}{h} \text{ in the writer's notation}\right)$.

This may also be shown by use of the writer's "boundary function" curves, in his Fig. 10. Mr. Engel perhaps misunderstood these curves and used the wall-to-wall radius instead of the crest-to-crest radius. If the latter is used, and a drag coefficient for the circumferential rings of 1.9 is assumed, the critical value of $\frac{L}{e}$ is found to be 4, when $\frac{e}{r}$ is 0.22 $\left(\text{that is } \frac{e}{r_o} = 0.28\right)$ and 10 when $\frac{e}{r}$ is 0.03 $\left(\frac{e}{r_o} = .031\right)$. If these two values (for the highest and lowest values of

Mōbius' relative roughness) are averaged, one can say that for this type of roughness, the critical spacing discriminating between hyper-turbulent and semi-smooth flow is approximately 7e.

At this critical spacing, the writer's equations show that the friction factor has its maximum value for the given type of roughness element. This inference is satisfactorily confirmed by Mr. Engel's discussion of the data of Mōbius, Nunner, and Koch, that similarly showed the critical spacing to be at about 7e or 8e.

In light of the preceding analysis, Mr. Engel's Table I, comparing Mōbius' friction factors with values computed from the writer's equation of normal turbulence, is seen to be irrelevant. The flows were not characterized by normal turbulence, or even by hyper-turbulence, but were rather semi-smooth turbulent flows.

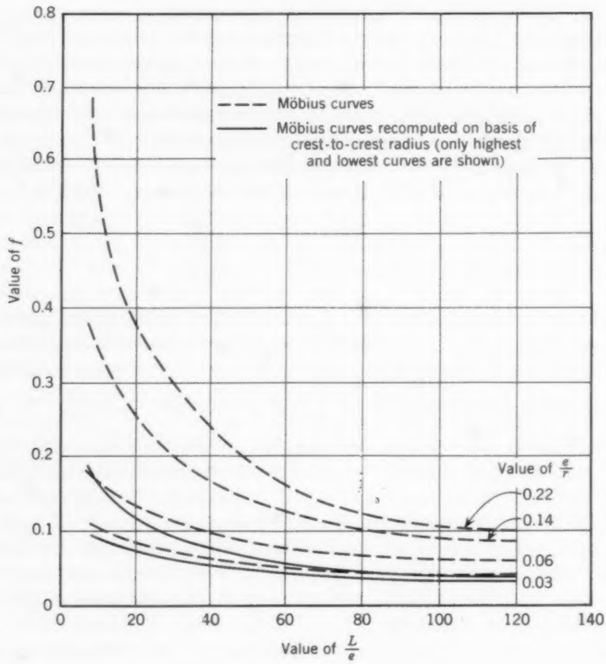

FIG. 11.—f VERSUS L/e

It is impossible to compute values for exact comparison because the value of Reynolds Number for the Mōbius curves was not given by Mr. Engel. However, it will be noted from Fig. 11 that the range of friction factors, when computed on the basis of crest-to-crest diameter, was only from about 0.04 to 0.18, even for the pipe of largest relative roughness. The element characteristic E $\left(= C_D \frac{p}{P} \frac{h}{\lambda}\right)$, for use in the semi-smooth equation, is in this case 1.9 (1) $\frac{e}{L}$.

Assuming a Reynolds Number of about 100,000, and an $\frac{L}{e}$ value of 100, so that E = 0.019, the writer's Fig. 5 then indicates a friction factor of 0.04, that

is exactly the same value indicated in Fig. 11 from the Mōbius curves. Similarly, at $\frac{L}{e} = 20$ and a Reynolds Number of 100,000, the semi-smooth equation gives $f = 0.13$ and the Mōbius data give f between 0.08 and 0.12. More exact correspondence would be obtained at a Reynolds Number of 300,000.

It is, therefore, concluded that the Mōbius data provide both qualitative and quantitative confirmation of the writer's equations. Because the Nunner and Koch tests were in line with those of Mōbius, it is clear that they also would verify these concepts.

Mr. Engel is correct in suggesting that, for large values of $\frac{r}{L}$, the value of $\frac{e}{r}$ would have to be correspondingly small $\left(\text{such that } \frac{e}{L} \leq 1\right)$ in order to prevent the hyper-turbulent flow from becoming quasi-smooth flow, that is, for the extreme case when $\frac{r}{L} = 5000$, $\frac{e}{r}$ should be less than $\frac{1}{5000}$ if hyper-turbulent flow is to prevail (assuming the roughness elements to be of infinitesimal width). This is, of course, quite feasible; the Moody curves have, for example, usually been plotted for a range of relative roughnesses extending to much lower values than this.

The curves representing hyper-turbulent flow were not extended to values of $\frac{r_o}{L} < 1$, however, because usually semi-smooth flow will prevail under these conditions. This is evident from the trends indicated on the discrimination function curves of Fig. 10.

The writer does not as yet see any need to specify a maximum value of e/r for which the functions are applicable, other than those already implied by the limiting conditions for the various flow regimes. The demarcation noted by Mr. Engel between the two types of resistance characteristics at about L/e = 8 is not related to any limiting e/r but, as discussed previously, represents the critical spacing distinguishing hyper-turbulent from semi-smooth flow, for the given roughness elements.

By the same token, it is unrealistic to suggest two different flow mechanisms for small and large orifice area ratios, respectively. In both cases, the basic phenomenon is one of flow separation, wake formation, generation of vorticity at the interface, and shedding of that vorticity from the wake into the main flow.

In conclusion, the writer would reiterate his appreciation to the discussers for their stimulating contributions to this paper. He heartily agrees with them that these are complex phenomena, well deserving of more intensive and systematic study than has yet been devoted to them. It is hoped that these new concepts will contribute materially to a more rational understanding of them, as well as to more effective design practices. The encouraging qualitative and quantitative confirmations indirectly brought out by the discussions should be of significant help in creating confidence in the concepts and in the functions embodying them.

ROLL WAVES AND SLUG FLOWS IN INCLINED OPEN CHANNELS[a]

Closure by Paul G. Mayer

PAUL G. MAYER,[43] M. ASCE.—Considerable interest in the author's study was reflected by the elaborate reviews and discussions. The discussers presented veritable studies of their own and their writings merit recognition as valuable contributions to the engineering annals.

Whatever differences of opinion were brought out had their origin either in the respective approach to the study of roll waves and slug flows or were a consequence of results obtained in the laboratory. Careful experimentation does not forstall results that are much dependent upon scale effects and limitations of equipment and techniques.

The study reported by the author was intended much more to elucidate the basic phenomena involved in unsteady open channel flows than to arrive at formulas immediately applicable to hydraulic engineering problems. The size of waves studies was, indeed, diminutive when compared to some waves encountered in steep open channels. They were, however, not of purely academic dimensions, but represent, at once, fluid mechanics problems encountered in sheet flows such as rain waves on inclined streets, condensate flows, paint applications, and many others.

Mr. F. Escoffier rightly pointed to his own mathematical analysis of waves of instability and presented a rigorous mathematical treatment. His reference to proto-type waves was interesting and made clear that actual wave magnitudes can be considerably in excess of those observable in the laboratory channel.

Messrs. Taylor and Kennedy similarly point to proto-type waves and question the author's conclusion as to the origin of roll waves and slug flows. Also, there is no question of the veracity of the original author's observations, it is indeed agreed that unsteady flows of the type discussed occured in flows definitely not laminar. Cornish's observations and Jeffrey's study illustrate this point.

The writers questioned also the use of the Airy equation. Since the initial disturbances are indeed quite small they satisfy the restrictions imposed on the wave equation by shallow water. The application of this equation was not implied to the terminal or fully developed roll waves and slug flows. Their Eq. (ii) for waves of finite magnitude may well be a suitable approximation for the propagation speed of roll waves and slug flows.

The writers are correct in stating that the relationship between Froude Number, Reynolds Number, and slope of Eq. 19 and shown in Fig. 18 represent plots analogous to pipe friction factor diagrams. It was thought desirable, however, to refrain from emphasing this obvious relationship and thus let the study become a mere testing program.

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[a] July, 1959, by Paul G. Mayer.

[43] Assoc. Prof. of Civ. Engrg., Georgia Instit. of Tech., Atlanta, Ga.

The writers also analyzed the author's data in terms of Binnie's study. Their conclusions proved interesting. The footnote in respect to chronological precedence was appreciated but it is hoped that progress does not depend first upon proper accreditation.

Messrs. Ishihara, Iwagaki, and Iwasa have presented information pertaining to their own laboratory experiments and subsequent analysis. Their publications in English were known to the author and had been consulted. Their numerous Japanese writings in this field, however, were beyond the linguistic skills of the author.

Their commentaries on the author's study were neatly summarized in 16 items. In the subsequent remarks the pertinent items are invoked, accordingly.

Item 1, whatever merit their comments may have in regard to some wave phenomena in hydraulic engineering, the influence of surface tension on the formation of roll waves was real and significant in the author's laboratory study. As proof, the addition of a surface active agent and hence the reduction of surface tension prevented wave formation under conditions normally leading to roll waves.

Item 2 appears more of a problem in semantics. Unsteady flows could be obtained in the region in question if the flow was externally disturbed. Transitional problems are generally known to depend, to a large degree, on external influences.

Roll waves and slug flows, as described by the author, were distinctly different phenomena. The designations may seem somewhat misleading because of prior usage. Very little gain is envisioned in the suggestion of an all descriptive designation and by putting them back into one basket.

Item 5 and 7 could be referred to Item 1. The author fails to appreciate the writers interpretation of his conclusions. It appears that a modified Darcy-Weisbach relationship would suit their ends and no heed needed to be paid to free surface phenomena. In replying to Messrs. Taylor and Kennedy, the author has already stated his sentiments.

Items 9 and 10 are related to Item 3. Referring to the equations, criteria, and data mentioned the writers overlooked the fact that in the author's laboratory experiments supercritical laminar flow was existent.

The previous commentators and the author are agreed that wave phenomena in hydraulic engineering may differ in magnitude, mode of origin, and other ramifications. From the foregoing discussions pertaining to the author's study one may conclude that considerable effort has been exerted in order to elucidate the phenomena of fluid flow in open channels. The exchange of scientific data and findings across international boundaries and over language barriers can be considered a significant aspect of this paper.

CONSISTENCY IN UNITGRAPHS[a]

Closure by Bertram S. Barnes

BERTRAM S. BARNES,[1] F. ASCE.—The writer is indebted to Mr. Clark not only for his excellent comments but equally for his support of the writer's basic argument: the need for more systematic study of the behavior of runoff from natural basins. Some questions have been raised and a few points of disagreement are noted. The writer, because of his absence from the country at the time, unfortunately could not provide certain specific information that Mr. Clark wished to have before preparing his discussion.

The stage-discharge relation at Rangamati is well defined by a large number of current-meter measurements. Hourly readings of the gage were made during daylight hours. These readings clearly define the minor tidal effect at low water, which in any case becomes negligible when 24 hr is used as the time unit for discharge. Levels that were run to observed flood marks along the main river show that the slopes of the high-water profiles are about the same in the lower reach, from Barkal down to the Rangamati gage as in the reach above Barkal to the headwaters. It seems likely that the odd manner in which certain major tributaries enter the main stem from the right and the left, in pairs, is the cause of the characteristic double peak of the unitgraph. Here a question is in order: How uncommon, actually, are multiple-peaked unitgraphs? By using a precise method of derivation, might we not find them much more often?

A matter of greater interest to the writer is the range of the elapsed time between the two peaks of the unitgraph; from about 26 hr for an excess of 3.4 in. in three days to roughly 100 hr for an excess of 17.2 in. in four days. Because in a larger flood, both peaks are retarded as well as spread apart, the effect can hardly be explained by Seddon's law.[2] It seems more likely that it results from pondage other than that in the main channel. The flood wave in the main stem may actually reverse the direction of flow in the lesser tributaries so as to impound a fairly large part of its volume in each ravine that it crosses, thus both retarding and lengthening the wave. Mr. Clark has suggested that the Karnafuli may not be truly a unitgraph stream. Certainly it is not a typical one. However, its discharge hydrographs can be reconstructed from a unitgraph if a variable lag directly related to the total flood volume is applied to adjust the unitgraph. The question then arises: Is the variable lag peculiar to the Karnafuli or is it merely more prominent in that basin?

It is a fairly common practice to steepen a unitgraph arbitrarily when it is to be applied to a rainfall sequence of great intensity. The theory is that more intense rains are likely to generate a higher percentage of overland flow and

[a] August, 1959, by Bertrom S. Barnes.

[1] Head Hydrol., Internatl. Engrg. Co., Inc., San Francisco, Calif.

[2] "River Hydraulics," by J. Seddon, Transactions, ASCE, Vol. 43, 1900, pp. 217-229.

therefore should produce steeper unitgraphs. The peak of the unitgraph is often raised 20% to 25% as a margin of safety, which in effect shortens the lag. In the Karnafuli, however, the lag increases in the greater floods. Both peaks of the unitgraph are lowered and the elapsed time between them is greatly increased.

The apparent close relationship between the unitgraph lag and the total flood volume merits further study, especially to determine whether a similar effect (or an opposite one) can be detected in other basins. For this purpose, unitgraphs must be derived, not from observed physical features of the basin nor from estimates of rainfall and retention, but uniquely from the streamflow hydrographs themselves. Unitgraphs that do not fit the record with unmistakable precision are hardly consistent enough to define small variations in their lag. This statement in no way overlooks the obvious merit of those procedures, such as Mr. Clark's, that do not require the complete analysis of an actual flood hydrograph and can be used in the derivation of design floods for poorly gaged or ungaged basins.

Mr. Clark is quite correct in saying that the rainfall was not uniform over Karnafuli Basin during the four-day rain of July-August, 1947. Nevertheless it probably maintained the semblance of a standard pattern. During the rainy season, masses of air with a consistently high dewpoint are propelled by the monsoon across the series of high transverse ridges that characterizes the basin. In the mountain zones of California a rather similar situation occurs during the winter months, and heavy rains there are distributed in a characteristic orographic pattern. It is logical, then, to expect that Karnafuli Basin also has its own rainfall pattern, and the highly satisfactory definition of the unitgraphs derived by the writer lends support to that belief. Of the eight floods selected for analysis, two showed almost perfect agreement of their unitgraphs from the forward and reversed runs. Four others were regarded as "excellent" or "good" and the remaining two, although the agreement of their runs was only "fair," were yet consistent enough to prove the correctness of their rainfall sequences beyond any doubt. All eight would have given excellent reproductions of the hydrographs from which they were derived, but because of the large variations in their lag, they could not be applied to other floods without adjustment.

The writer seriously doubts "time of concentration," as ordinarily defined and applied, is the most satisfactory basis for relating basin characteristics to unitgraph lag-time. The concept seems to have originated with the "Rational Method" for the design of storm sewers, in which the idealized contributing basin was considered to be a circle or circular segment having a uniform slope along the radii toward a catchbasin at the center. It was assumed that the greatest concentration of flows at the outlet would occur as soon as the perimeter began to contribute water to the outflow. When applied to a natural basin, the time of concentration is the estimated length of time required for a drop of water to travel from the tip of the most remote panhandle to the point of outflow. Obviously, the physical significance of the term has been lost in its translation from a city sewer system to a river basin. The writer is convinced that much better parameters could be developed for estimating the lag if there were available a large stock of unitgraphs of which the lag, by the definition of Horner and Flynt, could be determined precisely.

Mr. Clark's comment that the development of a unitgraph from a compound event is "a mathematically indeterminate procedure in principle, capable of

indicating obvious errors" is perfectly true of the older procedures of this type. In the present case the statement must be challenged, because the writer's method is designed to define the unitgraph uniquely and to prove the accuracy of the rainfall sequence. The basic principle is the solution of two sets of normal equations, with the numbers representing the excess rainfall sequence being treated as trial constants. In the forward run there are as many equations as unknowns and the same is true of the reversed run. These two sets of equations must be solved independently. To combine them would establish the trial rainfall sequence as a part of the basic data. The unitgraph then would not be uniquely defined and the reliability of the trial sequence would remain in doubt.

Each pair of runs, then, has only two possible results; either the two derived unitgraphs will differ or they will agree. In the latter case, the ratio of the rainfall numbers will have been proved correct. With that sequence no other unitgraph can be derived, and only that sequence or a multiple of it can bring the two runs into coincidence. Variations of the rainfall pattern and errors in the discharge record and in the separation of the base of flow, of course, will always prevent a perfect agreement of the two runs, and the correct unitgraph will lie between the two. However, any actual bias from this source will be negligible or nearly so, unless the mutual fit of the two runs in extremely poor. No false semblance of accuracy will have been created.

The oscillation described by Mr. Clark is an outstanding defect of the earlier procedures, in which the swings were reduced in amplitude by makeshift adjustments, most of which introduced error into the results. In the writer's procedure the oscillating run is superseded, but never adjusted. As the correct sequence is approached, the oscillations disappear automatically and no error from that source can affect the results. Because the amplitude of swing is related to the amount of error in the trial sequence, the oscillation itself helps to point the way to the correct solution. A good example is seen in the first four frames of Fig. 3.

Mr. Clark calls attention to the difficulty of obtaining from protracted compound events, unitgraphs for unusually short unit durations. It is difficult, by any procedure, to handle trial rainfall sequences of more than four or five terms. However, a unitgraph first derived with a four-term sequence can be treated, in turn, as a compound event and another unitgraph derived from it. It is necessary to complete the outline of the first unitgraph by sketching, in order to obtain the necessary intermediate points from the second breakdown. After the resulting, or second, unitgraph is derived it should be checked and refined by applying its rainfall sequence directly to the original net flood event. Another method is to apply the first unitgraph, with all but one term of its rainfall sequence, to reconstruct the major part of the original net event. The remainder of the event can then be used to derived a unitgraph for a fractional duration. A third method, simpler but possibly less accurate, is to reduce the first unitgraph to the dimensionless form described by the writer, and then reconvert it to dimensioned form for the shorter duration. The well-known "S-curve hydrograph"[3] also is often used for that purpose, but the results are likely to be inaccurate because of the severe oscillation that occurs with that method.

The writer has never found any merit in the distribution graph, except possibly for the computation of lag. The distribution graph is not dimensionless except with respect to its runoff. Because it has a fixed lag in hours it cannot

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[3] "Channel Storage and Unit Hydrograph Studies," by W. B. Langbein, Transactions, AGU, 1940, Part II, pp. 620-627.

be used to adjust a unitgraph to another river point or to a longer or shorter unit duration. Its basic principle, of course, is employed in the first step of the conversion of a unitgraph to dimensionless form.

Mr. Clark points out the significance of the recession factor with respect to one of the two storage-discharge functions used in routing his instantaneous unitgraph. Actually, Table 1 shows this relationship, since the storage factor is identical with his recession-period storage function. The writer, however, prefers to use individual factors for the recessions of ground-water flow, interflow and overland flow, respectively. Mr. Clark treats the storage factor as a measure of the valley-storage characteristics of the stream, by which the instantaneous unitgraph is converted into a unitgraph of outflow. In that way he has derived a unitgraph for Karnafuli River that would have given a reasonably good reproduction of at least three of the observed floods, having runoff volumes in a range from about 8 to 11 in. of excess rain. However, the lag of the writer's unitgraphs for Karnafuli River varies from 45 hr for a flood runoff of 3-1/2 in. to 105 hr for a runoff of 17 in., there being no apparent change in the recession factor for interflow and none, therefore, in the recession-period storage-discharge function. This phenomenon seems as hard to explain in the simple terms of open-channel storage as the large difference, found in nearly all streams, between the recession factors for interflow and for overland flow.

Previous to the Karnafuli study, the writer no doubt would have agreed with Mr. Clark that unitgraphs should be derived from multiple-unit events only as "a necessary expedient." However, nearly all unitgraphs are derived for the express purpose of synthesizing multiple-unit events. If observed unit events had been available for the Karnafuli study and no analysis of compound events had been made, the results obtained would almost certainly have been misleading. The writer now believes that multiple-unit events should be selected as source data for the derivation of unitgraphs, in preference to unit events or in addition to them, simply as a matter of sound practice.

SPILLWAY DESIGN FOR PACIFIC NORTHWEST PROJECTS[a]

Closure by Marvin J. Webster

MARVIN J. WEBSTER,[1] F. ASCE.—The author is greatful to Messrs. Michel and Gagnon for contributing to the discussion of this important subject. The information they have added relative to Mr. Lemoine's work is appreciated.

The cavitation factor referred to by the writers and proposed by Mr. Lemoine should prove to be very useful; not only in spillway design but also in the design of any water passage where danger of cavitation occurs. The present policy of the United States Army, Corps of Engineers in the design of hydraulic structures is to limit negative pressures to -20 ft of water. This would give a cavitation factor considerably lower than the value 0.38 obtained by the writers from the results of model studies on Chief Joseph spillway.

More data and observations are needed, especially on prototype structures, to determine the limit of negative pressures that will not produce damage from cavitation. It is hoped that piezometers can be installed in several new prototype structures so that data can be obtained and observations made to add to the very little information available at the present time.

It is interesting to note that extensive model studies were made on ogee type spillways during the design[2] of Banas Dam, Dantiwada Project, Bombay State, India. Pressures slightly below atmospheric were observed on the crest during model studies with maximum head simulated.

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[a] August, 1959, by Marvin J. Webster.

[1] Chf., Hydr. Design and Lab. Section, U. S. Army Engr. Dist., Portland, Corps of Engrs., Portland, Ore.

[2] Central Water and Power Research Station Poona, Annual Research Memoirs, 1959, Government of India.

FLOOD CONTROL ASPECTS OF CAUCA VALLEY DEVELOPMENT[a]

Closure by Phillip Z. Kirpich and Carlos S. Ospina

PHILLIP Z. KIRPICH,[1] F. ASCE, and CARLOS S. OSPINA,[2] F. ASCE.—Señor Julio Escobar-Fernández has provided an excellent summary of the principles necessary to ensure success of regional development programs. These are days of intense competition between the two great powers for the loyalty and support of the peoples of the underdeveloped nations. The recognition and understanding of these principles by engineers and others engaged in the planning of development programs is therefore a matter of primary national concern.

As the first program of its type in Colombia, the Cauca Valley development is now receiving close attention on the part of the National Government and of international lending agencies. In particular, the Planning Unit of the Central Government, mentioned by Señor Escobar-Fernández, is giving great emphasis to the program with the aim of setting it up as a model for similar regional programs elsewhere in Colombia.

A year and a half has passed since the original paper was written. It is of interest to report that the flood-control and drainage works completed thus far, at the Aguablanca and Roldanillo-La Unión-Toro pilot projects, have demonstrated their value and this is overcoming landowner skepticism and opposition. The traditional attitude among farmers has been one of suspicion towards the costly engineering planning required. Now, slowly, this is changing toward a recognition of the necessity of engineering investigations. This is most gratifying to the engineers who have spent several years on this work.

Of the various hydrologic problems treated in the original paper, Mr. Gordon R. Williams has discussed the spillway design flood for the Timba project. Two key features are: value of 5-day precipitation and relation between 5-day precipitation and peak. With respect to the former, Mr. Williams questions the writers' method of determing the maximum probable precipitation, by extrapolation of an 18-yr rainfall record up to a frequency of 1000 yr. The writers believe Mr. Williams has a valid point. Actually, they followed this procedure because of lack of data and because no means were available to obtain data from other, similar climatic regions. Fortunately, however, there is time to attempt to improve the procedure. CVC's first major hydroelectric project, Calima, is now entering the construction stage. Timba is not programmed to start before 1964. Before design of the Timba spillway is finalized, additional and further data on the relation between peak daily discharge and 5-day precipitation will be obtained. Some additional data on rainfall has already been obtained. This data, plotted in Fig. 11, is from study of all available daily rainfall records for southwestern Colombia. The longest record, at Bogotá, is for

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[a] September, 1959, by Phillip Z. Kirpich and Carlos S. Ospina.

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[2] Partner, OLAP, Bogota, Columbia.

74 years. The maximum 5-day precipitation, as a percentage of the mean annual, is 16%. It seems probable that the flood finally adopted for design of the Timba spillway will have a smaller peak but a greater volume. The net effect on structural dimensions will of course have to be reinvestigated.

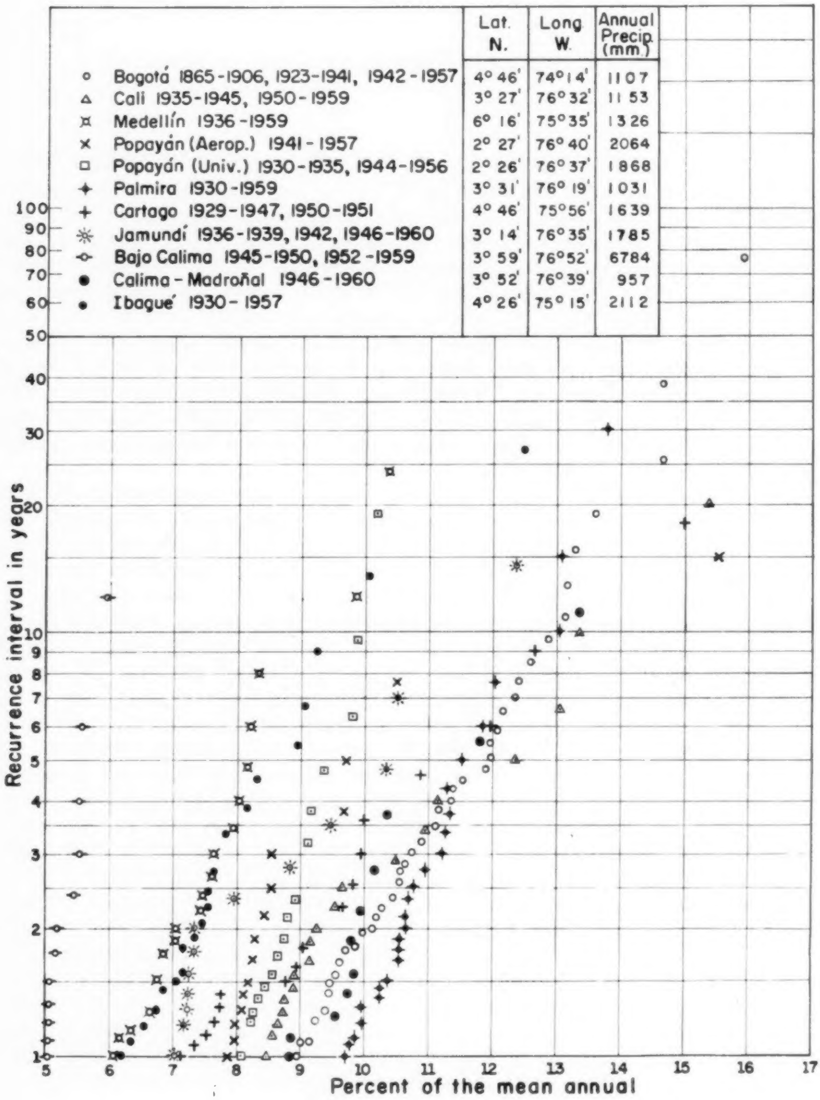

FIG. 11.—FIVE-DAY PRECIPITATION

FRICTION FACTORS IN CORRUGATED METAL PIPE[a]

Closure By M. J. Webster and L. R. Metcalf

M. J. WEBSTER,[1] F. ASCE, AND L. R. METCLAF,[2] F. ASCE.—Mr. Diskin has introduced values of equivalent pipe roughness computed from maximum values of the friction coefficient shown on Fig. 17 of the writers' original paper. The writers had computed values of the equivalent roughness (ϵ) for all of the data taken with full-pipe flow in the 3-,5-, and 7-ft-diam corrugated metal pipes, but did not include these values in the original paper. Values were found to range from 0.128 to 0.188 for the 3-ft pipe, 0.166 to 0.199 for the 5-ft pipe, and 0.155 to 0.189 for the 7-ft pipe. Values of ϵ were found to vary both with the Reynolds number, R_e, and with pipe diameter, thus indicating that the corrugated metal pipe type of roughness is not completely described by height alone.

The data on standard and helical corrugated pipe that were presented by Mr. Chamberlain are a welcome addition to the original paper. It is of particular interest to note that the data for 12-in.-diam standard corrugated pipe confirms the extrapolation of curves showing the values of "f" and "n" on the writer's Fig. 17.

Mr. Bilonok refers to Manning's "n" as a coefficient of absolute roughness. To avoid confusion with other measures of roughness, the writers prefer to think of "n" as a roughness coefficient. Mr. Bilonok states that values of "f" computed from his Eq. 6 are in agreement with those obtained by the writers. This agreement is not surprising as he has merely reversed the computation made by the writers. The writers first computed "f" and then used those values to compute "n". Mr. Bilonok has taken the values of "n" and computed "f" using the same relationships as used by the writers in their computations.

Mr. Bilonok states that "The exact form and position of "f" versus Reynold's number for a given pipe diameter and roughness is not clear (Fig. 11)...." He evidentally is speaking of values outside the range of Reynold's numbers tested, as the form and position of "f" within the range of Reynold's numbers shown on Fig. 11 seem clear enough. He also says that the writer's Tables A and B indicate that the Reynold's number varies as velocity of flow only and that the water temperature changes have no significance. Actually, all values of Reynold's number were computed using the proper value of kinematic viscosity for the water temperature observed. The significance of temperature is minor only because of the relatively narrow range of temperature observed.

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[a] September, 1959, by Marvin J. Webster and Laurence R. Metcalf.

[1] Chf., Hydr. Design and Lab. Section, U. S. Army Engr. District, Portland, Corps of Engrs., Portland, Ore.

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Mr. Ree has presented a method of computing composite Manning's "n" values for paved-invert corrugated metal pipe which should prove very useful where the percentage of paving is other than that for which data are available.

The writers wish to thank all of the discussers for their comments and for the additional data which have been contributed. It is hoped that the tests on which the writer's paper were based maybe be extended by someone to cover a wider range of Reynold's number.

REVISED COMPUTATION OF A VELOCITY HEAD WEIGHTED VALUE[a]

Closure by Joe M. Lara and Kenneth B. Schroeder

JOE M. LARA,[1] M. ASCE, AND KENNETH B. SCHROEDER,[2] M. ASCE.—Messrs. Aldridge, Benson, and Peterson all comment on the matter of facilitating the computation of the alpha coefficient. The writers offer no objection to such a procedure. It is pointed out, however, in Table 1 of Mr. Benson's discussion the sum (374.3) of Column 8 probably should have been shown as a separate entry to complete the data necessary for the alpha computation.

Although it was not the authors' intention of presenting anything in the way of a "new" or "revised" formula as apparently interpreted by Mr. Anand, his proof that the old and the new formulas are essentially the same is of mathematical interest. He also misunderstood that another objective of the paper was to illustrate the entire cycle of computations required for a slope-area determination of the discharge and not just to show only an alpha computation. Mr. Anand's idea of applying the value of K equal to 1.486 a $r^{\frac{2}{3}}$ to hydraulic computations is excellent although it is the authors' express hope that he yields to the use of K_d for some of the slope-area computations particularly for cases as used in the example. In this example only the water surface elevations at the cross sections were observed and it was necessary to assign an "n" value to arrive at the estimated discharge. The use of Mr. Anand's K value is ideal if several discharges are to be determined.

Messrs. Benson and Peterson introduced the formula for a direct solution of the discharge by using the same basic data as observed for the example. Such a formula does eliminate the need of a trial and error computation which heretofore had been used by the writers. An error was committed in the analysis of Bernoulli's Theorem regarding eddy losses in a contracting reach. The correct formula for friction head is

$$h_f = f + 1.10 \left(h_{v_1} - h_{v_2}\right)$$

The writers gratefully acknowledge both Mr. Bitoun and Mr. Peterson for pointing this out.

The writers agree with Mr. Kolupaila that determining the Coriolis coefficient is a nebulous affair. Although the 1.19 value of alpha appears low as compared to those quoted in Mr. Chow's book, it is still a common approach

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a September, 1959, by J. M. Lara and K. B. Schroeder.

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2 Hydr. Engr., U. S. Bur. of Reclamation, Federal Center, Denver, Colo.

to arrive at the value by this means. The coefficients quoted in Mr. Chow's book range from 1.50 to 2.00 for overflooded channels. However, it is the opinion of the writers that the supporting data used in the analysis to arrive at these values were insufficient as a check of Mr. Kolupaila's original work in this field[3] showed the hydraulic information was available for only one river. Mr. Kolupaila recommends the procedure of taking more point velocities along each vertical to make a more realistic determination of the Coriolis coefficient. This procedure is fine, but the data gathered for many of the slope-area computations do not include such observations generally because the hydrographer was not present during the occurrence of the peak flow.

The writers thank Mr. Kolupaila for calling attention to the typographic omission noted under 10, page 72, ΔQ is correct instead of Q appearing in the numerator.

Mr. Steinberg's two methods of determining the velocity head were also interesting. It is noted, however, that in Column 5 of Table 1, the value for Q 8.03 should be 1,348 instead of 134.8. Mr. Steinberg may also find that the more simplified formula presented by Messrs. Benson and Peterson is better suited to slope-area computations.

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3 "Methods of Determination of the Kinetic Energy Factor," by S. Kolupaila, Port Engineer, Vol. 5, No. 1, 1956, pp. 12-18.

EDDY DIFFUSION IN HOMOGENEOUS TURBULENCE[a]

Closure by G. T. Orlob

G. T. ORLOB,[50] M. ASCE.—The discussions have each added significantly to a clearer understanding of eddy diffusion through application of techniques of fundamental fluid dynamics, by more illuminating mathematical treatments of the basic relationships, and through the presentation of additional physical evidence in support of the experimental results.

Mr. Hino, using the assumptions traditionally applied in analysis of shear flows, has correctly concluded that the "constant" indicated in Eq. 21 is not a universal number but rather, a function of the roughness of the bottom. He suggests, rather conservatively, that this effect, indicated in Eq. 30, is not great but will become evident when wider ranges of bottom roughness are investigated. While the present experiments did not encompass a wide range of absolute bottom roughnesses, it is nevertheless possible to demonstrate from the experimental data the significance of friction characteristics of the bottom.

Recognizing that

$$\frac{U}{U_*} = \sqrt{\frac{f}{8}} \qquad \text{. . . . . (56)}$$

in which f is the Darch-Weisbach friction factor, and substituting in Eq. 30 there results

$$D_z = \text{Constant} \left[f^{5/6} \left(1 - \frac{1}{K}\sqrt{\frac{f}{8}}\right)^{-1/3} \right] E^{1/3}\, L_a{}^{4/3} \qquad \text{. . . . (57a)}$$

or

$$D_z = \text{Constant}\; \phi(f)\; E^{1/3}\, L_a{}^{4/3} \qquad \text{. . . . . (57b)}$$

in which K = 0.4.

Because the values of f were determined for each experimental run in which D_z, L_a, and E were also evaluated it is possible to test the significance of ϕ (f) in Eq. 57b. Fig. 19 presents the results of thirty-three separate runs plotted in the fashion of Fig. 11 without regard to variations in the friction factor. It can be seen that for the larger values of $E^{1/3}\, L_a{}^{4/3}$, there is fairly good agreement with Eq. 21, but for the lower values there is a significant and consistent departure. However, when the effect of bottom friction, ϕ (f),

[a] September, 1959, by G. T. Orlob.

[50] Assoc. Prof., Civ. Engrg., Univ. of Calif., Berkeley, Calif.

is introduced for the same thirty-three observations, as in Fig. 20, reasonably consistent agreement is obtained throughout the full range of data. This tends to give general support to the basic assumptions of Mr. Hino's analysis.

The reason for the proportionately greater correction in the lower range of values is apparent upon consideration of variations in f and D_z with Reynolds Number (**R**). In the experiemnts reported f was generally dependent on **R** ranging from as high as 0.3 at **R** = 2000 to as low as 0.09 for **R** greater than about 35,000. At the higher Reynolds Numbers, the friction factor was apparently independent of **R**, the flow being identified as "wholly rough." As might be expected, the coefficient of eddy diffusion also was greatly dependent on **R**, the lowest values of D_z occurring at low turbulence scales and low Reynolds Numbers.

In the wholly rough range f can be considered dependent on relative roughness of the boundary alone, as indicated in the adaptation of the Nikuradse pipe flow equation to open channel flow,

$$\frac{1}{\sqrt{f}} = 1.14 + 2 \log \frac{4y}{e} \qquad (58)$$

in which e is an equivalent uniform sand grain roughness. In the present experiments application of Eq. 58 to observations made at Reynolds Numbers greater than 35,000 indicates that e was about 0.089 ft for the 1/2-in. mesh and about 0.110 ft for the 1-in. and 1 1/2-in. meshes. Relative roughnesses, e/y, for these observations ranged from about 0.34 to 0.43, the higher values resulting from runs on the coarser meshes.

The effect of a much wider range of relative roughness on the eddy diffusion coefficient can be shown by the transforming Eq. 57b with the aid of Eq. 58 into

$$D_z = \text{Constant}\ \phi\left(\frac{e}{y}\right)\ E^{1/3}\ L_a^{\ 4/3} \qquad (59)$$

which applies only in the "wholly rough" region at high Reynolds Numbers. Taking the constant of Eq. 57 as 0.085 there results, for various values of the parameter, e/y, the series of curves which are shown superimposed on Fig. 19. The line representing Eq. 23 in the figure has a position that corresponds to a relative roughness of 0.345, about the average of the ten observations that were made at Reynolds Numbers greater than 35,000.

The influence of the frictional characteristics of a smooth channel bottom on eddy diffusion can be examined with the aid of Eq. 59 and a knowledge of the frictional resistance of the surface. The channel bottom of the present experiments was found to have an equivalent uniform sand grain roughness of about 0.0004. For a maximum practical depth of flow in the experimental channel of 0.25 ft the relative roughness would be approximately 0.0016. From Eq. 59 it will be noted that the change in relative roughness from 0.345 (Eq. 23) to 0.0016 for constant E and L_a reduces D_z in the ratio of about 5-to-1.

Mr. Ichiye contends that the validity of Eq. 7 has been well established, apparently basing this conclusion on the fact that independent sets of field observations could be fitted with a single line on a logarithmic plot extended to cover eight orders of magnitude in eddy diffusivity. Unfortunately, at certain scales the eddy diffusivity values determined by Olson and Ichiye (7) as shown in Mr. Gunnerson's Fig. G-4, vary in the ratio of nearly 6-to1 or 7-to-1. While

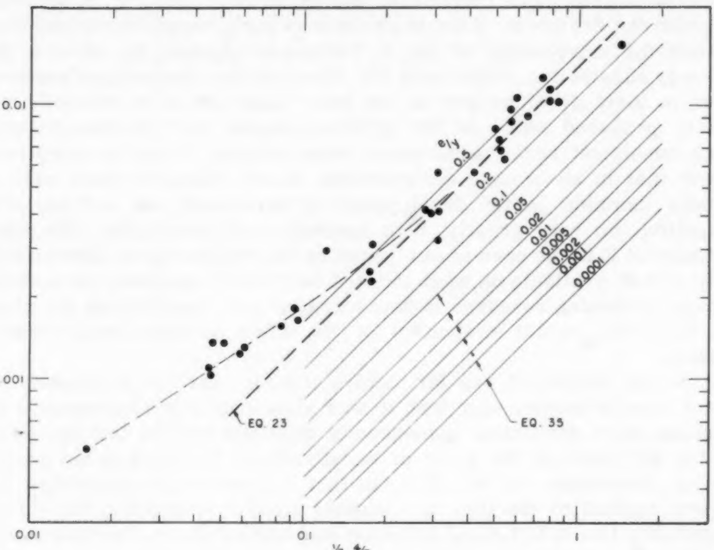

FIG. 19.—EDDY DIFFUSION—NO FRICTION CORRECTION

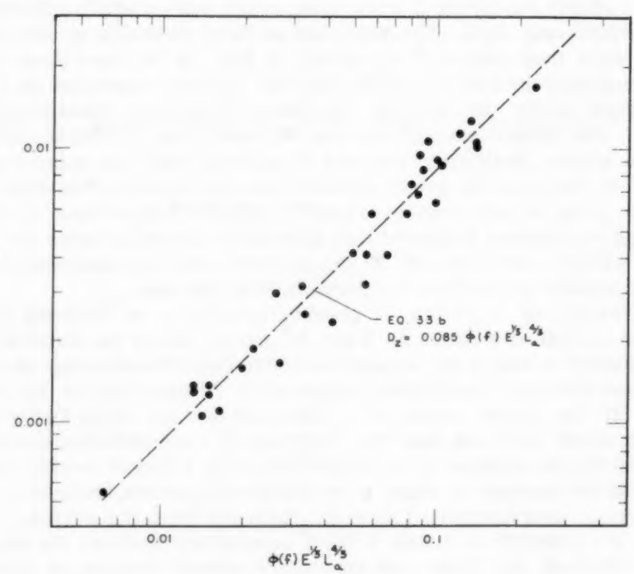

FIG. 20.—EDDY DIFFUSION—CORRECTED FOR FRICTION EFFECT

there is strong support in these observations, as there is in Fig. 8, for variation of D_z with the 4/3 power of the scale there is considerable doubt that they substantiate the universality of Eq. 7. Failure to appraise the effect of the rate of energy dissipation, which both Mr. Hino and Mr. Ichiye have correctly included in their developments of the basic eddy diffusion relations, unquestionably produced some of the apparent scatter in field observations even though consistent scale parameters were adopted. There is every reason to suspect that in an oceanic environment, where transient winds and complex currents combine in the development of turbulence, that the rate of energy dissipation can vary greatly, both spatially and temporally. The magnitude and range of E in the open ocean cannot be determined from information presently (1960) available on wind-induced turbulence and wave characteristics. Perhaps continuing research in oceanography will soon provide the necessary data that will permit evaluation of its effect on large scale oceanic eddy diffusion.

It is not suggested, as Mr. Ichiye implies, that the turbulence regime studied was isotropic, only that it was statistically homogeneous in two dimensions. Such statistical homogeneity does not require that the coefficient of eddy diffusion or the scale of turbulence be the same in the direction of flow as transverse to the flow. In fact, cursory experimentation with dye droplets applied to the flow in a manner similar to that of Elder,[51] indicated a possibility that longitudinal diffusion was somewhat greater than transverse diffusion. In order to determine more accurately the ratio between these two quantities for the two dimensional turbulence studied, a photographic technique was employed to position floating particles in both x and z directions.

Particles were released at a fixed point on the turbulent stream through a hole in a slowly revolving (1 rpm) disk, which subsequently actuated a single flash stroboscopic light. The positions of fifty particles in each dispersion pattern were thus recorded on sheets of film by the open flash method and were accurately located by projecting the various negatives on a screen at an enlarged scale. By varying the interval between release and flash the shape of the dispersion pattern was followed over a distance of 1-to-15 ft from the source. Statistical analysis of particle positions with respect to the centroid of the particle group showed that the distributions were of typical Gaussian form in both transverse and longitudinal directions. Calibration of the timing mechanism indicated that statistical variations in the interval could be kept within $\pm$ 0.1% for 95% of the observations, and thus would not be expected to significantly affect the particle distributions.

The results of a series of observations made at Reynolds Numbers of 18,000 to 19,000 are shown in Figs. 21 and 22, where the familiar relationships between σ and x for dispersion in the transitional range is evident for both transverse and longitudinal dispersion. Comparison of the two curves in Fig. 21 for large values of x indicates that the ratio, σ_x/σ_z, tended to approach about 1.58 and that the longitudinal eddy diffusion coefficient was about 2.50 times as large as the transverse eddy diffusion coefficient. Fig. 22 illustrates the changes in shape in the dispersion pattern, defined by multiples of σ_x and σ_z, as a function of time (or distance) from the source.

It is not possible to make a valid comparison between the experimental results obtained by Elder and the author simply because of differences in

51 "The Dispersion of Marked Fluid in Turbulent Shear Flow," by J. W. Elder, Journal of Fluid Mechanics, Vol. 5, No. 4, 1959, pp. 544-560.

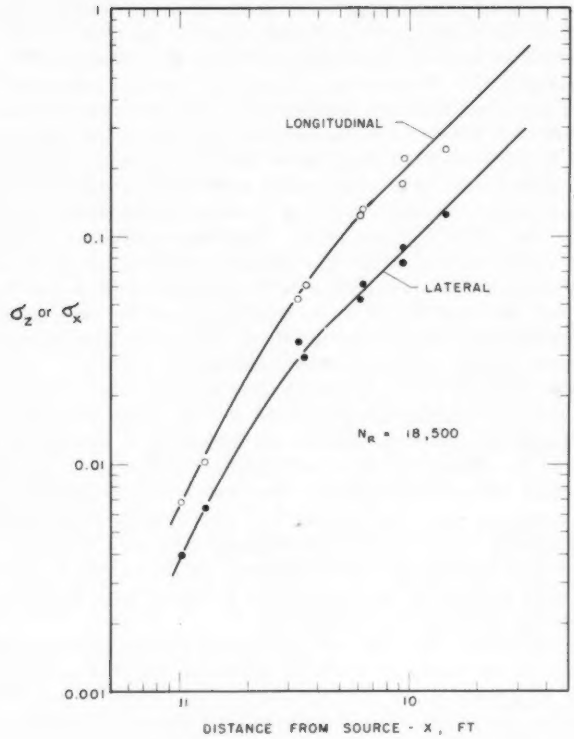

FIG. 21.—LONGITUDINAL & LATERAL PARTICLE DISPERSION

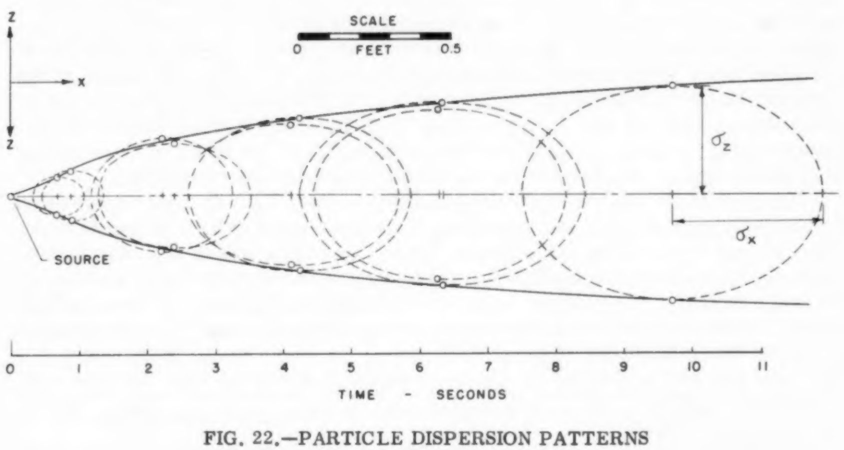

FIG. 22.—PARTICLE DISPERSION PATTERNS

technique and lack of comparable data. In the present studies dispersion was examined only at the surface of the stream and the particles were not carried below the surface as was the dye mixture that Elder employed.

Elder separated the longitudinal dispersion curves, obtained from photographs of dye patterns, into two components, one which he attributed to turbulent mixing and the other to diffusion into and out of the laminar sublayer. It is implicit in the method of separation that the turbulent component represents the combined effects of diffusion at all levels in the flow outside of the sublayer, irrespective of the variation in eddfy diffusion with position in the flow. Kalinske and Pien[52] have shown that the coefficient of eddy diffusion does in fact, vary widely with the depth of flow in an open channel. Such variations in eddy diffusion together with changes in advection velocity with depth could well account for much of the difference between longitudinal and lateral dispersion that Elder observed when simply viewing the entire phenomenon from one vantage point, directly overhead.

The results obtained by such a method cannot be used to debate the question of the isotropy of turbulence either pro or con. Even if the regime studied had been isotropic in planes parallel to the boundary, a similar conclusion as to the ratio of the lateral diffusion coefficient to the "apparent" longitudinal coefficient would have been reached. For these same reasons it is not appropriate to compare the longitudinal and lateral eddy diffusion coefficients for pipe flow unless observations are confined to a very limited region along the centerline. For obvious practical reasons observations in this region are likely to be very difficult if not impossible to obtain with sufficient accuracy to demonstrate the isotropy of turbulence.

Mr. Ichiye has confirmed that the statistical theories of turbulence can be used to describe the form of the σ -x curve (or the σ -t curve) close to the origin, or at relatively large distances from the origin, but that between the two extremes the curve can be described only by knowing the functional or experimental form of R_ξ . In fact, to obtain a solution near the origin it is also necessary to assume the functional form of R_ξ , as may be seen in Eq. (f) which was, in fact, prescribed originally by Taylor[53] as a series, the higher order terms of which have been neglected to secure the parabolic form adopted in the writer's derivation. Such a simplification leads to the conclusion that close to a point source in a turbulent stream with a mean velocity $\bar{u}$, σ_z is proportional to x or t. This conclusion cannot be justified by argument based on the physical nature of the diffusion process, nor is it supported by experiment or field observation as can be seen from Fig. 5 and the data presented by Mr. Gunnerson. The basic concept of a "Four-thirds Law", which all parties to this discussion appear to agree on, is in contradiction to this conclusion.

The reason for selection of the functional form of R_ξ , so well documented by previous experiment, was precisely to overcome the failure of the writer's Eqs. (e) and (i) to define the shape of the σ_z -x curve unless R_ξ was known. In the laboratory it is virtually impossible to generate turbulence at sufficiently large scales for practical study of diffusion in Batchelor's "equilibrium range." Moreover, practical limitations in the size of channels or water tunnels preclude the possibility of developing the σ_z -x curve to the point where σ_z becomes proportional to $x^{1/2}$, ($D_z(\infty)$ = constant) except at very low Reynolds

52 "Eddy Diffusion," by A. A. Kalinske and C. L. Pien, Ind. Engrg. Chem., Vol. 36, No. 3, 1944, pp. 220-222.

53 "Diffusion by Continuous Movements," by G. I. Taylor, Proceedings, London Math. Soc., Vol. 20, August, 1921, pp. 196-211.

Numbers. It will be noted, however, that the method employed in the experiments overcomes these limitations to a certain degree and permits description of the turbulence phenomena without working to either extreme of the σ_z -x curve.

In the open ocean, on the other hand, most investigators have been concerned with diffusion relatively near the source, where neither Eq. 16 or Eq. i describes the phenomenon. Most observations have been directed to proving that the width of dispersion pattern from a source increases as the four-thirds power of its width, as suggested by Mr. Gunnerson's Eq. G-5.

Consideration of the limitations of boundaries on the scale of turbulence and on the exchange of momentum suggests that there is a limit to the scale of turbulence, which cannot be shown by Eq. G-5, and that there must also be a transitional region between the regions of expanding scale and fixed scale. Mr. Gunnerson's observation that the field from the existing outfall widened to one mile in 4 hr, but after 24 hr had attained a width of only 2 miles tends to confirm this concept. If, for example, it may be assumed that eddy diffusivity was constant when a width of one mile was attained and that the initial boil width was attained 0.1 hr after sewage discharge at the bottom, then the field width should have enlarged to 2.43 miles. On the other hand, if the 4/3 law had been continuously applicable over the entire period, the field should have reached a width of 14.7 miles (Eq. 11 for n = 3).

It is apparent that the limiting eddy diffusivity was actually less than that indicated by a one-mile scale. Mr. Gunnerson's notation that diffusion "is to the seaward side only" is further indication of boundary limitations on diffusion. It is probable that in this instance the dispersion pattern was very much like that shown in Fig. 13 and 22, the "4/3 Law" applying only close to the source, while at distances greater than one mile the pattern was of parabolic form with k = constant.

The writer would like to take exception to Mr. Ichiye's citation of unpublished material,[8] particularly when an impression is given that this work is a primary authority. The writer can only properly direct his attention to material published in an available medium at the time of the discussion, because others can neither be defended nor criticized.

It is not often that a paper brings together such a complementary combination of experimental, theoretical, and practical experience as has been presented here. The author is grateful for constructive criticism offered by the writers and is encouraged that future research will shed more light on some of the challenging problems posed.

HYDRAULIC DESIGN OF SLUTTED SPILLWAY BUCKETS[a]

Closure By G. L. Beichley and A. J. Peterka

G. L. BEICHLEY,[1] M. ASCE, AND A. J. PETERKA,[2] F. ASCE.—The writers welcomed the discussions submitted by Messrs. McPherson and Brodbeck because their questions and reactions are probably typical of those who will use the material in the paper. Mr. McPherson's detailed comparison of the performance characteristics of the slotted-type and solid-type roller buckets is a valuable contribution for his analysis will aid design engineers faced with the decision of which bucket type to use. Mr. Brodbeck's discussion of bucket performance versus operating conditions emphasizes the fact that the final bucket design should be thoroughly investigated and analyzed in the light of every condition that may prevail during normal and emergency spillway operation.

In Mr. McPherson's comparison of solid bucket and slotted bucket performance, he has emphasized points which are in full agreement with the writers. For example, he shows that the slotted bucket is particularly suited for lower ranges of tail-water depths within the tail-water limits and that for a given height of dam, the need for slots, which produce a quiet water surface and a less violent ground roller, becomes less as the tail-water elevation approaches head-water elevation. Therefore, the slotted bucket is particularly suited for medium-and high-head installations.Mr. McPherson raises several questions regarding statements made,testing techniques employed, and methods of presenting and interpreting the data. The remainder of this closure is devoted to explaining, justifying, or modifying the areas of the paper in question.

The testing flume, as stated in the paper, has a motor-driven tailgate geared to raise or lower the tail water at will. Although "sweepout" tests were made with falling tail-water and "diving flow" tests were made with a rising tail water, the action in both cases was inspected for both rising and falling tail water. In fact, this inspection was used in part to determine the 0.2-ft and 0.5ft depth corrections for sweepout and diving flow conditions. It may, therefore, be stated that the corrections are sufficient to compensate with certainty for either a rising or falling tail water in both cases.

Mr. McPherson's opinion that unsymmetrical spillway gate operation should not be used with slotted buckets, as well as with solid buckets, is correct. However, if unsymmetrical operation becomes necessary such as during emergency discharges through a partially completed structure, a slotted bucket is more desirable than a solid one because debris swept into the bucket is more easily washed out of the slotted bucket. In a solid bucket, debris tends

[a] October, 1959, by G. L. Beichley and A. J. Peterka.
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[2] Hydr. Engr., Bur. of Reclamation, Denver, Colo.

to roll around and erode the concrete until a near maximum flow lifts it out over the bucket lip. In a slotted bucket, it is reasonable to expect that much smaller discharges would move debris out through the slots. The comparison of solid and slotted buckets made by the writers was not entirely based on material presented in the paper. Actually, tests were made on buckets of 3 different radii and for 3 different lip elevations from one of the radii. In each of these cases, the bucket shape was a true arc ((called radial exit by Mr. McPherson). These tests were made before the slotted bucket had been developed, in an attempt to provide a satisfactory solid bucket for use on Angostura Dam. It was these tests that led to the development of the slotted bucket. After the slotted bucket had been developed, but before the generalization tests were made, a single solid bucket shaped to a true arc and having a 45° exit was again tested to substantiate and prove beyond a doubt in the writers' minds that the solid bucket is objectionable in many installations because it produces a concentrated jet, a standing wave (called a boil by the writers), and a ground roller that moves loose bed material up to the bucket lip. It was after these visual studies that the writers concluded that the slotted bucket is superior to the solid bucket. The remainder of the comparison was based on a review of the results obtained during the Grand Coulee Spillway model studies. Much of the comparison data could not be presented because the data were visual and photographic in nature.

The range of bucket sizes and tail-water elevations covered was quite extensive, although not "complete" in the strict sense of the word. As to tail-water elevations covered, comprehensive data for flow-tail-water-roller relationships between the limits T_{min} and T_{max} are not available because the limiting conditions, T_{min} and T_{max}, are determined in such a way that any tail water between these limits results in satisfactory performance. As to bucket sizes covered, McPherson states, in effect, that small Froude numbers have been tested for only small h_1/R values (low dams) and large Froude numbers for only large h_1/R values (high dams). The writers contend that large Froude numbers are associated with high dams, small Froude numbers are associated with low dams, and there are no other practical combinations to be covered. The buckets tested had radii ranging from 1/10 to 3/10 of the spillway height and each was tested for a complete range of tail-water depths. Thus, bucket design data are included for Froude numbers (computed for the flow at the point on the spillway face corresponding to tail-water elevation) ranging from 4 to 10. For Froude numbers below 4, the tail water is usually near spillway crest elevation, the total energy in the flow is relatively small, and a modified type of bucket dissipator is often more desirable. It was not the intention of the writers to include buckets used on low dams where the tail-water depth approaches the height of the spillway crest. Damsites where a Froude number of 10 can be exceeded are rare ($F = 9.7$ for Grand Coulee) and would ordinarily demand individual model tests.

The curves of Fig. 21 were obtained from a model 5 ft high, measured from the bucket invert to the spillway crest as was pointed out by Mr. McPherson. However, 4 different bucket radii were used to provide 4 different ratios of bucket radius to spillway heights, making the surface profile characteristics curves as acceptable as any of the other data presented. The writers would like to state, however, that the curves give the boil height only for the design flow with tail-water depth between T_{min} and T_{max}. Since the ratio A/T is only

slightly changed by using T_{min} or T_{max}, the change in boil height is not of practical concern. As the discharge is decreased below the design flow, the ratio of A to T approaches unity.

Mr. McPherson is correct in questioning the use of the slotted bucket on approach slopes other than the 10 on 7 slope used throughout the model tests. To be strictly correct, an approach slope of 10 on 7 should be used in the prototype. On the other hand, the writers believe, based on their experience with many bucket tests, that the slopes could be modified to 10 on 6, or 10 on 8, without affecting the bucket performance adversely. It is also entirely possible that the slope could be modified to even a greater degree. Certainly, the bucket can be used with flat slopes if the spillway face is made steep just before maximum tail-water elevation is reached.

The inconsistency between Fig. 22 in the writers' paper and Fig. 15 in Mr. McPherson's Reference 5 is of more academic than practical interest because the purpose of the figures is to provide a first approximation of the velocity in making exploratory calculations. For large heights of fall, the use of H or H/2 does not affect the velocity to a practical degree and for low heights of fall neither expression is entirely correct. Fig. 22 was used by the writers because of the slight additional safety factor inherent in using H rather than H/2.

Table M1 prepared by Mr. McPherson indicates another method of comparing the conjugate depth required for a hydraulic jump with the tail-water depth required for a slotted bucket. However, the writers did not present the comparison in this way because the minimum and maximum tail-water requirements in most cases place the bucket invert at extremely different elevations and using an average tail-water depth for computation of the Froude number at the bucket invert elevation is not correct. However, both the writers' and or McPherson's tables illustrate the only point the writers wished to make-a bucket-type dissipator requires a greater depth of tail water than an hydraulic jump stilling basin.

The extrapolation to obtain the bucket size for Angostura Dam to which Mr. McPherson objects is the result of discharging more water through the bucket than is recommended. In the Introduction, Item C, it is stated that individual model tests should be conducted when the discharge per foot of width exceeds 500-600 cfs. The extrapolation to which Mr. McPherson objects is to a unit discharge of 901 cfs.

The extrapolation of the curve in Fig. 15 for the design flow computations in Table 3 should read 0.6 instead of 0.5. The bucket radius is then computed to be 56 ft, instead of 47 ft. Mr McPherson's curve in Fig. M-2 indicates the radius should be 50 ft.

The actual bucket radius used in the existing structure is 40 ft, but this was determined from a separate model study of the Angostura spillway conducted prior to the general study by the writers. The smaller bucket derived from the individual tests illustrates the point the writers tried to emphasize throughout their paper—the design curves will produce installations that are safe for prototype operation. In the general study, it was difficult to establish exact limits especially in the lower range of Froude numbers. Therefore, to obtain the smallest structure and the minimum tail-water depth, an individual model study is recommended. The values shown for the various structures in Table 4 were obtained from the design curves in the paper and are not intended to represent, necessarily, the existing structures.

Mr. McPherson expresses the hope that the writers will present other data on other structures. If other data on other structures had been available, they would have been presented in the paper. The Superior-Courtland and Cambridge Diversion Dams mentioned by Mr. McPherson are low structures with tailwater elevation near crest elevation. The slotted buckets for these structures were developed prior to the general study conducted by the writers and no laboratory data useful for design purposes are available. The portions of the Angostura model study data which are useful for general design purposes were summarized and presented in the paper. The corrections suggested by Mr. McPherson and listed at the end of his discussion are typographical in nature and should be made as he has indicated.

PERFORMANCE OF FLOOD PREVENTION WORKS DURING THE 1957 FLOODS[a]

Closure by Charlie M. Moore

CHARLIE M. MOORE,[1] F. ASCE.—Mr. Blaisdell is correct in his supposition concerning the functioning of the principal spillways, in that they did function very satisfactorily and with little damage below the outlet end of the spillways. Perhaps this is due to the use of relatively small pipe diameters, low discharge rates, and the fact that the pipes discharged, in most instances, on reasonably good erosion-resistant soils.

The scour holes shown in Figs. B1 and B2 of Mr. Blaisdell's discussion are typical of the scour holes that have developed below many of the structures. Because scouring was anticipated in design the cantilever section of pipe was placed well out from the toe of the embankment on all structures. Some soils of course erode more readily than others, and this must be considered in design. Where highly erodible soils exist at the end of a proposed conduit, it may be necessary to relocate the conduit or install rock riprap or other control measures.

Whether or not future capacity flows will increase the size of the scour hole to dangerous proportions is of course unknown, as pointed out. However, inspections of the structures after every storm of any significance, and annual maintenance inspections of all structures, have revealed no appreciable enlargement in most cases.

The questions raised by Mr. Blaisdell as to the future performance of cantilevered outlets are well founded. He points out that there "must be limitations as to the size of cantilevered outlets that can be used with soils of different erodibilities in order to prevent the scour hole from endangering the dam." This statement appears to be true based on experience with several larger floodwater retarding structures constructed since 1957 in the arid and semiarid areas of Texas. In these areas the erodibility of the soils is higher and the chance of establishing vegetation for erosion control is much lower. The drainage areas are considerably larger which requires higher peak flow discharges through the principal spillways.

Where the major portion of floodwater retarding structures in this area contain principal spillways of 18-in. to 24-in. diam pipes, the structures mentioned above have pipe diameters ranging from 31 in.to 57 in., maximum operating heads ranging from 26.2 ft to 74.3 ft, and maximum anticipated discharges ranging from 125 cfs. to 580 cfs. These structures discharge on SM, CL, SM-SP, SM-SC and CH soils. The most erodible is of course the SM, SM-SC and SM-SP soils, and with the ML soils are the ones offering the most problems.

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[a] October, 1959, by Charlie M. Moore.

[1] Design Engr., Engrg. and Watershed Planning Unit, SCS, Fort Worth, Tex.

One large structure built in Oklahoma on which a scour hole is being corrected has a 34-in. diam pipe, 74.3 ft maximum operating head, and a 250 cfs. maximum discharge. This structure discharges on SM, CL-ML, and SM-SC soil classifications which afford very little resistance to erosion.

In all of the preceding cases some form of outlet protection will be needed.

In light of this more recent experience it is thought that where the maximum pipe discharge exceeds 125 to 150 cfs. and the erodibility of the soil is high, some type of energy dissipator should be provided. Where soils have a high erodibility we anticipate using the so-called "plunge basin" in which the size of the scour hole will be predetermined and riprap used. This protection

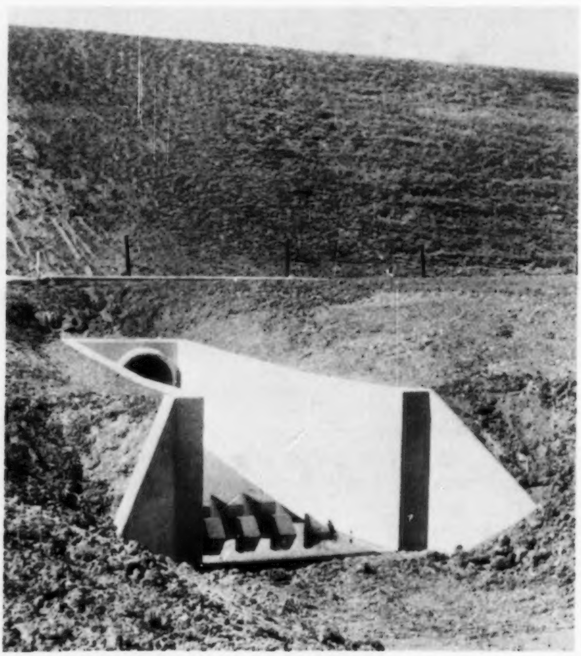

FIG. 1.—ST. ANTHONY FALLS STILLING BASIN

will be used in connection with the cantilever support. A limited amount of criteria is available for the design of such basins although some information is contained elsewhere.[2]

Where soils are less erodible and tail water depths will not be changed appreciably due to erosion of the channel below, the SAF stilling basin as developed by Mr. Blaisdell is a very appropriate device for dissipating the velocity. Such an installation as shown in Fig. 1 was made on Rowlett Creek Site 4 near Dallas. This structure is in soils similar to the ones shown in Figs.

[2] "Design of Small Dams," by the Bureau of Reclamation.

B1 and B2 of Mr. Blaisdell's discussion. The stilling basin was installed because of the relatively large anticipated peak rate of discharge due to the subsequent installation of a 48-in. diameter pipe through the embankment.

This basin will accommodate a maximum discharge of 408 cfs from the 48-in. and a 22-in diam pipe running parallel through the embankment. The basin has length of 12.0 ft, sidewall height of 9.5 ft, and is 12.0 wide. The floodwater retarding structure controls the runoff from a drainage area of 7,291 acres of Blackland soils and is founded in CH-CL soils.

There are very good design criteria for the SAF stilling basin as developed by Mr. Blaisdell which the designer may use. However, criteria for predicting the size and extent of scour holes and for the design of plunge basins are sadly lacking. The writer agrees that this would be a very fertile field and a worthwhile project for research. A systematic research program to study the development of scour holes below the many floodwater structures built in the watershed protection program throughout the country would no doubt provide data and criteria for use in future design. It would also answer important questions raised by Mr. Blaisdell in his discussion.

WAVE-INDUCED MOTION OF BOTTOM SEDIMENT PARTICLES[a]

Closure by P. S. Eagleson and R. G. Dean

P. S. EAGLESON[19] A. M. ASCE, AND R. G. DEAN,[20] A. M. ASCE.—The authors are indebted to Mr. Miller and his co-workers at the Woods Hole Oceanographic Institution, Woods Hole, Mass., for their persistent efforts to evaluate recent theoretical idealizations of beach processes in the light of their observations on natural beaches.

All too often, the geologist and engineer have chosen to go their separate ways in an area of common interest with the result that the best resources of both disciplines are seldom brought to bear on the same problem.

Mr. Miller correctly points out that the assumption of $D_{e/k} = 1$ at all points on an equilibrium beach is unrealistic. A few words concerning the necessity for this assumption are in order.

The final (equilibrium) size-frequency distribution of surface sands at any position on a natural beach under a given wave system depends upon the initial size-frequency distribution at all points along the beach and upon the final (equilibrium) beach profile. However, the equilibrium relationship between local beach slope and particle size is shown by the authors to be dependent upon the local value of the ratio, $D_{e/k}$, which in turn depends upon the local equilibrium size-frequency distribution.

A convenient and sensible way out of this dilemma is to obtain a first approximation to the equilibrium size-slope relationship using the value $D_{e/k} = 1$ everywhere.

If, as Mr. Miller points out, the offshore mode of particle motion is unlikely to be of significance on natural beaches, and if $D_e < D_i$ everywhere, then at any point all sizes smaller than D_e will be absent from the surface sands. Under such conditions $D_{e/k}$ will be closest to unity near the breaker and will get progressively smaller in the offshore direction.

Once approximate equilibrium sizes are located using $D_{e/k} = 1$ this assumption may be modified and more accurate values of D_e obtained through a second iteration.

Mr. Miller states that field observations indicate that seaward motion offshore of the equilibrium location ("null point") does not occur in nature. This is not surprising since, even under the ideal and controlled conditions of the laboratory the offshore motion was found to be weak and erratic. In this zone the driving force is predominately gravitational and is thus quite small. The slightest irregularity in the frictional resistance of the bed is sufficient to stop the motion.

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[a] October, 1959, by P. S. Eagleson and R. G. Dean.

[19] Asst. Prof., Hydr. Engrg., Mass. Instit. of Tech., Cambridge, Mass.

[20] Student, Mass. Instit. of Tech., Cambridge, Mass.

HYDRAULIC CHARACTERISTICS OF GATE SLOTS[a]

Closure by James W. Ball

JAMES W. BALL,[1] M. ASCE.—The discussions have added much to the scope and usefulness of the material presented in the paper, and indicate the need of expanding and clarifying certain parts. The discussers have added several valuable references, and the writer expresses appreciation for their important contributions.

The data contained on Fig. 11 cannot be compared directly with that of the other figures of the paper because of differences in data test facilities, and relative piezometer locations. Fig. 11 was presented mainly to show the similarity of data obtained from two separate test programs, and particularly to show that the low-pressure area had been moved from the downstream edge of the slot, Slot A, to just downstream of the P.I. when the downstream corner was offset outwardly with 12:1 converging walls downstream, Slots Band C.

The definition of h_o in the figure is in error. Actually it is similar to that used in Fig. 68 of Reference No. 2. The definition should be "the pressure in test facility about 9 in. upstream from slot." The curve for Slot B was plotted from Fig. 68 for comparison with Slot C with the scale changed (by the ratio of the 12:1 slope lengths) to correspond to that of the Bureau test facility. Also, the sharp dip in the curve immediately downstream from the P.I. of Slot B was drawn in, using the curve for Slot C as a guide. This was done because of the more complete piezometer coverage in the larger test facility. This explains the lower dip of the curve and why the distance of the P.I. for Slot B in Fig. 11 is 6.1 in. instead of the prototype dimension of 13.5 in. given on Fig. 68. It is still questionable that the minimum pressure has been indicated in either of the curves, a condition which was suggested in one of the discussions. The minimum pressure may be much lower than the curves indicate, particularly if pressure fluctuation is considered. Some scale effect is still not ruled out but the proof of this would have to be determined from tests on different scale, geometrically similar test facilities.

The reference point, h_o, for the data on the other figures of the paper is specified as the back of the gate leaf. A clearer definition would be "pressure immediately downstream from the slot." A value of $\frac{h_x - h_o}{h_v}$ for this station was obtained by projecting to it the hydraulic grade line determined in the system well downstream from the slot. The pressure values obtained in the slot areas were then adjusted by the amount appropriate to make a similarly sloped grade line pass through zero (atmospheric) at the downstream slot corner. Zero, or

[a] October, 1959, by James W. Ball.
[1] Hydr. Engr., Bur. of Reclamation, Denver, Colo.

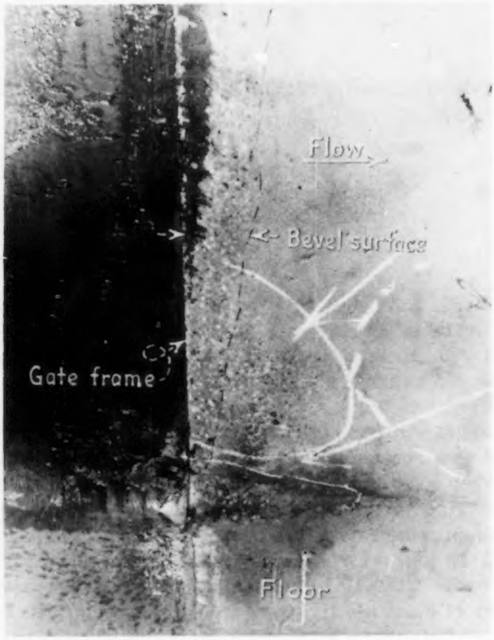

FIG. 21.—BEVEL-GROUND IN CONCRETE SURFACE AT END OF GATE FRAME

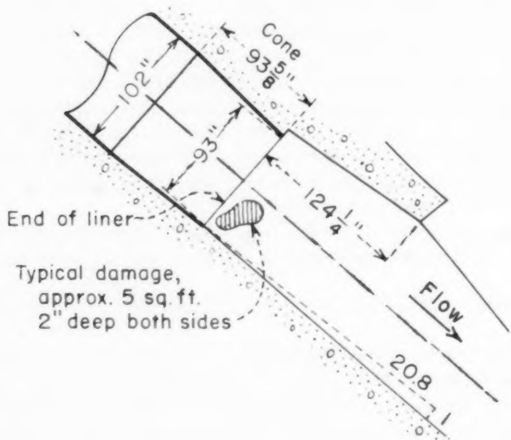

FIG. 22.—BREAK IN SLOPE AT END OF METAL LINER OF OUTLET CONDUIT

atmospheric, pressure was used simply as a datum of convenience. The pressure datum is not affected by either the leaf-wall aperture or seat. The term h_v in the expression given above is the velocity head corresponding to the average velocity at the upstream edge of the slot.

The writer's conclusion that the omission of the 0.056 W radius on the downstream corner had no adverse effects came from tests made on the water-test facility in which the pressure curves downstream from the P.I. were identical for both the sharp and rounded corners. The dimensions of the model slot with "the outwardly offset, 1-in. rounded corner with a 12:1 convergence" were: Width 4-1/2 in., depth 3-3/4 in., radius of rounded corner 1/4-in. The rounded corner was not tested in the air facility. Because of the difference in datum in Figs. 11 and 12 of the paper, conclusions drawn from these two figures might be in error. This fact may change some of the comparisons made in Tables M1 and M4 of one of the discussions.

An interesting observation made during the analysis of data from the several test facilities was that the pressure factor was usually slightly more negative with water than air. Only a small number of tests could be compared so no conclusion was drawn from the observation. However, the machined shapes in the water facility probably had sharper intersections than the planed and carefully sanded surfaces of the air facility. Also, different relative locations of piezometers may have been a contributing factor.

The question as to whether or not cavitation damage with 12:1 convergent walls is necessarily associated with the minimum boundary pressure downstream of the P.I. can be best answered by a photograph (Fig. 21). An offset into the flow of about 1/4-in. occurred in the concrete at the end of the metal gate frame in the Palisades Dam Outlet Works during construction. This offset was removed by grinding the concrete to grade at the gate frame, and forming a sloping surface or bevel which varied from about 4:1 to flat in the direction to flow. After several months exposure to velocities of about 100 fps damage occurred at slope intersections even less severe than 12:1.

There have been other observations which show that cavitation can be induced by breaks in slopes flatter than 12:1. In one case, where a metal cone lining was placed at the end of an outlet conduit through a dam, the break in slope at the end of the cone was 20.8:1 (Fig. 22). Severe cavitation-erosion occurred just downstream from the P.I. (Fig. 23). The remedy used was the slot-type groove shown on Fig. 24 which was based on the data given in the paper. Preliminary reports from the field indicate that this treatment, which was applied in the 1959-60 winter season, was very effective. The writer is convinced that cavitation downstream from the 12:1 slope for slots similar to B and C, Fig. 11, is associated with the minimum boundary pressure downstream from the P.I. It is evident from photographs and the laboratory tests that the break in slope at the P.I. can produce a reduction in pressure which would cause vapor cavities to form immediately downstream from the break, and that these cavities would move almost immediately into a steeply rising pressure grade where they would implode to cause cavitation-pitting.

From the test data presented by several investigators, it can be concluded that the gate slot design using 12:1 convergent wall sections downstream may have two separate sources of cavitation: the vortex flow within the slot, and the break in slope at the P.I. There is still some question as to whether or not the cavitation action within the slot, if it does occur, is of a damaging nature. The absence of signs of cavitation on the floors of the gate frames at Palisades Dam seems to indicate that cavitation either does not occur, or occurs

within the fluid, away from the flow boundary, and does not produce damage. This fortunate situation may not be true of all gate slots.

An observation made during tests in the water tunnel of Fig. 4, from which Fig. 20 was developed, may have some bearing on the preceding statements. Piezometers located as close as 3/64-in. from the offset corner did not show vapor pressure with pressure cells when the offsets were large and vapor

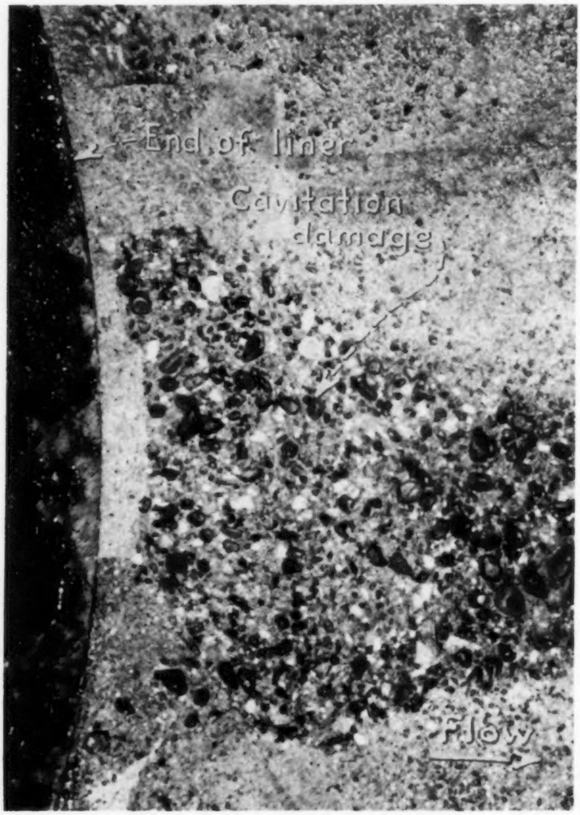

FIG. 23.—CAVITATION-EROSION IN CONCRETE SURFACE DOWNSTREAM FROM BREAK OF SLOPE AT END OF METAL LINER OF OUTLET CONDUIT

flashes were first noted. Furthermore, vapor pressure was not indicated until an envelope of vapor existed downstream from the corner. When the envelope was small the pressure fluctuated greatly with momentary values equal to the vapor pressure. As the vapor envelope increased, the pressure became steady at vapor pressure. For small offsets, the pressure cell showed intermittent values of vapor pressure almost simultaneously with the vapor flashes.

The preceding observations were the basis for the statement that "the pressure on the surface just downstream of the downstream slot corner need not be of vapor pressure magnitude for cavitation to occur at the slot." The same

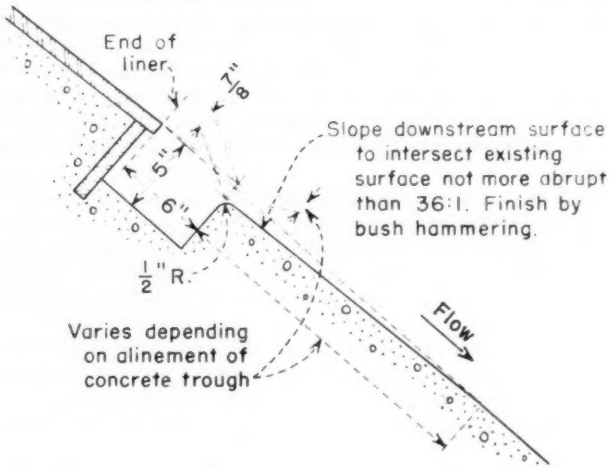

FIG. 24.—GATE SLOT-TYPE GROOVE AT END OF METAL LINER OF OUTLET CONDUIT

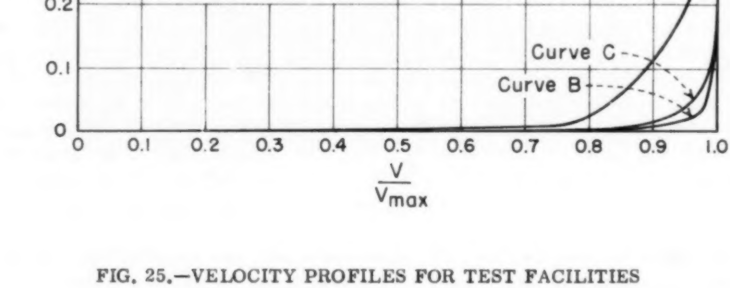

FIG. 25.—VELOCITY PROFILES FOR TEST FACILITIES

indications were present when the in-line wall slot was tested in the water tunnel shown on Fig. 4.

The pressures for the gate slot tests were recorded with open manometers; thus, the values obtained were average and would not indicate instantaneous

lows which could reach vapor pressure and cause cavitation, even though the average pressure was much above the vapor pressure.

There have been cases when pressures obtained by open manometers in model tests have been only slightly subatmospheric and yet cavitation has occurred in the field structures. A recheck of model pressures by pressure cells in these instances has shown that intermittent pressures equivalent to vapor pressure were occurring, thus confirming the cavitation on the field structure. Cavitation can thus occur before the pressure recorded by an open manometer shows vapor pressure. In fact, under certain flow conditions and amount of sharp-cornered offset into the flow, tests have indicated that cavitation may occur off the surface even though surace pressures do not momentarily reach vapor pressure.

The writer has observed cases where cavitation has taken place within sudden enlargements downstream from gate values without inflicting damage to the enlargement surfaces.[2] In other cases not yet reported damage could be inflicted to the walls of the enlargement either by lowering the back pressure to give a certain value of the cavitation index, or by reducing the relative size of the sudden enlargement.

Minimum boundary pressures may not be a true measure of the cavitation potential in certain designs. However, if rapid fluctuations in pressure cause instantaneous occurrences of vapor pressure, cavitation will occur and the minimum boundary pressures may be most important. Operation of gates using a slot design in which minimum boundary pressures (average water column) are near atmospheric, $\frac{h_x - h_o}{h_v} = o$, have not shown damage from boundary conditions or from conditions within the slot.

The water tunnel shown on Fig. 4 was used also to obtain the data shown on Fig. 20. The offset was placed 6 in. from the upstream end of the test section. A piezometer in the top of the test section, and 3 in. upstream from the offset, was used to measure the piezometric head. The pressure head H was obtained by projecting the hydraulic grade line to the offset station and correcting it to the elevation of the offset corner. Details of the test procedure, and background for Fig. 20, were not included in the paper because the information, along with tests on chamfered and rounded offsets and other surface irregularities, are expected to be the subject of a future paper.

When the test section was planned it was thought that the height might be too small for the larger offsets. However, the pump facilities did not make a larger section feasible. Moreover, the smaller offsets were of major interest because data were needed to determine sizes of offsets that can be tolerated in concrete surfaces subjected to high velocities, such as downstream from high-head outlet works gates. Even though the trend of little change in critical velocity with increase in offset size is logical for the larger offsets, there is still some question as to whether or not the test section size was adequate for the larger offsets.

The water for the test section was taken from, and returned to, the laboratory storage channel. Thus, the facility was not the usual closed recirculating-type water tunnel generally used for cavitation studies and the water contained

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2 "Cavitation Characteristics of Gate Valves and Globe Valves Used as Flow Regulators Under Heads Up to About 125 ft," by J. W. Ball, Transactions, of ASME, Vol. 79, No. 6, August, 1957.

considerable air that could not be removed. Initially, the facility contained a straight 6-in. wide by 3-in. high rectangular approach 4 ft 11 in. long which gave the centerline velocity profile (Curve A, Fig. 25) in the approach 9 in. upstream from the offset. The curves of Fig. 20 are based on the average velocity just downstream from the offset. This velocity was thought most appropriate for the use to be made of the data at that time. The ambient pressure

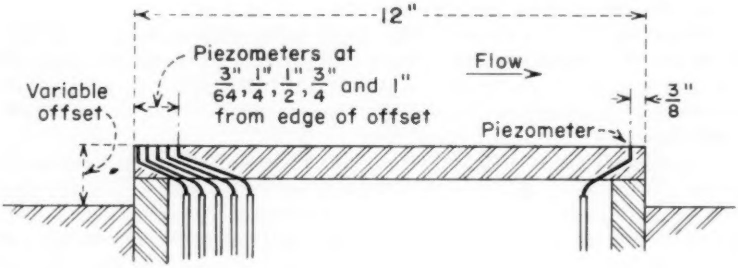

FIG. 26.—PIEZOMETER LOCATIONS—OFFSET STUDIES

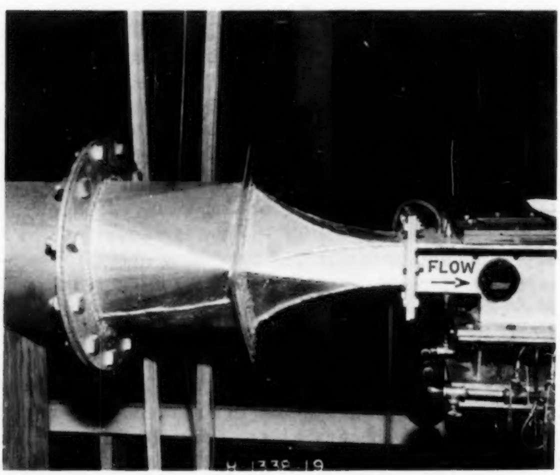

FIG. 27.—SPECIAL UNIFORM-VELOCITY TRANSITION

was varied by adjusting a valve in the downleg at the right in Fig. 4. Piezometers were located as shown on Fig. 26. Incipient cavitation for various velocities and pressures was usually determined by setting a discharge and then varying the back pressure until tiny vapor flashes formed in the flow at the offset corner. The flashes were viewed through circular plastic windows placed in

each side of the test facility at the offset station. Audible definition was also made during some of the tests, with good agreement between both methods.

The facility shown on Fig. 4 was rebuilt for recent research tests to obtain uniform velocity throughout the cross section in the approach to the offset. The quite uniform velocity 3 in. upstream from the offset Curve B, Fig. 25, was obtained by placing a special transition at the upstream end of the test section 6 in. from the offset station (Fig). 27). Results obtained from this facility, using the average velocity in the section just upstream from the offset, were quite similar to those on Fig. 20.

The replotting of the data from Fig. 20, in other coordinates, as in Fig. T1 of one of the discussions, is very interesting. Whether or not the data would be more useful in one form or another depends on what use is to be made of it. Experience has shown that the information as presented on Fig. 20 is very useful to designers.

The discussion from a designer's point of view points out that the ideal conditions determined from laboratory tests are often difficult, if not impossible, to apply to field structures. It is therefore desirable to design hydraulic structures so that slight and acceptable variations from the construction standpoint are not critical. The writer believes that this condition was met in the gate slot design developed from the test data that formed the background for the paper.

It is unfortunate that the high-head tests on a large model, and other tests conducted by the laboratories of the Societe Grenobloise d' Etudes et d' Aplications Hydrauliques, Grenoble, France, could not be described in detail. The limited data given is extremely interesting and adds much to the paper. A gate with an upstream seal and a forced contraction just upstream from the leaf has been used successfully by the Bureau. However, in this case, the capacity was reduced, the coefficient of discharge based on full open area at the gate being reduced from about 0.97 to 0.80.

The spreading of jets is definitely a part of the gate slot problem. The jet starts to spread as soon as it leaves the upstream corner of the slot. This fact is clearly illustrated in a paper[3] presented at the Montreal meeting of IAHR in 1959. The offset downstream corner of the design developed by tests in the Bureau laboratory takes advantage of this spreading action to keep acceptable pressures on the flow surfaces downstream from the slots. The writer is firmly convinced and agrees with one of the discussers that the spreading action is important. However, the pressure patterns are just as important, and both should be considered together in any analysis of the slot problem.

The vortex patterns shown in the figures of one of the discussions are of much interest. They serve to explain some of the trends and variations noted by the author when analyzing the test data taken from the various test facilities. Photographs of vortex action and wet-paint flow patterns on the slot floor obtained in one of the early tests were deleted from the original paper in order to make it conform to the publication word limitation. These indicated some very unusual and interesting flow conditions. No definite points of discontinuity were noted for the W/D ratios observed. As suggested in one discussion, this may have resulted from insufficient observations in the region of W/D=2.

In the case of the offsets in the cover-plate joints of the 48-in. water tunnel where slot cavitation occurrence was evident and presumably occurred only at

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3 "Hydro-Dynamics of Flow Across the Width of a Gate Slot," by Taher Abu El Wafa and C. T. Advani, Eighth Congress, IAHR, Montreal, Canada, August, 1959.

velocities above 45 fps, it would be of interest to know the magnitude of the ambient pressure in the tunnel at the time this condition was noted. The curves of Fig. 20 show that cavitation will not occur on a 1/16-in. offset when the pressure is near atmospheric and the velocity is 45 fps.

The boundary layer thickness was a consideration in all the gate slot tests. In tests which concerned specific designs, the facility was constructed and installed so as to produce a rather thin boundary layer at the slot. The velocity profile taken at a station 12 in. upstream from the slot, Curve C, Fig. 25, was typical of the air test facility. The thin boundary layer was considered to be on the safe side when applied to field structures. In later tests on the offsets, the idea was to minimize the effects of boundary layer and subject the offsets to an easily obtained known velocity.

The discussion of boundary-layer thickness effects on pressure distribution, its changes, and varying magnitude with varying W/D ratios is extremely interesting. It is obvious that the discusser has delved much deeper into basic research on slots than did the tests described by the writer. This information will be valuable in any future similar study performed to extend present knowledge. The data presented in the discussions concerning boundary-layer thickness and its effects on the wall pressure magnitude are proof of the reasoning used in the laboratory tests described by the author, that thin boundary layers in the test facilities would tend to introduce a factor of safety when applied to the full-sized field structure.

The initial approach in the case of offsets into the flow (Fig. 20) was to determine the ambient pressure and velocity at the offset for incipient cavitation and then to use this along with computed near-boundary velocities to determine the criticalness of various sizes of offsets in different structures. This approach was used to suppy the designers and construction engineers with information which would permit them to determine the tolerances required for flow surfaces subjected to high-velocity flow. Actually, the designer feels that it is on the safe side to use curves like those presented on Fig. 20, assuming that the offset is subjected to the average flow velocity. This, of course, is the case where the boundary layer is well established. This factor of safety varies in different structures and decreases as the boundary-layer thickness decreases. The designers consider the data of Fig. 20 very helpful for both open channel and closed conduit flow.

It is hoped that this closing discussion will answer all the questions set forth in the several discussions. References to specific discussions were not made because of the similarity to many of the questions and comments contained in the various discussions.

ERRATA

Abcissa of Figure 20 should be changed to "Average velocity just downstream from offset."

h_o on Figure 11 should be defined as "Pressure head in test facility about 9 inches upstream from slot."

The ratio 24:1 in the title of Figure 10 should be 12:1.

Definition of h_v in Figures 6, 8, 9, 10, 12, 13, 14, 15, and 17 should be "Velocity head at upstream edge of slot."

MOUNTAIN CHANNEL TREATMENT IN LOS ANGELES COUNTY[a]

Closure by William R. Ferrell

WILLIAM R. FERRELL,[1] A. M. ASCE.—The author is indebted to Messrs. Newhall and Henry for their forward thinking discussion of the author's paper. In answer to their questions the following is presented.

With respect to the possibilities of increasing the amount of deep percolation of ground water resulting from the installation of channel stabilization structures, an increase in dam height, as suggested, is not believed justified inasmuch as a stabilization system is designed primarily for erosion control, and economy of continuous stabilization versus foundation and construction problems related to increased heights goes out of proportion very rapidly with increased height above 17 ft. There are, however, special situations which necessitate larger dams. One such structure is being constructed (as of 1960) in Sawpit Canyon where, due to the lack of a suitable intermediate dam site, a 30-ft dam is being constructed. This, of course, will increase possibilities of augmenting water conservation through increased percolation opportunity. The economy and problems being encountered with this structure are being carefully observed for future design criteria. At the present time (1960) no consideration has been given to the drilling of wells or water galleries for the promotion of deep percolation. This is an atea that should possibly be given future consideration at special locations.

Messrs. Newhall and Henry commented relative to the possible use of the finer material that will be deposited in the District's debris basins and debris dams as a result of erosion control works for top soil and other nursery uses. In this regard, the District at the present time has many permittees excavating this material for lawn and garden purposes, and this type of use is being encouraged to the utmost.

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[a] November, 1959, by William R. Ferrell.

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THE VORTEX CHAMBER AS AN AUTOMATIC FLOW-CONTROL DEVICE[a]

Closure by R. C. Kolf and P. B. Zielinski

R. C. KOLF,[1] M. ASCE, AND P. B. ZIELINSKI,[2] A. M. ASCE.—The discussions indicate that a real interest exists in the subject matter of the paper. The suggestions offered are greatly appreciated. Both writers apparently "second the motion" for continued vortex research because of the probable engineering applications as well as the necessary increase in understanding of fluid behavior.

Several of the questions raised by Mr. Amein need clarification. First, although the description of the Helmholz vortex helps in the understanding of the vortex phenomenon as it occurs in nature, the idealizations involved in the mathematical concept do not indicate a weakness in the proposed treatment of vortex flow through orifices. In this presentation it is assumed that no torque is supplied to the rotating fluid through moving boundaries, (as in the forced vortex situation) but that fluid friction does exist. However, the curves and equations presented in this paper are based on dynamic similarity and, therefore, the variations in the velocity distribution are accounted for implicitly. It should also be noted that the superposition of a radial flow on the free vortex does not change the type of velocity or pressure distribution except in the region near the orifice (significant water surface draw down). In both types of flow the product of velocity and radius is a constant.

In all of the test runs, a high degree of vorticity was induced by the inlet conditions. Within the boundary ring, the radial component of the velocity vector was found to be negligibly small compared to the tangential component (the resultant vector making less than a 5° angle with the tangent). Therefore, the direction of the velocity vector, for this particular apparatus, was changed by an amount approximately equal to angle θ. This change in direction necessitates only a small force, which is supplied by the very slight increase in water surface elevation inside the boundary. Although the magnitude of the velocity vector is slightly reduced inside the vanes (kinetic head converted to elevation head) this difference for the apparatus described was detectable in the third significant figure only, thus justifying the computational convenience of assuming that the velocity within the entrance vanes is equal in magnitude to the velocity immediately inside the vortex chamber. Because of the nearly tangential conditions within the chamber, however, the term $\cos\theta$, should not appear in Mr. Amein's equation.

The observation that a high head will "drown out" a vortex is correct. This can be seen on Fig. 2, in which, for a constant circulation and orifice diameter,

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[a] December, 1959, by R. C. Kolf and P. B. Zielinski.

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the coefficient of discharge is shown to increase with increasing head. The design of a diversion system or "counterflow brake" however, may have features which produce an increasing circulation with increasing head.[3,5]

The discussions have been valuable in pointing out the required directions of further study. It is hoped that the basic method presented will provide the means for establishing the feasibility of utilizing the vortex phenomenon in engineering design as well as measuring the effects of existing vortex conditions.

TOLKMITT'S BACKWATER AND DROPDOWN CURVE TABLES[a]

Discussion by Chesley J. Posey, Monir M. Kansoh, and Serge Leliavsky

CHESLEY J. POSEY,[34] F. ASCE.—Presentation of the Tolkmitt tables is a timely contribution to American literature. The recent development of theoretically complete integration methods, together with accurate tables for the functions required, would seem to have rendered the older integration methods of Bresse and Tolkmitt quite obsolete. However, as one uses the newer methods he becomes aware that the numerous double interpolations which must be done carefully in order to obtain the necessary accuracy are very laborious. Preliminary determinations of parameters is time-consuming and evaluation of terms after the interpolated functional values have been obtained takes still more time and affords additional opportunities for error. One might as well use the step method for uniform channels advocated by J. C. Stevens, Past-President, ASCE.[35] In acquiring a more precise evaluation of the effects of friction and cross sectional shape the direct integration methods have lost the advantages of convenience and easy detection of errors.

The mathematician who finds his solution too complicated searches for an "approximating function." Now it turns out that the best approximating functions for the backwater and dropdown curves are those of the type of Bresse and Tolkmitt. To make sure that the approximating function gives a good approximation one needs certain rules - "cautions" to be observed in the selection of the coefficients C and n. If these cautions are carefully observed, the older integration methods will give good results. If through misunderstanding or carelessness they are improperly applied, the results can be very poor indeed. It is, no doubt, the ease with which the unwary can obtain poor results that brought the older methods into disrepute.

The value of the normal depth (d in Figs. 1 and 2) is the most important parameter in obtaining a good approximation, and it should be based upon the best information obtainable. In descending order of reliability, sources of this information are (1) direct measurement in the actual channel, at the desired discharge (2) interpolation between two such direct measurements (3) a short extrapolation beyond two or more such measurements (4) a computed value based upon measured or estimated values of "n."

The value of the coefficient C given by Eq. 5 should be based on the average of the values at each end of the reach. The writer's experience in selecting the "best C" for use with Bresse's function indicates that the most accurate results

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[a] May, 1960, by R. D. Goodrich.

[34] Prof. and Head, Dept. of Civ. Engrg., Univ. of Iowa, Iowa City, and Dir., Rocky Mountain Hydr. Lab., Allenspark, Colo.

[35] Discussion of "Back-Water and Drop-Down Curves for Uniform Channels," by J. C. Stevens, Transactions, ASCE, Vol. 103, 1938, p. 990.

are obtained by using a constant value throughout the length of the curve. Attempting to vary C to gain precision not only complicates the computations, but actually results in poorer approximation. Even for narrow and deep rectangular cross sections, C chosen on this basis will give results that are nearly identical with those given by the step method. Good results are obtained even when the cross section differs appreciably from rectangular.

TABLE 3.—TABLE FOR TRIANGULAR SECTIONS, PART I

X	Φ (X)	X	Φ(X)	X	Φ(X)	X	Φ (X)
1.000	∞	1.037	0.38638	1.145	0.17021	1.52	0.03958
1.001	1.03277	1.038	0.38177	1.150	0.16560	1.54	0.03727
1.002	0.91737	1.039	0.37729	1.155	0.16118	1.56	0.03513
1.003	0.84235	1.040	0.37294	1.160	0.15695	1.58	0.03314
1.004	0.78929	1.041	0.36870	1.165	0.15289	1.60	0.03130
1.005	0.74815	1.042	0.36457	1.170	0.14900	1.64	0.02799
1.006	0.71454	1.043	0.36054	1.175	0.14526	1.68	0.02512
1.007	0.68612	1.044	0.35662	1.180	0.14166	1.72	0.02261
1.008	0.66159	1.045	0.35280	1.185	0.13820	1.76	0.02040
1.009	0.64002	1.046	0.34907	1.190	0.13487	1.80	0.01846
1.010	0.62074	1.047	0.34543	1.195	0.13166	1.85	0.01636
1.011	0.60328	1.048	0.34187	1.200	0.12857	1.90	0.01454
1.012	0.58737	1.049	0.33839	1.21	0.12270	1.95	0.01297
1.013	0.57276	1.050	0.33499	1.22	0.11722	2.00	0.01161
1.014	0.55927	1.052	0.32840	1.23	0.11210	2.1	0.00937
1.015	0.54676	1.054	0.32209	1.24	0.10730	2.2	0.00765
1.016	0.53508	1.056	0.31604	1.25	0.10279	2.3	0.00631
1.017	0.52411	1.058	0.31023	1.26	0.09855	2.4	0.00525
1.018	0.51380	1.060	0.30464	1.27	0.09455	2.5	0.00439
1.019	0.50406	1.064	0.29407	1.28	0.09078	2.6	0.00371
1.020	0.49485	1.068	0.28422	1.29	0.08722	2.7	0.00315
1.021	0.48614	1.072	0.27502	1.30	0.08386	2.8	0.00270
1.022	0.47787	1.076	0.26638	1.31	0.08067	2.9	0.00232
1.023	0.46995	1.080	0.25826	1.32	0.07765	3.0	0.00202
1.024	0.46237	1.084	0.25059	1.33	0.07478	3.2	0.00151
1.025	0.45511	1.088	0.24335	1.34	0.07205	3.4	0.00115
1.026	0.44816	1.092	0.23650	1.35	0.06946	3.6	0.00090
1.027	0.44149	1.096	0.22999	1.36	0.06699	3.8	0.000710
1.028	0.43507	1.100	0.22381	1.37	0.06464	4.0	0.000569
1.029	0.42889	1.105	0.21648	1.38	0.06240	5.0	0.000213
1.030	0.42294	1.110	0.20957	1.39	0.06026	6.0	0.000091
1.031	0.41719	1.115	0.20304	1.40	0.05822	7.0	0.000047
1.032	0.41164	1.120	0.19686	1.42	0.05441	8.0	0.000025
1.033	0.40626	1.125	0.19099	1.44	0.05092	9.0	0.000014
1.034	0.40106	1.130	0.18541	1.46	0.04772	10.0	0.000009
1.035	0.39602	1.135	0.18011	1.48	0.04478	15.0	0.000004
1.036	0.39113	1.140	0.17505	1.50	0.04207	∞	Zero

As the author points out, tables are now available for rectangular, parabolic, and circular cross sections. In computing backwater and dropdown curves to compare with the experimental results obtained in the National Science Foundation-sponsored 400-ft variable slope flume of the Rocky Mountain Hydraulic

TABLE 4.—TABLE FOR TRIANGULAR SECTIONS, PART II

X	Φ (X)	X	Φ (X)	X	Φ (X)	X	Φ (X)
0.00	0.00000	0.76	0.79182	0.945	1.15578	0.980	1.36010
0.10	0.10000	0.78	0.81840	0.950	1.17581	0.981	1.37013
0.20	0.20001	0.80	0.84638	0.952	1.18431	0.982	1.38067
0.25	0.25003	0.81	0.86096	0.954	1.19314	0.983	1.39179
0.30	0.30009	0.82	0.87601	0.956	1.20232	0.984	1.40356
0.35	0.35022	0.83	0.89159	0.958	1.21188	0.985	1.41605
0.40	0.40050	0.84	0.90778	0.960	1.22187	0.986	1.42938
0.45	0.45103	0.85	0.92465	0.962	1.23233	0.987	1.44368
0.50	0.50201	0.86	0.94231	0.964	1.24331	0.988	1.45908
0.52	0.52257	0.87	0.96389	0.966	1.25486	0.989	1.47580
0.54	0.54326	0.88	0.48053	0.968	1.26706	0.990	1.49408
0.56	0.56411	0.89	1.00144	0.970	1.28000	0.991	1.51418
0.58	0.58515	0.90	1.02386	0.971	1.28677	0.992	1.53660
0.60	0.60642	0.905	1.03572	0.972	1.29376	0.993	1.56198
0.62	0.62795	0.910	1.04809	0.973	1.30099	0.994	1.59127
0.64	0.64981	0.915	1.06103	0.974	1.30847	0.995	1.62631
0.66	0.67204	0.920	1.07461	0.975	1.31622	0.996	1.66897
0.68	0.69471	0.925	1.08891	0.976	1.32429	0.997	1.72316
0.70	0.71791	0.930	1.10404	0.977	1.33268	0.998	1.79931
0.72	0.74174	0.935	1.12012	0.978	1.34142	0.999	1.93243
0.74	0.76636	0.940	1.13730	0.979	1.35055	1.000	∞

Laboratory, K. P. Singh found that none of the existing tables permitted a satisfactory approximation for the case of a triangular cross-section.[36] He therefore computed tables for the case. To fill this gap, which again is not a serious one, it seems appropriate to reproduce Mr. Singh's tables herewith (Tables 3 and 4). In Part I (Table 3)

$$X = \frac{d + z}{d} \qquad \text{(38)}$$

and

$$\Phi (X) = \int_0^X \frac{dX}{X^{5.33} - 1} \qquad \text{(39)}$$

In Part II (Table 4)

$$X = \frac{d - z}{d} \qquad \text{(40)}$$

and

$$\Phi (X) = \int_0^X \frac{dX}{1 - X^{5.33}} \qquad \text{(41)}$$

The equation into which the tabular values are to be substituted is

$$L = \frac{d}{s} (X) - d \left(\frac{1}{s} - \frac{C^2}{\sqrt{2g}} \right) \Phi (X) \qquad \text{(42)}$$

36 "Study of Backwater Curves in a Triangular Channel," by Krishan Piara Singh, thesis presented to the University of Iowa, Iowa City, Iowa, in August, 1958, in partial fulfilment for the degree of Master of Science.

The symbols are the same as the author's except that L differs from L_z in being measured from an arbitrary origin which could be located if it were deemed desirable to do so. The simplest procedure is to compute the value of L corresponding to z = h and then subtract it from any others for which the value L_z is desired.

MONIR M. KANSOH.[37]—Mr. Goodrich presented his paper with the idea of making Tolkmitt's tables available to American engineers because, although "only briefly discussed" by Bakhmeteff, the formulas and tables by G. Tolkmitt are, to Mr. Goodrich's knowledge, "only to be found in the German texts".

It would have been more useful, then if the paper made a contribution to the English literature by including, as well, the derivation of the formulas as done by Tolkmitt or as tried by Mr. Goodrich himself. This would have helped also in adjusting the formulas and in detecting, more, readily, any misprint as happened in Eq. 3. In this equation each term should have a length dimension to represent a distance. The term $f\left(\frac{d-h}{d}\right)\left(1 - \frac{s\,C^2}{g}\right)$ must then either be multiplied by (d/s) or included in the preceding square bracket with the term $f\left(\frac{d-z}{d}\right)$.

Unless the derivation of this equation is presented, to prove the reverse, the writer believes that the term $\left(1 - s\,C^2/g\right)$ is not the proper one, in the case of channels of parabolic cross section. This term is correct, for any bed slope, in only one case, when it is assumed, as done by Bresse, that the channel has a very wide rectangular cross section. This is shown in the following derivation which the writer tried in order to arrive at the same results pictured by the first four equations given in the paper. Unfortunately, the results were not the same. The derivation proceeds as follows:

Most of the texts give, for the water surface profile, an equation of the following or of some modified form:

$$\frac{d\,Y}{d\,x} = \frac{s - i}{1 - \dfrac{Q^2\,T}{g\,A^3}} \qquad (43)$$

known as the varied flow equation, in which Y is the depth of flow; x is the distance between two cross sections; s denotes the bed slope or inclination; i is the slope of the total head line (virtual slope) $= \frac{V2}{C2\ R} = \frac{Q^2}{A^2}\,C^2\,R$, (on the assumption that the conditions of flow are approximately similar to uniform or normal flow conditions, that is, on the assumption that hydrostatic conditions prevail), R is the hydraulic mean depth or hydraulic radius A denotes the area of cross section A ÷ wetted perimeter P; C is the Chezy coefficient (function of R); T is the top width; and g is gravitational acceleration.

The distance x may be measured horizontally or along the bed, the slope s being usually very small that cos s almost = 1.00.

The term $\frac{d\,Y}{d\,x}$ looks preferable than $\frac{d\,d}{d\,l}$ or $\frac{d\,Y}{d\,l}$, and so the letters Y and x are chosen to represent the depth of flow at any section and the distance between two sections.

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37 Prof. of Hydr. & Water Power Engrg., Alexandria Univ., Alexandria, United Arab Republic.

Under normal conditions where no obstructions or interruptions take place, the flow is usually uniform and the depth can be distinguished by the symbol Y_u or Y_n.

A cross section at which the flow is normal or uniform may be described by the parameters Y_n, A_n, P_n, R_n, C_n, T_n. In this case the bed and the water surface, in the longitudinal section, are parallel to the total head line or s = i. Chézy equation can here be put in the form:

$$Q = A_n C_n R_n^{\frac{1}{2}} s^{\frac{1}{2}} \quad \ldots \ldots \ldots \ldots \ldots \ldots (44)$$

Generally,

$$i = \frac{Q^2}{A^2 C^2 R} = \left(\frac{A_n C_n}{A C}\right)^2 \times \frac{R_n}{R} \quad s \quad \ldots \ldots \ldots \ldots (45)$$

A cross section at which the flow is critical may be described by the parameters Y_{cr}, A_{cr}, P_{cr}, R_{cr}, C_{cr}, T_{cr}. Here holds the relation:

$$Q^2 = g\, A_{cr}^3 / T_{cr} \quad \ldots \ldots \ldots \ldots \ldots \ldots (46)$$

In a channel of a parabolic cross section, the top width = constant x $Y^{\frac{1}{2}}$ and the area of flow = 0.66 T Y = constant x $(Y)^{3/2}$.

Tolkmitt assumed a shallow wide parabolic cross section so that the wetted perimeter P might be substituted by T with the result that:

$$R = 0.66\ T\ Y/T = 0.66\ Y \quad \ldots \ldots \ldots \ldots (47)$$

while

$$C = \text{const x} \left(R^{0.15} \text{ to } R^{0.25}\right)$$
$$= \text{const x} \left(Y^{0.15} \text{ to } Y^{0.25}\right) \quad \ldots \ldots \ldots \ldots (48)$$

According to Eq. 7, which is a good contribution, Tolkmitt's assumption is applicable only where the ratio T/Y exceeds 16. Thus the term:

$$A^2 C^2 R = (0.66\ T\ Y)^2 C^2 (0.66\ Y) = \text{const.} \left(Y^3 C^2 Y\right)$$
$$= \text{const.} \left(Y^4\right)\left(Y^{0.30 \text{ to } 0.50}\right) = \text{const.} \left(Y^{4.30 \text{ to } 4.50}\right) \quad \ldots (49)$$

Using Tolkmitt's second assumption of a constant Chézy coefficient, thus

$$A^2 C^2 R = \text{const.} \left(Y^3 C^2 Y\right) = \text{const.} \left(Y^4\right) \ldots \ldots (50)$$

In the same time

$$A^3/T = \text{const.} \left(Y^{9/2}\right)/Y^{\frac{1}{2}} = \text{const.} \left(Y^4\right) \ldots \ldots (51)$$

Returning back to the varied flow equation and attempting to arrive to Tolkmitt's formulas (1, 2, 3 and 4), using his two assumptions just mentioned, it can be said that:

$$\frac{dY}{dx} = \frac{s - i}{1 - Q^2 \dfrac{T}{g\,A^3}} = \frac{s - \dfrac{A_n^2\ C_n^2\ R_n\ s}{A^2\ C^2\ R}}{1 - g\,\dfrac{A_{cr}^3}{T_{cr}}\ \dfrac{T}{g\,A^3}} = s\ \frac{1 - \left(\dfrac{Y_n}{Y}\right)^4}{1 - \left(\dfrac{Y_{cr}}{Y}\right)^4} \quad \ldots . \ (52)$$

This can be modified as follows:

$$s\,dx = \frac{1 - \left(Y_{cr}/Y\right)^4}{1 - \left(Y_n/Y\right)^4}\ dY = \frac{Y^4 - Y_{cr}{}^4}{Y^4 - Y_n{}^4}\ dY$$

$$= \frac{\left(Y^4 - Y_{cr}{}^4\right) + \left(Y_n{}^4 - Y_n{}^4\right)}{Y^4 - Y_n{}^4}\ dY$$

$$= \frac{\left(Y^4 - Y_n{}^4\right)}{Y^4 - Y_n{}^4}\ dY + \frac{\left(Y_n{}^4 - Y_{cr}{}^4\right) dY}{\left(Y - Y_n\right)\left(Y + Y_n\right)\left(Y^2 + Y_n{}^2\right)}$$

$$= dY + \frac{\left(Y_n{}^4 - Y_{cr}{}^4\right) dY}{\left(Y - Y_n\right)\left(Y^3 + Y_n\,Y^2 + Y_n{}^2\,Y + Y_n{}^3\right)}$$

$$= dY + \frac{K\,dY}{\left(Y - Y_n\right)} + \frac{\left(L\,Y^2 + M\,Y + N\right) dY}{\left(Y^3 + Y_n\,Y^2 + Y_n{}^2\,Y + Y_n{}^3\right)} \quad \ldots\ldots\ldots . \ (53)$$

where

$$\left(Y_n{}^4 - Y_{cr}{}^4\right) = K\left(Y^3 + Y_n\,Y^2 + Y_n{}^2\,Y + Y_n{}^3\right) + \left(Y - Y_n\right)\left(L\,Y^2 + M\,Y + N\right) \ldots \ (54)$$

Putting $Y = Y_n$, then

$$\left(Y_n{}^4 - Y_{cr}{}^4\right) = 4KY_n{}^3 \quad \ldots\ldots\ldots\ldots\ldots . \ (55)$$

from which

$$K = \left(Y_n{}^4 - Y_{cr}{}^4\right) / \, 4Y_n{}^3 \quad \ldots\ldots\ldots\ldots\ldots . \ (56)$$

Coefficient of Y^3 is $(K + L) = o$, that is,

$$L = -K = -\left(Y_n{}^4 - Y_{cr}{}^4\right) 4Y_n{}^3 \ldots\ldots\ldots\ldots\ldots . \ (57)$$

Coefficient of Y^2 is $\left(K\,Y_n - L\,Y_n + M\right) = 0$, therefore

$$M = Y_n\,(L - K) = -2Y_n\,K = -\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)/2Y_n^{\,2} \quad \ldots\ldots\ldots\ldots (58)$$

Coefficient of Y is $\left(KY_n^{\,2} - MY_n + N\right) = o$ and

$$N = -KY_n^{\,2} - 2Y_n^{\,2}K = -3KY_n^{\,2} = -\frac{3}{4}\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)/Y_n \quad \ldots\ldots\ldots (59)$$

Absolute term is: $\left(KY_n^{\,3} - NY_n\right) = \left(Y_n^{\,4} - Y_{cr}^{\,4}\right)$, that is,

$$N = +\,KY_n^{\,2} - \left(Y_n^{\,4} - Y_{cr}^{\,4}\right)/Y_n = -\frac{3}{4}\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)/Y_n \quad \ldots\ldots\ldots (60)$$

Therefore

$$s\,dx = d\,Y + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4\,Y_n^{\,3}\left(Y - Y_n\right)}\,d\,Y - \frac{\dfrac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)Y^2}{4Y_n^{\,3}} + \dfrac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)Y}{2Y_n^{\,2}} + \dfrac{3\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n}}{\left(Y + Y_n\right)\left(Y^2 + Y_n^{\,2}\right)}\,d\,Y \quad \ldots (61)$$

$$s\,dx = dY + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\frac{dY}{\left(Y - Y_n\right)} - \frac{Y^2 + 2\,Y_n\,Y + 3\,Y_n^{\,2}}{\left(Y + Y_n\right)\left(Y^2 + Y_n^{\,2}\right)}\,dY\right]$$

$$= dY + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\frac{d\left(Y - Y_n\right)}{\left(Y - Y_n\right)} - \frac{\left(3Y^2 + 2Y_nY + Y_n^{\,2}\right) - \left(2Y^2 - 2Y_n^{\,2}\right)}{\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)}\,dY\right]$$

$$= dY + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\frac{d\left(Y - Y_n\right)}{\left(Y - Y_n\right)} - \frac{d\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)}{\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)} + 2\,\frac{\left(Y - Y_n\right)\left(Y + Y_n\right)}{\left(Y + Y_n\right)\left(Y^2 + Y_n^{\,2}\right)}\,dY\right]$$

$$= dY + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\frac{d\left(Y - Y_n\right)}{\left(Y - Y_n\right)} - \frac{d\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)}{\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)} + \frac{2\,Y\,dY}{\left(Y^2 + Y_n^{\,2}\right)} - \left(2\,Y_n\right)\frac{dY}{\left(Y^2 + Y_n^{\,2}\right)}\right] \quad \ldots\ldots\ldots\ldots (62)$$

$$s\,x = Y + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\log\left(Y - Y_n\right) - \log\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right) + \log\left(Y^2 + Y_n^{\,2}\right) - \frac{2\,Y_n}{Y_n}\tan^{-1}\left(\frac{Y}{Y_n}\right)\right] + \text{Const}$$

$$= Y + \frac{\left(Y_n^{\,4} - Y_{cr}^{\,4}\right)}{4Y_n^{\,3}}\left[\log\frac{\left(Y - Y_n\right)\left(Y^2 + Y_n^{\,2}\right)}{\left(Y^3 + Y_nY^2 + Y_n^{\,2}Y + Y_n^{\,3}\right)} - 2\tan^{-1}\left(\frac{Y}{Y_n}\right)\right] + \text{a const, say } k'$$

$$= Y_n\left(\frac{Y}{Y_n}\right) + \frac{Y_n}{4}\left[1 - \left(\frac{Y_{cr}}{Y_n}\right)^4\right]\left[\log\frac{\left(Y - Y_n\right)\left(Y^2 + Y_n^{\,2}\right)}{\left(Y + Y_n\right)\left(Y^2 + Y_n^{\,2}\right)} - 2\tan^{-1}\left(\frac{Y}{Y_n}\right)\right] + k'$$

$$= Y_n\left(\frac{Y}{Y_n}\right) + Y_n\left[1 - \left(\frac{Y_{cr}}{Y_n}\right)^4\right]\left[\frac{1}{4}\log\frac{\left(Y/Y_n\right) - 1}{\left(Y/Y_n\right) + 1} - \frac{1}{2}\tan^{-1}\left(\frac{Y}{Y_n}\right)\right] + k' \qquad \ldots\ldots (63)$$

The ratio $\left(Y_{cr}/Y_n\right)^4$ can be put in another form as follows:

$$\left(\frac{Y_{cr}}{Y_n}\right)^4 = \left(\frac{Y_{cr}}{Y_n}\right)^3 x \left(\frac{Y_{cr}}{Y_n}\right) x \left[\left(\frac{T_{cr}}{T_n}\right)^3 x\ \frac{P_n}{T_{cr}}\right] x \left[\left(\frac{T_n}{T_{cr}}\right)^3 x\ \frac{T_{cr}}{P_n}\right]$$

$$= \frac{g\,x\left(\frac{2}{3}\ Y_{cr}\ \ T_{cr}\right)^3 \Big/ T_{cr}}{g\,x\left(\frac{2}{3}\ Y_n\ \ T_n\right)^3 \Big/ P_n}\, x \left[\left(\frac{T_n}{T_{cr}}\right)^3 x\ \frac{T_{cr}}{P_n}\right] x\ \frac{Y_{cr}}{Y_n}$$

$$= \frac{Q^2}{g\,A_n^{\,2}\left(A_n/P_n\right)}\, x \left[\left(\frac{T_n}{T_{cr}}\right)^3 x\ \frac{T_{cr}}{P_n}\right] x\ \frac{Y_{cr}}{Y_n}$$

$$= \frac{A_n^{\,2}\,C_n^{\,2}\left(A_n/P_n\right)s}{g\ A_n^{\,2}\left(A_n/P_n\right)}\ x \left[\left(\frac{T_n}{T_{cr}}\right)^3 x\ \frac{T_{cr}}{P_n}\ x\ \frac{Y_{cr}}{Y_n}\right]$$

$$= \frac{s\,C_n^{\,2}}{g}\ x \left[\left(\frac{T_n}{T_{cr}}\right)^3 x \left(\frac{T_{cr}}{P_n}\right) x \left(\frac{Y_{cr}}{Y_n}\right)\right] \qquad \ldots\ldots\ldots\ldots (64)$$

and

$$s\,x = Y_n \left(\frac{Y}{Y_n}\right) + Y_n\, x \left[1 - \left(\frac{s\,C_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3 \left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right)\right] x$$

$$\left[\frac{1}{4}\,\log \frac{\left(Y/Y_n\right) - 1}{\left(Y/Y_n\right) + 1} - \frac{1}{2}\,\tan^{-1}\left(\frac{Y}{Y_n}\right)\right] + k' \quad \ldots (65)$$

Using the same letters employed in the paper, the symbols: x, Y_n, Y and Y/Y_n are substituted by: L, d, $(d \pm h)$ or $(d \pm z)$ and X. Thus

$$s\,L = d\,X + d\left[1 - \left(\frac{s\,C_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3 \left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right)\right]\left[\frac{1}{4}\,\log\frac{X-1}{X+1} - \frac{1}{2}\tan^{-1} X\right] + k'$$

$$L = \frac{d}{s}\left\{X + \left[1 - \left(\frac{s\,C_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3 \left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right)\right]\left[\frac{1}{4}\log\frac{X-1}{X+1} - \frac{1}{2}\tan^{-1} X\right]\right\} + \frac{k'}{s}$$

$$= \frac{d}{s}\,(f\,X) + \frac{k'}{s} = \frac{d}{s}\,(f\,X) + k \quad \ldots\ldots (66)$$

For $X = \dfrac{d \pm z}{d} = 1 \pm \dfrac{z}{d}$,

$$f\,X = f\left(\frac{d+z}{d}\right) = \frac{d+z}{d} + \left[1 - \left(\frac{s\,C_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3\left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right)\right]$$

$$\left[\frac{1}{4}\,\log\frac{X-1}{X+1} - \frac{1}{2}\,\tan^{-1} X\right] \quad \ldots\ldots (67a)$$

and

$$f\,X = f\left(\frac{d-z}{d}\right) = \frac{d-z}{d} + \left[1 - \left(\frac{s\,C_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3\left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right)\right]$$

$$\left[\frac{1}{4}\log\frac{X-1}{X+1} - \frac{1}{2}\,\tan^{-1} X\right] \quad \ldots\ldots (67b)$$

Eqs. 67a and 67b do not agree with their corresponding ones: Eqs. 2 and 4, in the paper, which take the form:

$$f\left(\frac{d+z}{d}\right) = \frac{d+z}{d} + \left[-1-0\right]\left[\frac{1}{4}\log\left(\frac{d}{z}+X\right) - \left(-\frac{1}{2}\tan^{-1}X\right)\right] + \frac{\pi}{4} \quad (68)$$

$\left(\text{where: } -\frac{1}{2}\tan^{-1}X + \frac{\pi}{4} = \frac{1}{2}\cot^{-1}X\right)$ and

$$f\left(\frac{d-z}{d}\right) = \frac{d-z}{d} - \frac{d-z}{d} + \left[1-0\right]\left[\frac{1}{4}\log\left(\frac{d}{z}-X\right) - \left(-\frac{1}{2}\tan^{-1}X\right)\right] \quad (69)$$

The distance L_z between two cross sections, the depths at which are $d \pm h$ and $d \pm z$, is:

$$L_z = \frac{d}{s}\left[f\left(\frac{d+h}{d}\right) - f\left(\frac{d+z}{d}\right)\right] \quad (70)$$

and

$$L_z = \frac{d}{s}\left[f\left(\frac{d-h}{d}\right) - f\left(\frac{d-z}{d}\right)\right], \text{ or } \frac{d}{s}\left[f\left(\frac{d-z}{d}\right) - f\left(\frac{d-h}{d}\right)\right] \quad (71)$$

where $f\left(\frac{d \pm h}{d}\right)$ can be obtained from $f\left(\frac{d \pm z}{d}\right)$ after substituting h for z.

Eq. 70 is in agreement with its corresponding one Eq. 1, in the paper, which refers to the (rising) backwater curves.

Eq. 71 corresponding to dropdown curves do not agree with Eq. 3, in the paper, which has the form:

$$L_z = \frac{d}{s}\left[f\left(\frac{d-z}{d}\right)\right] - f\left(\frac{d-h}{d}\right)\left(1 - \frac{s\,C^2}{g}\right) - \frac{h-z}{s} \quad (72)$$

As mentioned before, the term $f\left(\frac{d-h}{d}\right)$ must be multiplied by (d/s).

The term $\left(1 - \frac{sC^2}{g}\right)$ seems, to the writer, to have no place here. It is to appear in Eqs. 2 and 4, and should take the form:

$$1 - \left(\frac{sC_n^2}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3\left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right) \quad (73)$$

unless C is constant such that $C_n = C$, and unless the channel is rectangular so that $T_n = T_{cr}$ at two sections having the totally different depths Y_n and Y_{cr}. In

addition the rectangular cross section must be shallow, in which case the term (Y_{cr}/Y_n) is eliminated during the development of the formula. Here the ratio (T_{cr}/P_n) can be equated to unity.

It is only with these assumptions, which were cleverly adopted by Bresse, that the expression:

$$1 - \left(\frac{sC_n^{\,2}}{g}\right)\left(\frac{T_n}{T_{cr}}\right)^3\left(\frac{T_{cr}}{P_n}\right)\left(\frac{Y_{cr}}{Y_n}\right),$$

may reduce to $\left(1 = \frac{sC^2}{g}\right)$.

The term $\left(1 - \frac{sC^2}{g}\right)$ may be used for parabolic channels, as recommended by Tolkmitt, assuming a constant coefficient C, but only for shallow parabolic sections where (T_{cr}/P_n) may be equated to unity. Such shallow parabolic channels were defined by the aid of Eq. 7 as those which have the ratio T/Y greater than 16 so that R may be 0.66Y.

Moreover, (Y_{cr}/Y_n) should equal unity which is satisfied only when the flow is simultaneously critical and uniform, that is, when the bed slope is critical. As a consequence, (T_n/T_{cr}) will be unity when $Y_n = Y_{cr}$.

This limits the applicability of Eqs. 1 to 4 and Tables 1 and 2 to shallow parabolic channels laid with critical bed slopes, the fact being established that the equations and tables given in the paper are all correct, and that the disagreement between them and between the equations derived in this discussion is due to some error in the last ones, otherwise, the tables would have to be recomputed.

The writer does not understand Mr. Goodrich's statement that Bakhmeteff's tables are sometimes not easily applied, and believes that these tables are a good achievement. They can be applied to channels of any form and to cross sections of any proportions including the sections considered by Tolkmitt for which the paper was devoted.

In addition, Bakhmeteff's tables are suitable whether C is assumed constant or variable. C is supposed to be a function of R and then to vary with the depth Y as shown, for instance, in Eqs. 5 and 6 which, by the way, correspond only to very wide shallow cross sections like, for example, rectangular ones in which:

$$R = \frac{A}{P} = \frac{TY}{T + 2Y} = \left(\frac{TY}{T}\right) = Y \quad \ldots\ldots\ldots\ldots (74)$$

and

$$V = C\,R^{\frac{1}{2}}\ s^{\frac{1}{2}} = C\,Y^{\frac{1}{2}}\ s^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots (75)$$

where

$$C = \frac{1.49\ R^{2/3}}{n\ Y^{\frac{1}{2}}} = \frac{1.49\ R^{2/3}}{n\ R^{\frac{1}{2}}} = \frac{1.49}{n}\ R^{1/6} \quad \ldots\ldots (76)$$

Another thing is worth mentioning. With a dropdown curve, the depth gradually dimishes, in the direction of flow, until it reaches the terminal depth Y_t or d-h, at the end of the channel. Usually, it is not possible for the depth to attain a value less than Y_t which, according to Dodge and Thompson, is less than 2/3 of the critical depth Y_{cr}. If, for a parabolic channel, Y_{cr} is assumed,

roughly, to be about 3/4 Y_n, it becomes apparent that the minimum depth which may be encountered in a parabolic channel cannot be less than 0.5 Y_n and that the minimum value of X or Y_t/Y_n or (d-h)/d exceeds 0.50. The last pair of columns in Table 2 is thus redundant and can with no harm be omitted.

Not only these two columns, but also a great part of the preceding pair of columns is to be dispenced with as well. The reason is simple. The equations and tables, in the paper are based on the assumption that hydrostatic conditions prevail. This is not satisfied when the flow experiences a sensible curvature. Such curvature is expected when the depth of flow approaches Y_{cr}. The tables are then not applicable for some distance before Y_{cr} and between Y_{cr} and Y_t at the end of the channel.

When the tables are applied, the values of f (X) corresponding to values of X less than Y_{cr}/Y_n, that is, less than about 0.75 should be excluded.

It is to be noted that the length of the surface curve should not be computed by means of one operation between the depths d and d $\pm$ h, but with as many steps as possible, the required length being the sum of many short reaches.

SERGE LELIAVSKY,[38] F. ASCE.—The author has stated that "The formulas and tables by G. Tolkmitt were developed for channels of parabolic cross section and to the writer's knowledge are only to be found in the German texts by that author."

This statement calls for correction, because the writer has presented[39] Tolkmitt's tables, for both the backwater and dropdown curves, together with a complete explanation of their use (all in English).

The author states that "Since it appears that Tolkmitt's tables may fill a gap in the material available as an aid to the computation of backwater curves, and although admittedly the gap is not a serious one, they are submitted herewith." Now, in view of the preceding correction, the paper may have lost some of its importance. The tables were originally published, in German, in 1898.[40]

The writer had an opportunity to compare this formula with other backwater curves, when, in his early days in the Egyptian Irrigation Projects Department (1921), he was asked to fit a curve to the records of five or six gages on the Nile, within the limits of the Esna Barrage Backwater. He used the formulas of Rühlmann, Bresse, and Tolkmitt, with the coefficients being adjusted in each computation in such a way as to obtain the minimum standard error. These minimum values were found to correspond to the Tolkmitt's curve.

The writer wishes to add that, in addition to these three well-known formulas, he has also presented[39] the tables for backwater equations of various other authors: Schaffernak, Ehrenberger, Baticle, and so on. On the other hand, Bakhmeteff's and Mononobe's formulas are given in the form of curves, which, the writer finds, are more convenient than the tables for carrying out the computations in this particular case.

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38 Hydr. and Struct. Engr., Maadi, Egypt.

39 "Irrigation and Hydraulic Design," Vol. I (first published 1955, second impression 1959), p. 450 to 456.

40 Grundlagen der Wasserbaukunst, 1898, p. 113.

HOOD INLET FOR CLOSED CONDUIT SPILLWAYS[a]

Discussion by J. Ernest Flack, Harold W. Humphreys, and E. T. Smerdon, R. P. Beasley, and L. D. Meyer

J. ERNEST FLACK,[15] A. M. ASCE.—The author's contributions to design of culverts and closed conduit spillways have been significant. The design data relative to hood inlets should be of real value to those concerned with the design of small hydraulic structures.

With regard to the formation of a vortex over the inlet of a submerged culvert and to methods of inhibiting this formation it is interesting to note the similarity between the results obtained by the author[16] and those found in an investigation of box culvert inlets made at the Iowa Institute of Hydraulic Research some years ago.

In the Iowa tests, studies were made of flow conditions at the entrance to a standard culvert. The inlet had wing walls at 45° from the centerline of the culvert and a sloping headwall that conformed to an embankment and extended all the way to the toe of the embankment. For square edged submerged entrances a zone of separation formed just downstream from the crown. With the crown submerged two downward curving patterns developed in the upper corners of the inlet. The patter ı on the left side, looking downstream, turned counterclockwise and the pattern on the right side turned clockwise. If the water surface was above the top of the inlet a cross-flow developed over the top of the culvert. This circulation pattern combined with the curving pattern of flow that was rotating in the same direction to form a vortex. Generally, the counterclockwise vortex persisted and moved to the center of the inlet over the crown where it drowned out the clockwise vortex.

As the head increased, the area of the circulation pattern increased. At medium and high heads, $H/D \cong 1.5$, where H is the head above the invert and D is the height of the culvert, many of the vortices were intermittent. As the vortex gained strength the discharge was reduced because of air entrainment and the headwater depth increased. A strong vortex was often drowned out, apparently by a combination of headwater surface fluctuations and large scale eddies in the approaching flow. As the surface became calm, after a vortex was drowned out, the circulation was re-established. At higher heads, $H/D \cong 2$, the air cavity of the vortex was often disrupted near the culvert inlet. The vortex was then intermittent with quite long periods between formations of the vortex. At high heads vortices of opposite direction were observed circling around a weak central vortex. The core of the vortex, which had a high angular velocity, was of

[a] May, 1960, by Fred W. Blaisdell.

[15] Assist. Prof., Dept. of Civ. Engrg., Univ. of Colorado, Boulder, Colo.

[16] "Improved Culvert Inlet Design," by J. E. Flack, thesis presented to the State Univ. of Iowa, Iowa City, Iowa, in August, 1954, in partial fulfilment of the requirements for the degree of Master of Science.

relatively small diameter in all cases observed. The general circulation pattern outside the central core moved at relatively slow velocities.

The strength of the vortex did not appear to depend on separation at the culvert entrance. Strong vortices formed easily over well-rounded entrances with culvert flowing full. The existence of a barrier of some type in the path of the circulation pattern did have a great effect on strength of the vortex.

The zone of separation, if an isolated air cavity, depended on the vortex to supply air to it. At small slopes with the culvert flowing full downstream from the separation zone, the air cavity would not form, the culvert flowing alternately full and part full. As the head was increased to quite high values the culvert would prime itself when surface waves in the culvert touched the top of the culvert.

From these observations the over-all development of an improved culvert inlet proceeded by investigating methods of reducing separation and suppressing vortex action. The former could be accomplished by using a well-rounded entrance and the latter by preventing cross-flow over the culvert entrance. The triangular baffle aligned with the culvert was more effective than a headwall in suppressing vortex action because it served as a barrier to the cross-flow. Because the headwall was tangent with this cross-flow it had to be very wide to disrupt the circulation.

It has been a general observation that vortices do not exhibit the same characteristics in large scale structures as in small models. This may be because of the differing intensity of turbulence in the prototype than in the model or because in the prototype surface waves are more prevalent or because of the presence in the prototype of floating debris. All of these tend to inhibit the circulation in the prototype.

As a field of research it would be interesting to evaluate the scale effect of vortices along with a comparison of the amount of air entrained and the persistence of the vorticity in the culvert itself between model and prototype culverts.

Intermittent flow, described by the author as a combination of air and water flow, is presumed to be predictable. However, tests on well rounded entrances indicate that generally this type of flow is least susceptible to model simulation,[17] because the initial cause of separation has been eliminated at the entrance and the separation point moves downstream an indeterminate distance. Such things as variation of the flow rate, a surface irregularity, or a variable degree of turbulence of the flow and, in the model, the influence of capillarity and surface tension, will shift the point of separation upstream or downstream.

If entrance velocities are small enough circulation and vortex action are not important. If larger anti-vortex devices are needed splitter walls work best because they are radial to the circulation. With very high entrance velocities, such as when severe separation takes place, the poorest condition prevails but prototype behavior is predictable.

With the hooded inlet the effect of the baffle plate is to cut off the circulation above the entrance from the main entering flow. The entrance area is greatly enlarged and entering velocities are small. A laterally convergent inlet, will, of course, produce the same results so that the velocity at the inlet vicinity will not be sufficiently great to produce more than desultory circulation and vortex formation.

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17 "Hydraulics of Box Culverts," by Donald E. Metzler and Rouse Hunter, Bulletin 38, Studies in Engineering, 1959, State University of Iowa, Iowa City, Iowa.

While it can be agreed that the Froude criterion for model and prototype during water flow only is valid and it may well be that during air-water flow the difference in headpool level due to dissimilarity in air flow quantities over that predicted by the model is relatively small when considering the prototype performance, nevertheless the irregular flow is undesirable. The head-discharge relationships at which mixed flow and alternately rising and falling headpool levels occur can not be predicted from models with well rounded entrances since the effects of surface tension and capillarity are not known.

With the point of separation fixed and related to the entrance submergence as it is in the hood inlet the air-water flow is predictable. Although the quantity of water discharge under such conditions is predictable the occurrence of alternately rising and falling headwater on a larger scale in the prototype could lead to undesirable pulsations.

HAROLD W. HUMPHREYS,[18] M. ASCE.—The author is to be commended for making this important contribution to the understanding of closed conduit spillway hydraulics. The paper should be of considerable value to the designer in comprehending the complicated flow phenomena encountered in this type of spillway.

The Illinois State Water Survey (ISWS), Urbana, Illinois, has performed tests on drop inlet closed conduit spillways that are pertinent to this discussion. The spillway consists of a 3 in. dia (D) barrel on a 30% slope. The barrel is 100 barrel diameters (100D) long, and the outlet discharges freely into the atmosphere. The drop inlets tested are five barrel diameters (5D) high and are circular, square, and rectangular in plan. The transitions between the drop inlet and the barrel have been well-rounded as well as square-edged.

The 100D long barrel is of sufficient length to obtain a well established friction grade line. This is a straight line obtained by the method of least squares for six piezometers located along the barrel at 10D intervals from stations 45D to 95D. Each piezometer station consists of four piezometers drilled at 90° intervals around the pipe periphery and connected to a common manifold. The piezometric heads for all six piezometer stations are obtained simultaneously by photographing a piezometer board.

The ISWS spillway barrel is on a slope that is considerably steeper than the friction grade line. The barrel flows full without submerging the outlet, and the barrel pressure is sub-atmospheric along most of its length. Therefore, this spillway meets the author's definition of a steep slope. The friction grade line and the hydraulic grade line coincide for most of the barrel length although they diverge near the barrel entrance. The local deviation between the friction and hydraulic grade lines is known as h_n and has been fully defined elsewhere.[19,20]

Typical values for the ISWS spillway of h_n divided by barrel exit velocity head, h_{vp}, are shown in Table 3 for a crown piezometer located near the barrel entrance. The location of the piezometer was measured along the barrel crown from the intersection of the crown and the inside edge of the drop inlet.

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[18] Head, Hydr. Research Section, Illinois State Water Survey, Urbana, Ill.

[19] "Hydraulic Fundamentals of Closed Conduit Spillways," by Fred W. Blaisdell, Proceedings, ASCE, Vol. 79, Separate No. 354, November, 1953.

[20] "Hydraulics of Closed Conduit Spillways, Part 1, Theory and Its Application," by Fred W. Blaisdell, St. Anthony Falls Hydraulic Laboratory Technical Paper No. 12, Series B, Minneapolis, Minn., January 1952, Revised February 1958.

Harris[21] has reported that h_n can be as low as $-1.27\ h_{vp}$ for a flush square-edged intake to a pipe.

The drop inlet is geometrically quite different from the hood inlet, and the h_n values reported in Table 3 should not be applied to a hood inlet spillway. They are presented to emphasize the fact that for closed conduit spillways on a steep slope the negative barrel pressure, as computed by extending the straight part of the hydraulic grade line to the barrel entrance, is not necessarily the lowest pressure in the barrel. Cavitation near the barrel entrance is a definite possibility. Therefore, computation of the local pressure should be included in the design to insure the elimination of cavitation. It is the writer's feeling that h_n/h_{vp} values for the hood inlet would be of considerable benefit to the designer.

Eq. 6 is correct for computing the spillway discharge for full pipe flow; however, applying this equation is difficult without having some knowledge of the variation of β. When the spillway operates under a relatively high head, β may be taken as 0.5 without incurring much error in computing the discharge, although the error becomes increasingly important as the operating head decreases. French[22] and Rueda-Briceno[23] have reported results that should be helpful to the designer in selecting values of β. A comparison of β values for the hood inlet with those of French and Rueda-Briceno should be interesting.

TABLE 3.—VALUES OF h_n/h_{vp} FOR CROWN NEAR BARREL ENTRANCE

Series (1)	Drop inlet size (2)	Piezometer location (3)	Transition (4)	h_n/h_{vp} (5)
24	4D/3 round	0.46D	well-rounded	-0.10
28	5D/3 round	0.43D	well-rounded	+0.12
65	1.5D square	0.32D	well-rounded	+0.14
78	1.75D square	0.41D	square-edged	-0.36
75	D x 4D rectangular	0.43D	square-edged	-0.86
82	D x 2D rectangular	0.40D	square-edged	-0.79

An experimental program that has as many phases as the hood inlet closed conduit spillway requires either a large number of personnel or very efficient test apparatus. The author states he has been conducting tests on closed conduit spillways since 1941, and during this time he has undoubtedly devised techniques that would be helpful to others. The writer is of the opinion that a more detailed description of the experimental apparatus and procedure would be of considerable interest and benefit to other researchers.

21 "Hydraulic Flow Characteristics of a Square-edged Intake," by C. W. Harris, Bulletin No. 61, Univ. of Washington, Engineering Experiment Station, Seattle, Wash., March 15, 1932.

22 "Second Progress Report on Hydraulics of Culverts, Pressure and Resistance Characteristics of a Model Pipe Culvert," by John L. French, National Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C., unpublished, October 29, 1956.

23 "Pressure Conditions at the Outlet of a Pipe," by Daniel Rueda-Briceno, thesis presented to the State University of Iowa, Iowa City, Iowa in February, 1954, in partial fulfilment of the requirements for the degree of Master of Science

E. T. SMERDON,[24] A. M. ASCE, R. P. BEASLEY,[25] AND L. D. MEYER.[26]—This paper clearly indicated that modification of the inlet of culverts and tube spillways can radically change the performance of such a structure. Similar work by the writers have produced results which point to the fact that some aspects of the hood inlet may be objectionable and unnecessary. For example, the anti-vortex wall described by Mr. Blaisdell will be large in prototype installations and often require considerable structural bracing to prevent vibrations and mechanical damage. The fact that the simpler anti-vortex plate of Fig. 8 (f) works satisfactorily indicates that simpler devices for the control of vortices are possible.

Recently, data were published concerning an inlet for closed conduits which gives vortex control without requiring anti-vortex plates or walls which extend beyond the conduit barrel.[27] The inlet is shown in Fig. 18. This inlet, which is called the canopy inlet, primes at a low depth of submergence over the invert of the inlet giving increased capacity similar to that associated with the hood inlet. However, upon priming, there is a sudden change from weir flow to pipe flow with little or no so-called slug flow. Therefore, the outlet channel must always be designed to handle that discharge which corresponds to pipe flow for the particular installation.

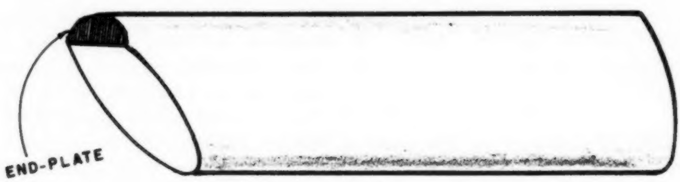

FIG. 18.—CANOPY INLET FOR CLOSED CONDUITS.

Mr. Blaisdell states that there seems to be no advantage in using hood lengths greater than 3D/4. In the canopy inlet, (on which a special anti-vortex device is not used), there is a definite relationship between the tube slope and the canopy length which is required for vortex control. As the tube slope increases the canopy length must also be increased to achieve satisfactory performance. The data in Fig. 7 indicate very severe reduction of capacity due to vortex action. Although, the hood length and tube slope for these tests were not given, it is very possible that the effect of these vortices would have been reduced if the hood length had been increased. The writers tests showed that the tendency for vortices to form in the hood inlet without anti-vortex device was reduced by increasing the hood length. The modification given in Fig. 18 further reduced vortex action, thus its desirability.

The writers realize that it may be hazardous to predict vortex action in large prototype installation from small scale model studies. Therefore, the canopy inlet tests included the field testing of a 1 ft dia corrugated metal structure

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[26] Agric. Engr., ARS, USDA, Lafayette, Ind.
[27] "Canopy Inlet for Closed Conduits," by R. P. Beasley, L. D. Meyer, and E. T. Smerdon, Agricultural Engineering, Vol. 41, No. 4, April, 1960.

105 ft long. The invert of the outlet was 14.5 ft lower then the invert of the inlet. The angle and velocity of the approaching flow were varied in an attempt to create a condition favorable for intense vortex action. Some small discontinuous vortices were noticed with some of the flow conditions, but in no instance was the vortex large enough to cause any measurable reduction in the capacity of the structure. Therefore, the writers feel that the canopy inlet, which does not require a special anti-vortex wall, also shows promise as an inlet for closed conduit structures, particularly agricultural.

There is apparently a minor error in Eq. 14a. This equation as written does not correspond to the so-labelled line in Fig. 15.

UNIFORM WATER CONVEYANCE IN ALLUVIAL MATERIAL[a]

Discussion by Gerald Lacey, Peter Ackers, G. Kalkanis,
John F. Kennedy, T. Blench, and Claude Inglis

GERALD LACEY.[4]—The authors' figures and equations embrace a wide range of channels of greatly varying characteristics. That so heterogeneous a collection of data should on analysis show so close a resemblance in respect to exponents to those obtained from considerations of regime theory is indeed remarkable. The authors' equations, whatever may be advanced in respect of regime relationships, are patently empirical and useful on that account if used with discretion.

The Bibliography (Appendix II), is significantly incomplete in respect to some of the papers published earlier on stable channels and regime; the authors' review of current methods of design and their comments on the "inadequacy of regime methods" bear evidence that they have relied too implicitly on commentators rather than on a close study of the original papers, the valuable discussions thereon, and the authors' replies.

For the record, the writer should explain that he was not, in 1927 "commissioned by the Governments of England and India to systematize all data that had been collected," nor was Laliavsky, a very recent commentator, correct in stating that the writer was "officially instructed by the irrigation authority by which he was employed to put some order into the mass of data available and produce if possible a standard design method."

The facts are that the post of Irrigation Research Officer having been created, the writer was the first to fill it. His instructions were to collate all the literature and published data relevant to irrigation science then available. The first task that he set himself was the analysis of all the authoritative hydraulic data to which he had access. It was at a later date, following the favourable reception accorded his first published paper, that he was officially instructed to prepare a "Technical Paper" for departmental use in the design of irrigation channels.

Figs. 1 and 2 would have been improved had they plotted the water surface width, W_s, the mean depth, D_m, instead of the average width, W, and the averaged bed depth, D. Not only are the parameters W_s and D_m much easier to measure, but in all large channels, and in rivers in particular, they are almost identical with the wetted perimeter, P, and R, the hydraulic mean depth, respectively.

Had this principle been adopted in plotting Fig. 1, the manner in which the value of the hydraulic mean depth steadily approached the value of D_m, with increase in depth, would have become evident immediately. Clearly, the plot must give an asymptotic curve.

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[a] May, 1960, by D. B. Simons and M. L. Albertson.

[4] Cons. Engr., London, England.

Similarly, if in Fig. 2 the water surface width had been plotted, instead of the mean width, a better correlation could have been obtained. The straight line plot is unconvincing. The plot should have been asymptotic and would have given an equally good fit.

From the relationship

$$A = P\,R = W_s\,D_m$$

we have

$$\frac{R}{D_m} = \frac{W_s}{P}$$

and both functions have a limiting value of unity. In Fig. 1 the authors should have drawn a smooth curve asymptotic to the line, $R = 0.90\ D$. Similarly, in Fig. 2 a smooth curve should have been drawn asymptotic to the line, $W = 0.90\ W_s$.

These difficulties have arisen solely because the authors have adopted the Blench parameters of mean width and "averaged" depth. The writer considers the concept of a system of channels consisting of trapezoidal cross sections and constant side slopes somewhat illogical and prefers the simpler and more rational system with a constant ratio of horizontal bed width to water surface width. The writer has, for the last five years, been designing irrigation channels in Iraq with a ratio of bed width to water surface width of 0.80, that he based on his experience in India. The authors' data, as plotted in Fig. 4, provides justification of a remarkable character for the adoption of this eminently practical procedure. It takes account of the well known principle of exaggeration in the vertical scale of small channels and provides a useful working rule.

TABLE 4.—VALUES OF W AND W_s

W, actual (1)	W_s, computed: By authors (2)	W_s, computed: By writer (3)
20.0	16.4	18.0
100.0	90.0	90.0
300.0	274.0	270.0
500.0	458.0	450.0

The writer's equation is $W = 0.90\ W_s$, and that of the authors' $W = 0.92\ W_s = 2.0$. Table 4 shows the close agreement for all mean widths exceeding 20 ft.

The writer's formula, $V = 16.0\ R^{2/3}\ S^{1/3}$, that the authors have displayed in Figs. 7 and 8, is unusual as it contains no rugosity coefficient. The principle is that in any loose bedded natural channel, free either by deposition or scour to develop its own depth and slope, the rugosity is implicit in the depth and the slope the channel demands. For this reason the equation is very useful for determining the flood discharges of torrents with gravel, shingle, and boulder beds when no other means are available. One defect is that there is no factor involving the sediment load, that is almost invariably impracticable to measure; nevertheless, the equation has been tested and used with success, notably in New Zealand.

This general equation can be derived directly from two equations of the writer:

$$V = \frac{1.3458\ R^{3/4}\ S^{1/2}}{N_a} \quad \ldots\ldots\ldots\ldots\ldots \quad (31)$$

and

$$N_a = 0.0225\ f^{1/4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

and the authors' Fig. 7 purports to show its agreement with the observed data. This figure has defects for which the authors are in no way to blame,

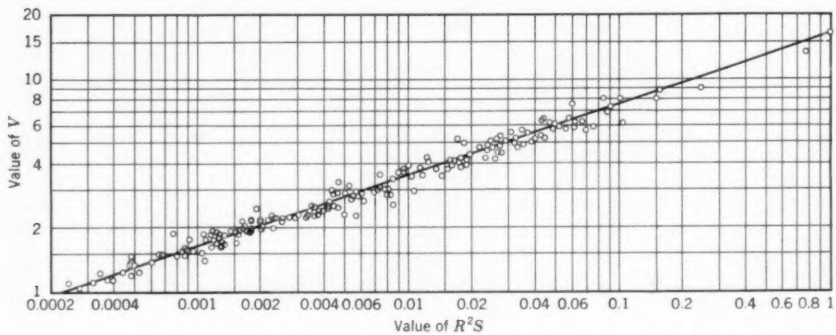

FIG. 13

because they copied it faithfully from Sir Claude Inglis' work on the behavior and control of rivers who had, in turn, relied on a 1930 work of the writer (28)[3]. The equation was first given by the writer in a 1930 discussion (26), and the data was first plotted by the writer in 1934 (27). Fig. 13 is a reproduction of this figure and deserves study.

The defect of Fig. 7 is that it was an actual re-plot to a different scale of the writer's data. It will be noted that the vertical scale logarithmic of the figure is twice that of the horizontal scale, so that the visual scatter is doubled, this is not a matter of great moment, but, unfortunately, the figure contains two highly discordant observations on the right hand side. Of these, the first was the mean of no less than 53 observations over a 25 mile length of the Lower Bari Doab, the data being

V	R	$S \times 10^3$	$R^2\ S$
3.530	8.843	0.1223	0.00956

but the printer preferred to insert the figures 0.0956 in the last column. The writer's observation, as plotted, is thus subject to an error of 1000% owing,

[3] Numerals in parenthesis, thus (1), refer to corresponding items in the Supplementary Bibliography.

in the first instance, to a misprint. The second discordant plot is due to a like cause.

The overall fit of the writer's early general equation, as shown in Fig. 8, is very satisfactory. It would indicate that the depths and gradients as finally recorded were dictated by deposition of sediment by the channels and not by the engineer. It must also be remembered that with this equation the roughness coefficient is implicit and the missing variable is the sediment charge. The correct general equation is probably of the form

$$V = \text{const. } R^{1/2} \left(R^{1/2} S\right)^n \quad \ldots\ldots\ldots\ldots\ldots \quad (33)$$

the value of n being 0.50 for very fine sediment, 0.333 for medium sand, and less for the coarse non-cohesive material. An analysis on these lines might be worth while. The Imperial Valley canal data clearly show, as the authors have emphasised, that for a given value of $R^2 S$ the mean velocity V is a function of the charge. In terms of the Inglis-Lacey equations, the expression would be

$$V = \text{const. } X^{1/6} R^{2/3} S^{1/3} \quad \ldots\ldots\ldots\ldots\ldots \quad (34)$$

in which X is the charge in parts per million.

The scatter in the Blench diagram, Fig. 9, is rather great, and the contention by the authors that the exponents in the upper range could be replaced by constant values of $V^2/(g\, D\, S)$ is justified. It would certainly be interesting to attempt, for the same data, an evaluation of the function

$$\frac{V}{(R\, S)^{1/2}} = \text{const. } R^n \ldots\ldots\ldots\ldots\ldots \quad (35)$$

this would give a value of n of 1/6 for the Manning rigid boundary equation. For a moving boundary, the value is 1/4.

The authors' review of the writer's 1930 equations is incomplete and does not contain the slope equation that he presented in his reply to the discussion. Eq. 8 is inaccurate, as it assumes that the Manning equation is applicable to a moving bed, and in Eq. 7 the power of f should be 2 and not unity. It is evident that they have not studied Etcheverry's admirable presentation in his work on Land Drainage (32), first published in 1931. Particular attention is drawn to his appreciation that the channel should be "formed in its own silt."

The authors state that the Punjab Irrigation Research Institute equations represent years of painstaking and statistical analysis of data. Table 5 compares the writer's early equations with those of the Institute. It must be borne in mind that the writer's equations covered sediment of a very wide range of fineness, but they were derived in part from Kennedy's researches in the Punjab, and Kennedy channels as a standard of reference were assigned by the writer a silt factor, f, of unity.

In the Research Institute equation for the slope, m indicates the diameter of the bed sand particle in millimetres. In his paper the writer had suggested that the factor f varied as the square root of the diameter of the particle; this would transform f in his equation to $m^{.833}$, as compared with the Institute value of $m^{.86}$.

The defect of the early equations is that they contained no coefficient for the charge. This affects the width as well as the other relationships. Within any given canal system, the charge is however tolerably constant. The second defect is that, so far as the cross section is concerned, width and depth are preferable to wetted perimeter and hydraulic mean depth.

In these channels the discharges remained practically constant as do the grade of bed sediment and the charge. The channels were self-formed with bed and berms of the same material as that transported. Modern regime theory postulates that in such circumstances

$$W = \text{const. } Q^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

$$D = \text{const. } Q^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

from which two equations (and only two are needed)

$$V = \text{const. } D^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

indicating a constant Froude number. Further, since the increase in the slope

TABLE 5.—COMPARATIVE TABLE

Function (1)	Writer (2)	Institute (3)
V	$1.17\ f^{1/2}\ R^{1/2}$	$1.11\ R^{1/2}$
P	$2.67\ Q^{1/2}$	$2.80\ Q^{1/2}$
R	$0.47\ (Q/f)^{1/3}$	$0.47\ Q^{1/3}$
$S \times 10^3$	$\frac{0.55\ f^{5/3}}{Q^{1/6}}$	$\frac{2.09\ m^{.86}}{Q^{.21}}$

of the smaller channels is none other than the exaggeration in the vertical scale we are led to the ineluctable conclusion that

$$\frac{D}{W} = \text{const. } S = \frac{\text{const}}{Q^{1/6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

In his latest paper (30) the writer has suggested that the slope equation requires modification in small channels in which the incidence of the sides in destroying energy is highest. The equations as stated are, however, the basis of all modern regime theory. Leopold and Maddock have demonstrated that sandy bedded river systems exhibit a relation between the discharge and the width consonant with regime theory. A manifest difficulty with flashy rivers is the selection of a discharge as a basis of reference. In any one flashy system, if the flood frequency is the same throughout, the average annual discharges are possibly the best compromise. For a great river like the Mississippi, in the lower part bank, full stage is the best criterion.

In all river systems with sandy beds, the sand is in general finer the further one proceeds from the source, that is, the greater the discharge. If, then, one considers the equation

$$D = \text{const. } \left(\frac{Q}{f}\right)^{1/3} \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

a little reflection will show that the power of Q in any such river system should be greater than 1/3. The average power of Leopold and Maddock is 0.40.

It would be rather naive, and "fallacious," as Leopold and Maddock have stated, to take observations of any river at one site and expect the widths to vary as the square root of the discharge; the fact that they do not so vary has been mistakenly advanced by Leliavsky as proof that the square root relationship is illusory.

Sir Claude Inglis recently made a great advance by the introduction of charge into regime relationships. The writer would however plead, when accused of ignoring charge, that as early as 1936 he agreed (31) that on the Imperial Valley canals "the fine heavy silt charge demands a velocity of the same order as that demanded by a smaller charge of coarse silt in India."

If we consider the equations for the Miami River system that Pettis produced, to which the authors' obliquely refer, and of which Leliavsky makes use, we must remember that they are not comparable to basic regime equations. There is no evidence of constancy in the grade of material transported or in the charge; the description of the Miami bed material at Dayton and Hamilton, clay sand and gravel, bed and greater part sides gravel sand and clay, as given in the Miami Conservancy Report, suggest that the bed is not of loose material, and that there might be a considerable difference between silting and scouring velocities, as Pettis (no adverse critic of regime theory) suggested. Leliavsky, however, states boldly that any attempt to widen the

TABLE 6.–COMPARATIVE TABLE, MIAMI RIVER SYSTEM AND REGIME EQUATIONS

Function (1)	Pettis Miami River (2)	Writer regime (3)	Leliavsky's version (4)
V	$0.8\ Q^{.2}$	$0.8\ (Q\,f^2)^{1/6}$	$0.80\ Q^{1/6}$
V	$1.25\ R^{2/3}$	$1.17\ f^{1/2}\ R^{1/2}$	Omitted by Leliavsky
W_s	$2.45\ Q^{.5}$	$2.67\ Q^{.5}$	$2.67\ Q^{.50}$

range of application of the writers exponents (that, incidentally, are those also of Sir Claude Inglis and Blench) is a failure. Of this, Leliavsky is persuaded because "the Pettis exponents," for a river system the writer must interpolate," are quite different." Why indeed should they not be since the differences in the exponents as Leopold and Maddock have shown betray the character of the river? Table 6 shows part of the comparative table given by Leliavsky, bearing in mind that only two equations are required. Note the somewhat extraordinary omission by Leliavsky of the silt, or sediment factor, f.

Leliavsky makes no comment on the fact that the Pettis equations for a river system are tolerable confirmation of the width-discharge relationship. Also, because Leliavsky omitted the silt factor, f, the fact that a systematic variation within the river system might account for a change in the exponents is not immediately apparent in the table presented by him.

The Pettis equation for the velocity, in terms of the hydraulic mean depth, deserves examination on its own merits for it indicates that

$$\frac{V^2}{R} = \text{const. } R^{1/3} \quad \ldots\ldots\ldots\ldots\ldots \quad (41)$$

that suggests that as one proceeds down this river the grade becomes coarser, or else there is an increase in bed load.

Leliavsky states that the application of regime theory to the design of channels is confined to India alone, is not accepted for practical application by canal designers in America, or in European countries such as Italy and Spain in which irrigation is practiced on a large scale; he states that the same is also the case with Egyptian, Russian, South African, and Australian designers. The authors state that, although popular in India, regime theory has never been extensively used anywhere else in the world. The writer contends that use, or refusal to use, is not a test of validity. So far as the United States is concerned, lack of use arises in part from lack of understanding of what regime theory really implies and to the limited field of application, other than the control of great rivers.

As to the comments of Leliavsky, the writer has yet to learn that great new irrigation canals remain to be constructed in Italy or Spain. In India, Pakistan, Burma, Iraq, and the Soudan practical use is made of regime theory.

As to Russia, Leliavsky is in error. Girshkan, in a recent paper (33) describing problems arising in the design of irrigation systems and the empirical methods used, commented:

> "It should be pointed out that by solving the mutually known generalizing functions of G. Lacey which he suggested for stable channels in alluvium and taking into account some of his remarks on depositing particles, rectilinearity of sections, channels, etc., we shall have for depth of flow, wetted perimeter, wetted cross-section and width of the canal expressions very close to those cited."

Similarly, Sharov in his treatise on the operation of hydraulic working systems (34), states that the coefficient of roughness of a canal is given in metric units by the equation

$$N = \frac{R^{3/4}\ S^{1/2}}{V} \qquad (42)$$

that is identical to what the writer suggested 30 yr ago. Further, Sharov when considering permissible seepage losses in canals in cumecs per kilometre expressed as a percentage, gives a table for the equation

$$p = \frac{3.0}{Q^{1/2}} \qquad (43)$$

There is, thus, no doubt of the practical use of regime theory so far as Russian workers are concerned. As to the validity of the basic regime exponents, time has gone far to confirm rather than refute them. It must be realized that regime theory is a new science, and for those who have devoted a lifetime to it, there is the comfort that it offers as many pitfalls to its critics as it does to its exponents—exponents in the sense of those who expound the theory, and not those of the Leliavsky type.

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PETER ACKERS.[5]—The authors have made a very useful examination of all data available to them on the stable dimensions of alluvial canals that is critical and unbiased, yet makes no attempt at providing theoretical bases for the empirical equations that result. The study is thus of particular value to the engineer faced with the task of alluvial channel design, for it provides him with general corroboration of the regime concept of the 'Indian' school, with additional coefficient data to suit other types of channel. The first of the three objectives of the authors' research has thus been very satisfactory achieved, though our understanding of the fundamental physical aspects of the subject is not materially improved.

The writer prefers the expression 'regime concept' to the more generally used phrase 'regime theory,' for, although attempts have been made to provide the empirical equations for stable channels with a scientific background, they still remain essentially experimental in character. The existence of preferred channels dimensions for a given discharge and charge, in a given material, has been established in nature, but no completely satisfactory theoretical basis for the form of these relationships between the geometrical parameters and discharge has yet been propounded. This remains one of the most vexed problems in free-boundary hydraulics, so that the word 'theory' is rather a misnomer at the present time.

In their historical review of the development of the regime concept, the authors do not mention the significant recent developments by White (35), (36)[3], Inglis (37), (38), on Lacey (39), introducing the sediment charge as an independent variable. Unfortunately, the experimental data in support of the form of their proposed 'charge' equations has not yet been published. Furthermore, these recent equations are not specific, because coefficient values have never been stated, and this detracts from their value. Lacey's most recent group of

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primary equations is as follows:

$$V \propto \frac{g^{\frac{1}{2}} R S}{d^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

$$\frac{V^3}{W_T} \propto g^{3/2} \ d^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

and

$$\frac{V^2}{g \ D_m} \propto I^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

in which $D_m = A/W_T$, and $I = X \ V_s/(\gamma \ g)^{\frac{1}{3}}$ with X = sediment charge and V_s = terminal velocity of transported solids. Lacey now uses mean depth and water surface width in place of his original choice of hydraulic mean depth and wetted perimeter as significant parameters. Moreover, comparing Eq. 46 with Lacey's original

$$V = 1.17 \ \sqrt{f \ R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which $f \sim 8d$ (d in inches) it is seen that the median particle diameter no longer occurs explicitly in the basic relationship, being replaced by $X \ V_s$. Eq. 46 will be recognized as implying a constant Froude number for a canal system in a given material with uniform silt load. This is consistent with the authors' statement that "if a channel is to be stable . . . the Froude number . . . must in most cases be less than 0.3 for alluvial material in the sand-size range and finer."

The writer has been in charge of a basic investigation into the dimensions of stable channels by experiments on 'model' canals, at the United Kingdom's Hydraulics Research Station. Tests so far made in sand of 0.16 mm median diameter at discharges from 1 to 5 cusecs confirm that $V/\sqrt{(g \ D_m)}$ must lie between 0.27 and 0.33 for stability with an active sandy bed. Also confirmed is the authors' top limit of about 500 ppm for the transport rate in such channels. In the writers experience, at about 150 ppm minor shoaling begins that may lead to bank erosion, followed by meandering as the charge increases further. These high charges also cause an increase in F_r, though from results obtained to date the dependence on concentration does not appear to be so marked as Eq. 46 would predict. The authors' opinion would be welcomed as to why the Froude number should prove to be constant in canals of a given character. It is by no means obvious why the ratio of dynamic to gravitational forces in the fluid should be pre-determined in a free-boundary channel, as has has been demonstrated in both field and laboratory.

It is of interest to note that the data obtained at Wallingford for small streams are in general conformity with the authors' graphs. For example, our plot of R against Q has a slope of about 0.40, close to the recommended exponent of 0.36, passing through Q = 5 cusecs, R = 0.52 ft. The authors' line C, for "sand bed and banks" is generally consistent with the trend of the

Wallingford data, although point 16 and point 17 extracted from Simons and Bender for comparable bed material lie appreciably above this line. However, these sections probably had some bank cohesion, permitting greater depth. Our P versus Q plot would pass through Q = 5 cusecs, P = 7.7 ft, with a slope of about 0.45 compared with the authors' exponent of 0.512; it thus seems that the P-Q graphs may curve slightly away from the simple exponential law at low discharges. The relationship of A to Q, and by implication V to Q, obtained in the Wallingford tests, is in close agreement with the basic equation the authors recommend; $A = 1.076\ Q^{0.873}$, lying generally some 10% low. (Q = 5 cusecs A = 3.9 cusecs).

The authors, considering the apparent influence of sediment concentration on width, conclude from Fig. 3 that, "as a fine sediment load of the type found in the Imperial canals decreases, the stable wetted perimeter, P, and consequently stable width, W, decreases." The Imperial Canal data included in this diagram do show this trend, the points for high concentration lying above the mean line and those for low concentrations lying below it. However, the Imperial Canal data as a whole lie below the Punjab and Sind graphs. The high concentrations in the former system are apparently carried with lower canal widths than the low concentrations of the latter systems, discharges being equal. Is there any explanation of this apparent anomaly?

The comparisons of data with the regime equations for slope are characterised by greater scatter than is shown by the other geometric relationships that, as the authors suggest, may result from the difficulty of measuring small slopes in the field. Nevertheless, it is interesting to note that they were able to confirm the general validity of the Lacey equation, admittedly with some uncertainty, at the large-canal end of plot; but, they had to modify the Blench exponent from 0.25 to 0.37 to achieve a reasonable fit to his form of slope equation. Blench has supported the expression of $V^2/(g\ D\ S)$ as a function of $V\ W/\gamma$ by analogy with the usual friction factor/Reynolds number plot that for smooth pipes within a limited R_e range has an exponent of 0.25. The authors' data, thus, do not appear to support the arguments put forward by Blench concerning the physical significance of his equations. Lacey's inspired empiricism is proved adequate for design purposes.

The comparison of stable channel data on the basis of tractive force is highly illuminating. Fig. 11 is of interest, not so much in what it proves but in what it fails to prove. Surprisingly, it does not give support to the reasonable contention that there is a unique relationship between allowable tractive force and grade of bed material. The plot of field data shows a strong correlation with discharge, so that the tractive force in small stable channels is much less than that existing in large canals. The variation is, for example, from 0.025 psf at 146 cusecs up to 0.075 psf at 5676 cusecs in canals with 0.3 mm bed material. It follows then that mean overall boundary shear is not, by itself, an adequate criterion for channel stability.

A little algebraic manipulation will also show that the original regime equations were not consistent with the concept of a limiting boundary shear depending solely on bed material size, as follows:

$$P \propto Q^{\frac{1}{2}} \quad \text{(48a)}$$

$$R \propto Q^{\frac{1}{3}} \quad \text{(48b)}$$

therefore

$$A \propto Q^{5/6} \quad \text{(48c)}$$

and

$$V \propto Q^{1/6} \propto R^{\frac{1}{2}} \quad \text{(48d)}$$

But

$$V \propto R^{\frac{2}{3}} S^{\frac{1}{3}} \quad \text{(48e)}$$

therefore,

$$R^{1/6} S^{\frac{1}{3}} = \text{constant} \quad \text{(48f)}$$

Hence, boundary shear, γ R S $\neq$ constant. Yet, when the exponents as modified by the authors are used in the analysis;

$$P \propto Q^{0.51} \quad \text{(49a)}$$

$$R \propto Q^{0.36} \quad \text{(49b)}$$

therefore

$$A \propto Q^{0.87} \quad \text{(49c)}$$

and

$$V \propto Q^{0.13} \propto R^{0.366} \quad \text{(49d)}$$

But

$$V \propto R^{\frac{2}{3}} v\, S^{\frac{1}{3}} \quad \text{(49e)}$$

therefore

$$R^{0.30} S^{0.33} = \text{constant} \quad \text{(49f)}$$

Hence, boundary shear, γ R S $\simeq$ constant, but the Froude (Boussinesq) number, $V/\sqrt{(R\,g)}$ should vary. Again, there is a conflict of evidence. It is expected that the authors' exponents in the R-Q and P-Q equations, coupled with their confirmation of the Lacey slope equation, would yield the sought-after link with the boundary shear concept. Yet, unexpectedly, Fig. 11 does not provide adequate confirmation of this deduction.

A further interesting feature about Fig. 11 is that the groups of canals, within themselves, show a definite trend towards a coarser graded bed in the large members than in their smaller progeny, reflecting the natural tendency for the coarse material to settle early in the system. It follows, therefore, that the basic regime equations for canals recommended by the authors include as a hidden parameter the natural variation of d with Q, and, if this effect could be separated, the exponents of the main equations for a constant grade of bed material must differ somewhat from those normally quoted. This may explain the departure of the authors' equations from the primary relationship, that is, V^2/R or $\left(V^2/D_m\right)$ is constant for a given bed material.

The authors have performed a very useful service in making this exhaustive examination of stable channel data, confirming the soundness of the regime concept, and providing new coefficient values extending the utility of the original Indian equations. Confirmation of the latest developments incorpor-

ating the sediment concentration is still awaited, however, although, because of the uncertainty inherent in field measurements of sediment load and the narrow band of variation in some canal systems, such support from field data may not be readily obtained. Laboratory experiments such as those now in hand in Wallingford may provide some guidance, but it is to be hoped that those with access to field data will bear this important aspect of the subject in mind.

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G. KALKANIS,[6] M. ASCE.—Hydraulic engineers may be divided into three main groups in their attitude towards the Indian or Regime Theory. First, are those who believe that the theory has a general and universal application to the design of stable channels; second are those who do not question the validity of the theory, as long as the basic assumptions made in its derivation are satisfied; finally there are those who dismiss it altogether because they think that it does not offer sufficient evidence of successful application in places other than India. This paper presumably aims to create a movement from the third group to the second and, if possible, from the second to the first. Numerous canals in the United States were investigated, and the data obtained from measurement after being properly analyzed showed to be in good agreement with those of the Punjab Canals.

The authors should be congratulated for presenting their material in such a clear and convincing way; one may be confident that few individuals will remain in the third group after reading the article. However, the writer believes that the authors were not equally successful with the second group. The purpose of this paper is not to discuss the merits or the drawbacks of the regime theory, for it is hoped that that subject has been exhausted. This review will be confined to some aspects of the study presented by the authors.

The writer believes that establishing the Froude's number as a criterion of stability was unfortunate for at least two reasons.

1. It contradicts the implication advanced by the article that the regime theory and the tractive force theory are practically equivalent. This can be shown by using the Froude's number as a criterion of stability together with

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one of the friction formulas, such as Eq. 27. Two channels having the same bank and bed material will both be stable as long as the flows in both have the same Froude's number

$$\frac{V_1}{\sqrt{g\ D_1}} = \frac{V_2}{\sqrt{g\ D_2}} = .3 \quad \text{(50)}$$

But from Eq. 27

$$\frac{\dfrac{V_1}{\sqrt{g\ D_1}}}{\dfrac{V_2}{\sqrt{g\ D_2}}} = \frac{R_1^{1/6}\ S_1^{1/3}}{R_2^{1/6}\ S_2^{1/2}} = 1 \quad \text{(51)}$$

(Substitute R for D because their relationship is linear according to Fig. 1). Therefore

$$\frac{\tau_1}{\tau_2} = \frac{\gamma\ R_1\ S_1}{\gamma\ R_2\ S_2} = \frac{R_1^{1/2}}{R_2} \quad \text{(52)}$$

However, according to the tractive force theory, the two channels should have equal tractive stresses or $\tau_1 = \tau_2$.

2. It contradicts the statement that the three equations of the Regime theory constitute a complete system having a unique solution. If the preceding statement is correct any additional relationship between the basic parameters make the system overdetermined. The resulting inconsistencies invalidate the theory. To be more specific, the writer computed the values of the Froude's number for the twenty-four Simons and Bender channels listed in Table 3. It was found that in only two cases the value was higher than .21 with a miximum 0.25. The remaining had an average of 0.18 with a minimum of 0.105. This means that the designer, when faced with an actual problem, does not know what to use as a criterion of stability. If for instance, we were called to design channel 13 of Table 3 by following the authors procedure and using the recommended graphs, we should come out with a channel having exactly the same geometry as the existing one. However, the Froude's number of this channel is only 0.192. The question arises as to whether the channel is over-designed. In other words, could it be possible to accommodate the flow in a smaller channel that still would be stable? The answer is yes, because as long as the Froude's number is less than 0.3 no scour occurs. In order to reduce the cost, we decided to use a smaller channel. The writer believes that the modified design will not fit the recommended graphs as well as they do the Punjab canals. This means that the graphs were based on data that probably represented conditions of various degrees of stability.

The authors claim that there is some lack of understanding of the Regime theory in the United States. This is not entirely true, for the theory in itself is extremely simple. The majority of professional people are solidly grouped in the second group because they are not convinced that channels designed according to the theory are close to the state of incipient instability. The fact is that after outlining the theory once more and after pointing all its merits, the authors in their conclusion recognized this limitation, and implicitly, they joined ranks with the majority.

The successful design of a stable channel, with the help of the tractive force theory, depends to a great extent, on the accurate evaluation of the

TABLE 7.—SIMONS BENDER CANALS

No. (1)	$N_{F'}$, $\frac{v}{\sqrt{g D}}$ (2)	Measured Q_B, in tons per day (3)	ν, in sq ft per sec X 10^5 (4)	v_*, in ft per sec (5)	δ, in ft X 10^3 (6)	D = k s, in ft X 10^3 (7)	$\frac{k\ s}{\delta}$ (8)
1	.250	166.4	.94	.159	.69	2.0	2.90
2	.180						
3	.105	216.8	1.04	.112	1.08	1.0	.93
4	.138	286.2	1.01	.098	1.19	.3	.36
5	.200						
6	.129						
7	.128	80.0	.97	.111	1.01	1.0	1.00
8	.171						
9	.143						
10	.172						
11	.176						
12	.142	162.9	1.2	.187	.75	23.0	31.00
13	.192						
14	.151	358.8	1.03	.155	.77	2.0	2.60
15	.203						
16	.201	29.0	1.08	.122	1.03	.6	.58
17	.181						
18	.169	45.3	1.07	.131	.95	.7	.74
19	.234	21.3	1.01	.160	.73	2.0	2.74
20	.152	41.7	.95	.115	.96	.1	.10
21	.170	-					
22	.159	5.2	.91	.138	.77	1.5	1.95
23	.195	-					
24	.171	-					

[a] The method used in the computation is described in "The Bed-Load Function for tin No. 1026, U.S.D.A., SCS, September, 1950.

tractive forces. Under the well controlled conditions of laboratory work, the degree of accuracy is rather high so that regardless of method the computed values of the tractive forces are about the same. It is very interesting to see that in the prototype the values differ considerably depending on the method used.

The authors deserve high praise for the realistic presentation of this phase of the problem of channel friction. They show in Fig. 10 the distribution of the shear stresses along the perimeter as it has been obtained by four different methods. The first method is based only on velocity measurements,

COMPUTATION OF BED-LOAD[a]

x (9)	X, in ft × 10^3 (10)	$\frac{D}{X}$ (11)	$\left[\frac{\beta}{\beta_x}\right]^2$ (12)	ξ (13)	Y (14)	ψ_* (15)	ϕ_* (16)	Computed Q_B, in tons per day
1.20	1.29	1.55	1.26	1.00	.55	3.00	1.80	266.0
1.61	1.50	.67	.53	1.80	.82	3.39	1.50	226.0
1.16	1.66	.18	.65	20.00	.30	1.72	3.90	53.6
1.62	1.40	.71	.55	1.63	.83	3.30	1.50	94.0
1.00	17.70	1.30	.81	1.0	.55	15.80	.012	162.5
1.25	1.23	1.62	1.26	1.0	.56	3.18	1.63	493.0
1.47	1.42	.42	.43	6.0	.54	3.05	1.75	11.2
1.57	1.32	.53	.47	3.2	.70	2.30	2.70	40.5
1.22	1.46	1.37	1.25	1.0	.56	3.00	1.50	265.0
–								
1.39	1.07	1.40	1.00	1.0	.62	2.68	2.20	34.4

Sediment Transportation in Open Channel Flows," by H. A. Einstein, Technical Bulle-

whereas the other three make use of the computed energy gradient. Evidently, the presence of material in suspension is not the cause of the difference between the first and the other three methods. According to results of many investigations, the tendency for k is to become smaller, making the values of the shear stresses even smaller. The fact that the velocities were measured along the vertical, and not perpendicular to the boundary, ordinarily would not account for so much difference. The only reasonable explanation is the one given by the authors that attributes the low values obtained by the first method to secondary currents.

Sufficient evidence has not been furnished in support of the statement that . . . . "two theories are currently recognized because of their superiority over other available existing methods used to approximate the design of stable channels." On the contrary, the authors emphasize the fact that both theories may be used successfully, only in the case of clear or practically clear water flows. The fact that certain canals are used to supply clear water, as in the case of power canals, does not necessarily imply that the entire length of the canal should be designed for clear water flows. The decision about the most efficient location for taking the sediment out of the channel should be made in connection with other criteria such as comparative cost. Therefore, the writer believes that other theories, that do not restrict the design to clear water conditions only, are equally satisfactory as the ones preferred by the authors.

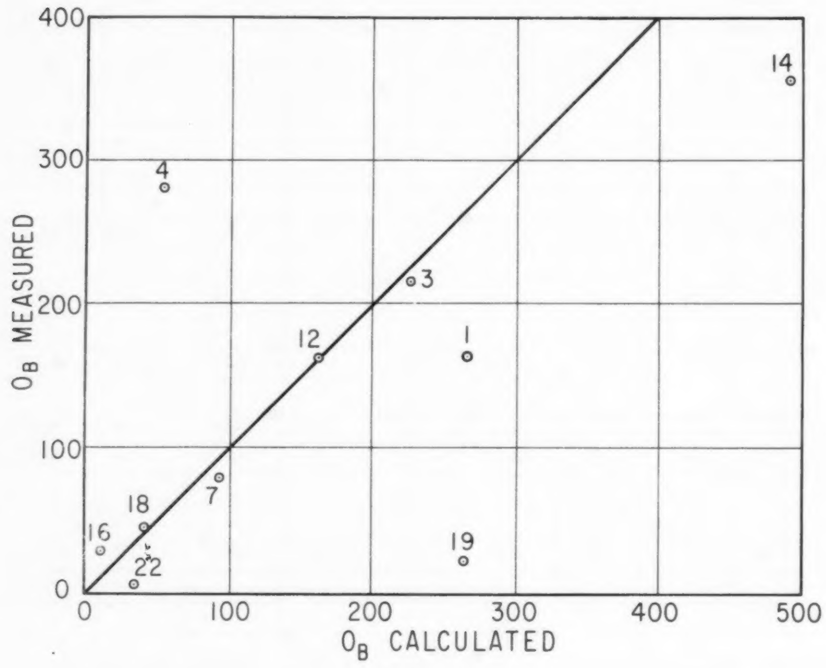

FIG. 14.—MEASURED VERSUS COMPUTED BED-LOAD IN TONS PER DAY

The reader would be more easily convinced if some of the other theories were tried and proven inadequate. In order to satisfy his curiosity, the writer tried Einstein's bed-load function on the twenty-four Simons and Bender Canals. With reference to Table 3 and to the writer's Table 7, the difference between the amount of total load and suspended load was assumed to represent the amount of bed-load. In eleven of the canals the total load was not given, and in the other two it was smaller than the suspended load. Therefore, the computations were carried out only on the remaining eleven. The computed values of the bed-load in tons per day are listed on Table 7. Channel 20 was omitted because of the cohesive material forming the bed and banks for which the

bed-load function has no application. The computed values of the other ten channels were plotted on Fig. 14 against the corresponding values measured by the authors.

Six of them seem to check very well, two fairly well, whereas the remaining two behave poorly. It is possible that the measured values of the bed-load of channel 4 were rather large. For this size of bed material (0.1 mm), the distinction between suspended and bed-load is not so easy. Moreover, one would expect the suspension load to be several times larger than the bed-load for such material. As far as channel 19 is concerned, no reasonable explanation for its erratic behavior was possible. If this channel is not the same as channel 1, at least it has the same hydraulic characteristics. It is surprising that the measured bed-load is so much different in the two channels. Maybe the authors are in a better position to offer the real reason for such behavior.

The intention of this discussion was not to criticize the work of Messrs. Simons and Albertson, for the writer believes that their work is of the highest quality. It is merely to point out that, due to the complexity of the problem, conclusions should be reached with caution.

JOHN F. KENNEDY,[7] M. ASCE.—The authors have observed that if a channel is to be stable the Froude number of the flow, $F = \frac{V}{\sqrt{g\ D}}$ must be less than 0.3. This requirement is apparently necessary to prevent the formation of stationary and translational waves which can cause detrimental scour of both the bed and banks of a channel. This type of instability has been discussed previously by the senior author (40). In the past decade several significant papers have been published which reported the results of theoretical and experimental studies of the mechanics of stationary waves in open channels and their formation. The purpose of this discussion is to summarize briefly the pertinent results of three of these investigations and examine the authors' criterion in light of them.

Several investigators have observed stationary waves in open channels at both subcritical and supercritical velocities (41),(42),(43). In 1954 T. Brooke Benjamin and M. J. Lighthill (44) presented the first comprehensive study of waves in running water in which they showed that a train of cnoidal waves (long waves of finite amplitude) is uniquely determined by the discharge, total head, and horizontal momentum flux of the flow. They did not prove explicitly that these quantities also determine a train of short gravity waves uniquely, but concluded that this is probably the case. In this paper they also presented a new derivation of cnoidal wave theory which clarified the physical significance of the different terms of the governing differential equation.

In 1955, Binnie, Davies, and Orkney (45) published the results of a model study of a channel to be used for experiments on ship models. In such a channel it is important that the flow be uniform in depth and free from gravity waves. Several different channel entrance configurations were used on the model channel, and in some runs a weir was placed at the channel exit to control the depth of flow. It was found impossible to prevent the formation of gravity waves for Froude numbers greater than 0.5 and less than about 1.3. From an approximate relation due to Kelvin the investigators concluded that the surface should be relatively free from gravity waves at Froude number

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less than about 0.6, which agrees fairly well with their observations. McCowan (46) has shown that the limiting form of the cnoidal wave, the solitary wave of J. Scott Russel, cannot occur at Froude numbers greater than 1.25. This value is close to the observed value of Froude number, F = 1.3, above which no gravity waves occurred.

A more complete explanation of the waves observed at subcritical velocities by Binnie, Davies, and Orkney was given by Benjamin (47) in 1956. Using the formulation of cnoidal wave theory developed by Benjamin and Lighthill (44) he showed that in a subcritical flow with a Froude number larger than some limiting value (not determined) it is impossible to produce a uniform stream unless the stream is absolutely uniform on entrance to the channel. In practice this condition is impossible to achieve. The experiments of Binnie, Davies, and Orkney indicate that this lower limiting value of Froude number is F = 0.5 for flows under practically ideal conditions.

In channels, translational waves probably cause at least as much detrimental scour as stationary waves. Banks are particularly subject to scour caused by the rise and fall of the water surface accompanying translational waves. Such waves are generated by quasi-steady disturbances, such as wind forces, and transients in the operation of the canal. Since the character of translational waves depends primarily on the nature of the disturbing forces, it does not appear possible to give a criterion for their prevention in terms of only the depth and velocity of flow. In fact detrimental scour could be caused by such waves even if the water were not flowing.

The wave investigations briefly discussed here indicate that the authors' requirement for the prevention of stationary waves, Froude number less than 0.3, is adequate. The limiting value observed by Binnie, Davies, and Orkney, F = 0.5, should not be approached under-field conditions because of the large number of disturbances such as channel bends, contractions, diversions, and wind forces which tend to induce gravity waves.

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T. BLENCH,[8] F. ASCE.—The writer congratulates the authors on their collection of regime data of U. S. canals, their presentation of other U. S. data, and their use of regime theory parameters to compare the results with a sample from an Indian canal system. Such work draws attention to the number of unmeasureable or doubtful variables producing regime in practice but, even for the very complex case of rivers (16), shows clearly the existence of simple laws of transport, erosion and resistance that would apply exactly to ideal cases. The authors, like Sir Claude Inglis (12) prefer to use "best fit" indices for practical problems. Lacey and the writer (48) prefer to use formulas that seem to express simple natural laws exactly, so retain the same simple indices for all cases and cover practical complications by coefficients only. The writer believes that both outlooks have advantages, especially when the former is adopted in full knowledge of the existence of the latter and its basic value.

Unfortunately, the economic impossibility of publishing new specialised information in North America so that it will reach a large public rapidly and in considerable detail has resulted in the authors being unaware of the major additions to regime theory since 1951. This does not detract from their valuable work, but it has caused considerable inadvertent misrepresentation of what regime theory can now do and what the writer has been alleged to advocate. For example the statement under "SUMMARY AND CONCLUSIONS" that "the range of conditions to which this (regime) theory applies has been expanded as a result of this study . . . ." is incorrect, since regime theory already covers the range as has been fully explained and published in 1957 (48), will appear in the second edition of a British Civil Engineering Reference Book (49) as the rewrite of the corresponding chapter in the first edition of 1951, and has been expounded in various papers since 1951 (52),(53),(55),(56),(57),(59). Obviously a clarification of events since 1951 is needed and, as the writer feels no disagreement with the way the authors have handled the facts at their disposal, he will attempt it in a form that must be very condensed since the subject as a whole requires book-length treatment. The writer is content to leave the readers and the authors to use it in the context of this valuable paper.

First, for regime references before 1952, reference (3), privately published, is out of print: the equivalent material is readily available in the first edition of reference (49), on pp. 1026-1070. The symbolism of reference (49) is that of the authors' paper but was altered to standard ASCE nomenclature in reference (5), which should preferably read Trans. 117, 1952, pp. 383 et seq.; a slight further amendment has been used consistently in all publications since.

Second, references since 1952 are best introduced while explaining physical essentials of regime theory in order of development so that they confirm, modify or amplify points raised in the paper.

The original Lacey equations were based on the data of self-adjusted canals having the properties (a) bed-load charge so small that it could not affect the

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[8] Prof. of Civil Engineering, Univ. of Alberta and Consulting Engineer.

equations appreciably (b) side erodibility fairly constant (c) formative discharges reasonably definite, and (d) suspended load sometimes reaching 1% by weight. Items (a) and (c) were advantageous as they removed two major indefinitenesses that would occur in rivers and item (d) fortunately was not of major importance for the analysis. Lacey believed firmly that the expression of the laws behind the self-information phenomena, when expressed in terms of simple circumstances, would, like other laws of physics, be of simple forms relating conventional dynamical entitles to each other. Therefore he fitted his data by smoothing lines that would be consistent with this belief and lie within what he called "the limits of statistical significance." For example, if he found that the best-fit line relating V to d for canals of one system was $V^{1.93}/d = A$, of another $V^{2.01}/d = B$, and so on, he would suspect that ideal conditions would have yielded $V^2/(g\ d)$ = a parameter depending somehow on the constitution of the water-sediment complex; however, for engineering use he never inserted useless terms like g, although he always thought with dynamically correctness. As Lacey used P and R for analysis his smoothing lines averaged out the relative and quite different effects of beds and sides. So his equations apply to idealized but physically possible self-adjusted channels of some average small (but not zero) bed-load charge, some average concentration of suspended load of some average quality, steady formative discharge and some average "relative importance of sides to bed." For the present explanation it is convenient to say that his discoveries could be summarized to "all other factors in self-formation, except discharge and bed-material size, being constant, P, R, and S adjust themselves to be proportional to the 1/2, 1/3 and minus 1/6 powers of discharge and the zero, minus 1/3 and plus 5/3 powers of a "silt-factor" f which depends in a very-poorly understood way on bed-material size." References (50) and (51) emphasise this outlook. Practically these results, so stated, could be used for channels far from ideal and even with large bed-load charge if engineering caution were used. In fact, the first two Q relations are quite consistent with the river results of Leopold and Wolman (16), see Fig. 15 as a sample, which can also summarize (within the limits of statistical significance) to the statement that "all other factors in self-formation, except Q, being equal, the breadths and depths at a suitable average Q vary as the square root and the cube root, respectively, of Q." With these relations established the many "other factors" are relegated to the coefficients applicable to special cases and, in most cases of practical interest, the engineer can estimate these coefficients for the circumstances or from related ones.

The contributions of King and the writer, referred to by the authors, before 1951 were to separate out the effects of sides and beds from the Lacey relations and in the process discover that the slope formula turned into a generalised Blasius one. This permitted them to generalize the three independent Lacey equations into forms that may be based on:

Bed-factor,

$$F_b = V^2/d \quad (53)$$

Side-factor,

$$F_s = V^3/b \quad (54)$$

and Slope equation

$$V^2/(g\ d\ S) = 3.63\ (V\ b/\nu)^{\frac{1}{4}} \quad (55)$$

Here b and d replace the authors' W and D and Eq. 55 is the correct form of Eq. 30. The bed-factor is roughly equal to the old Lacey f. The term F_s can

be converted to have the dimensions of the square of a shear stress by multiplying by $\rho\ \mu$, and it is then a standard form of rigid boundary hydraulics for "smooth boundary." These equations can be converted to agree with Lacey (preceding) that b, d and S vary as the 1/2, 1/3 and minus 1/6 powers of Q,

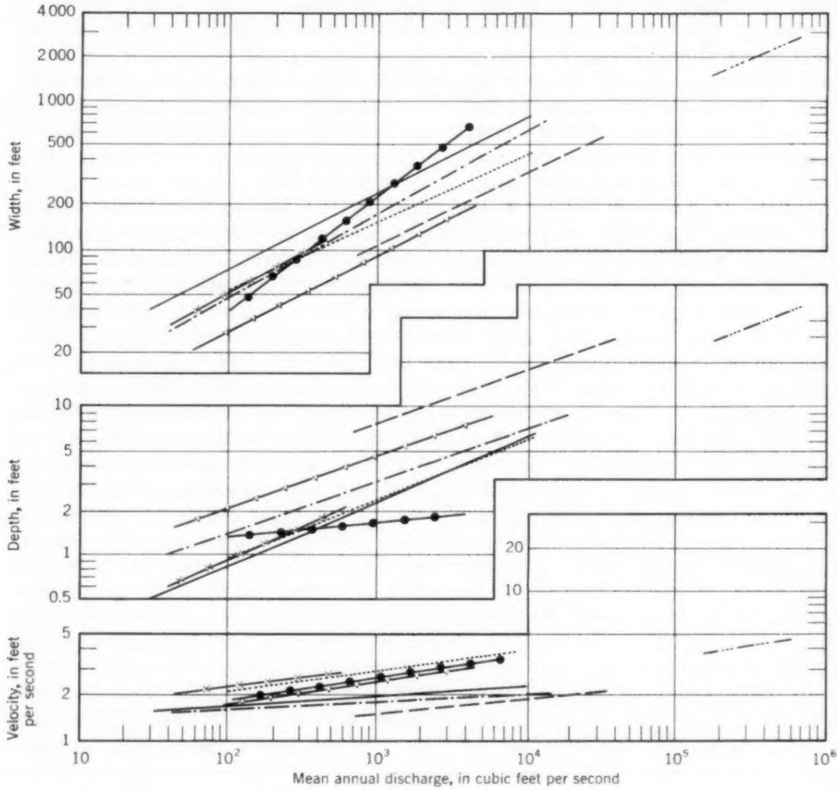

FIG. 15

but they show why his indices for f were found wrong in practice. In fact, the equations become:

$$b = \sqrt{F_b\, Q/F_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (56)$$

$$d = \sqrt[3]{F_s\, Q/F_b^{\,2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (57)$$

and

$$S = F_b^{\,5/6}\, F_s^{\,1/12} / K\, Q^{1/6} \quad \ldots\ldots\ldots\ldots \quad (58)$$

in which $K = 3.63 g/\nu^{\frac{1}{4}}$ (see Eq. 29). Generally, all indices of measureable quantities in the Lacey formulas are preserved in the new counterparts.

Note that F_b = constant, F_s = constant imply $b \propto Q^{\frac{1}{2}}$, $d \propto Q^{1/3}$, and conversely, irrespective of the physical meaning of F_b and F_s.

The data used by King and the writer at various times to justify these equations were those of several canals systems of both the Punjab and Sind; the authors' Punjab data are for channels of the Lower Chenab Canal System only. This fact is relevant to Fig. 9. A tremendous amount of information is needed to fix the ultimate fitting lines for such a figure, as large scatter must be expected. First, for example, there can be doubt on what is the dominant discharge. Second, some of the smaller Punjab channels ran consistently more than authorized supplies but research data sometimes showed the authorized discharges. Third, Punjab Main Lines were often not in regime, though "steady," but had been eroded in cohesive soil and had no sand on their beds, yet research engineers unwittingly included them. Fourth, the kinematic viscosity of the clean water varied from about 1.4 down to about 0.85 times 10^{-5} during the year, and turbidity produced a further marked alteration. Fifth, the modern use of current meters in small channels can give very bad results for Fig. 9; the meters are sensitive to turbulence but rated in still water, they record incorrectly when held within a foot of a boundary and, at one time in the Punjab, they were used on three verticals only in small channels and two verticals were within side influence. (The old-fashioned but slow method of velocity rods is far more accurate). Sixth, the inexpert measurement of depth in routine observations on a duned sand bed is usually inaccurate. Seventh, the slope S is for a reach of length of the order of a mile, but the sectional observations are for one section only and it may be unrepresentative of the reach. Eighth, $V^2/(g\ d\ S)$ is very sensitive to certain kinds of error in practical assessment of Q and d. On top of all this there is the chance that bed-load charge might be sufficient to upset individual points (requiring higher and higher fitting lines as charge increases) even on canals where, on the long term, it can be neglected. For these and other reasons the writer prefers to fit the data of Fig. 9 with a line representing Eq. 55, so having a slope of 1/4 instead of the authors' 0.37; having fitted this standard line he would agree to some parallel lines above it, in terms of additions to regime theory after 1951, to allow for various bed-load charges. A special reason for preferring a line that is based on extensive data and also is believed to have special dynamical significance (48),(49) is the writer's opinion that an engineer working with canals cannot achieve maximum expertness unless he relates his experience to some ideal which is believed to be dynamically correct. In fact, his expertness consists largely of having discovered the extent to which, and the general reasons why, actual occurrences deviate from the ideal. Advice on such matters (48),(49) recommend the use of Eq. 58. This is an algebraically derived form of Eq. 55 and has the merits of being in terms of factors the designer will use and of being exceedingly insensitive to everything except F_b which the practising engineer will soon learn to assess on lines laid down elsewhere (48),(49). He cannot, in any event, avoid the deviations that real conditions cause from any ideal formula.

Coming now to the period after 1951, the writer, under the impulse of Leopold and Maddocks' work (16) found time to undertake the often-deferred task of a major regime analysis of the classic Gilbert data (60) in the hope that it would provide the linkage between the regime equations that had left bed-load charge indeterminate and, therefore, unexpressed. The results were published (52) in accessible form first in the Proceedings of the International

Association for Hydraulic Research, 1955 (4). However, its essential findings were summarized, with some printer's errors, in the report on the Fontana Conference on Sediment Mechanics (53) held in 1954. Omitting the parts on supercritical flow, the results applicable to regime summarize to the following, for subcritical flow:

(a) An empirical relation between F_b and the bed-load charge C (weight of bed-load per unit time divided by weight of water per unit time) can be stated, as well as available data justify, in the form:

$$F_b = F_{b0} (1 + b\,C) \qquad (59)$$

in which F_{bO} is the value to which F_b tends as C tends to zero.

(b) Eq. 55 should be restated as:

$$V^2/(g\,d\,S) = 3.63\,(1 + a\,C)\,(V\,b/\nu)^{\frac{1}{4}} \qquad (60)$$

In these expressions a and b are constants depending on properties of the bed material. For example they are not the same for a natural river-bed sand (of log-normal distribution of definite dispersion) as for a uniformized sand such as used by Gilbert, nor would they remain the same if material density were altered.

Effectively Eq. 60 can now be regarded as a transport formula no better and no worse than the considerable variety of formulas already in existence and mostly depending on Gilbert's data or parts of his data that gave "consistent" results. Its original part retains its dynamical significance, but the (1 + a C) has no present dynamical explanation. It provides the link between the regime approach to transport and the laboratory approach. Another publication (48) devotes a chapter to worked problems on the use of the new equations for solving problems of channel flow with large bed-load charge. Fig. 16 (4) shows the effect the new formula has on a plot like Fig. 9 for the Gilbert data charge range (provided flow remains subcritical). It replaces the original one line by a set of parallel ones with the C values correct for Gilbert uniformized sediment. The authors have found themselves also impelled to fit parallel lines to Fig. 9, but the writer believes the lowest is probably a limit line to "experimental errors" whereas the upper might be influenced by charge. Also, he is willing to admit that the 3.63 in Eq. 60 could be for a small charge assumed wrongly to be vanishingly small but doubts if the true value would be much more than a couple of percentage points smaller.

With the new slope formula the author has managed to obtain reasonable estimates of bed load in a couple of engineering problems. One was prophesying the movement of sand into a trench cut into a river bed to receive a tunnel sunk in sections. Another was by comparison with the observed data and computations in reference (54) from various other formulas (after correcting for the effect of meandering on S, since a straight channel of slope S and given dimensions will not transport the same as a meandering one of the same S and dimensions). On the whole, after correcting slope for meandering rivers, the writer estimates that any transport formula probably gives an answer somewhere between about half and double the truth, which is quite useful accuracy. All formulas have to give similar answers within some range as they all depend on pretty well the same laboratory flume data and merely differ in algebraic form. The design of assisted cutoffs is also aided considerably by the new formulas. Boulder rivers in which boulder size is not too large compared with depth also analyze exceedingly well but data on their charges are lacking. Methods of dealing with such problems are worked out elsewhere (48), (49).

Reference (52) was followed by reference (55) in 1957, in which the result of analyzing all the Gilbert data, not just a major portion, plus some extraneous material that had a bearing on natural river bed sands was reported and special attention was drawn to the unfortunate gaps in experimental data in the hope that organizations with large resources would fill in the omissions; the hope remains unrealized. A paper containing the new findings was also presented, by request of the sedimentation committee at an ASCE Symposium in 1955 (56), but was not published. They were also outlined in discussion on reference (57).

There still remain the problems of obtaining better expressions than the linear ones in C to allow for bed-load charge and of allowing explicitly for suspended load which has always been known to be implicit in regime relations. The existence of a means for expressing charge, C, makes the suspended load problem somewhat easier, but it still remains exceedingly difficult since a natural suspended load contains many components. Moreover in practice, rivers and canals with large total load have the bed-load differentiated in relatively small (though highly important) quantity out of the total load and the differentiated bed portion will vary with stage if the bed is gravel. It

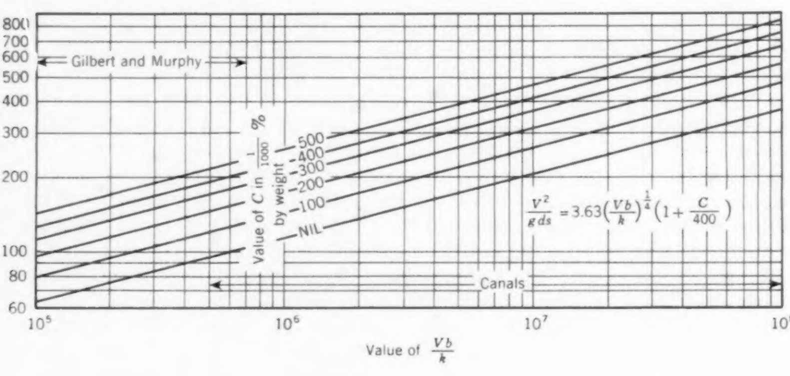

FIG. 16

is this fact that makes flume experiments with unnatural sands, or just a "total load" of some kind, so unrewarding in quantitative results when the total load is taken as a criterion. Suspended and bed-loads have different effects on the flow, so, unless they can be arranged to be controllable and measurable separately in experiments, no useful answers can be expected. Eq. 59 is probably the easiest step towards appreciating this, since F_{bo} depends on bed-material properties and, almost certainly, on the properties of the fluid which, with suspension in it, does not behave as clean water—it may even be non-Newtonian. Similarly it is probable that the (1 + a C) in Eq. 60 needs functional modification to allow for suspensions. In engineering practice suspensions do not usually affect design problems, since F_b is estimated from comparable circumstances so contains the effects of suspensions automatically. The difficulties arise in fundamental studies or in special works. The present popular puzzle about dual behaviour of sediment-bearing flow with apparently the same conditions ceases to be a puzzle when it is realized that the same total load does not necessarily mean the same bed and suspended

ones. The writer feels, accordingly, that the authors' references to total loads affecting data might be amplified by references to the presumably very small bed-loads compared with suspended ones and the different parts they play.

Regarding references to tractive force theory the writer believes it is applicable to beds that have not yet been moved, but are about to be set in motion; he uses it for problems of beds that are not to move. To him, the difference between flow with no bed movement whatever and flow where there is vanishingly small movement is as important as that between movement with no turbulence and with turbulence. When bed movement is occurring then the regime equations apply and the tractive force theory does not. In fact, when conditions are those of regime, it is the rate of working by gravity per unit mass per unit time that is fixed by the constitution of the water-sediment complex and the value of the side-factor (48),(49). This is a regime deduction of old standing recently appreciated in less specific terms by Bagnold (58). The writer used this relation recently as one of the three principles on which regime theory could be built if desired (59). As tractive force is proportional to g d S multiplication of Eqs. 57 and 58 shows that it is a function of discharge as well as of properties of water-sediment complex and side-factor which, for all cases where it is not constant because of constant discharge and fixed side erodibility, will depend on discharge and channel dimensions; this explains Fig. 11.

In terminating this necessarily long discussion, and leaving several details undiscussed, the writer would like to emphasize that he values the authors' work highly as an addition to badly needed field data requiring a lot of effort to collect, as a logical system of correlation, and as an ingenious set of proposals for design. The writer, personally, would normally use the latest regime methods of references (48) and (49), nowadays, but admits willingly that the recommendations under the heading "Summary" (with 3b modified to read "Fig. 9 representing a modification of the Blench-King regime equation") are comparable with ones he has accepted and even proposed in the past. The writer would be quite prepared to use the figures practically for initial results for the conditions they represent and admits also that they are excellent for engineers who have not acquired the expertness in full regime methods that only study and field experience can give. However, he has been compelled to fill in knowledge available since 1951 and feels somewhat in the position of an engineer who has to discuss empirical pipe formulas in terms of a knowledge of the friction factor diagram and the concept of roughness height. Such a discussor has to deal with total present knowledge, yet he knows that it requires some expertness to apply, that circumstances sometimes make it inapplicable, and that the so-called empirical formulas are really part of the whole and often the best way to express results within a range.

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CLAUDE INGLIS,[9] M. ASCE.—It is most gratifying to the writer, who has spent 50 yr studying and helping to develop and expand the regime concept, to find that so much interest is now being taken in the United States in the application of the regime theory to the design of channels in alluvial materials. However, the authors are mistaken in confusing the regime concept with silt-stable flow in channels excavated in scour-resistant material or subject to other forms of restraint (see the first line of their introduction). The regime concept, as developed in India by irrigation engineers, is of far wider scope and much greater significance than the design of silt-stable channels. Under natural conditions, regime is essentially a balance between accretion and erosion over a period of time, usually a year; it applies to rivers and tidal estuaries, as well as canals taking off from, and influenced by, the varying conditions in the rivers from which they draw their supplies. The changes that take place during the cyclic period are generally far too complex to be computed; however, if a channel is in regime it will return at the end of the period of the cycle to approximately the same condition as at the beginning of the cycle, a year or more earlier.

The idea that the design of a channel can be definitely linked with constant charge is, in fact, generally illusory under natural conditions, except where the charge is very small, as assumed by Mr. Lacey.

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[9] M.I.C.E. and F.R.S., formerly in charge of irrigation research, Bombay Presidency 1916-33, Dir., Indian Waterways Experiment Station, until 1946 and Dir., Hydr. Research Station, D.S.I.R., until 1958.

This does not mean, of course, that the charge is small throughout the year. In most rivers in Northern India the charge of material in movement is heavy during the flood season, and during a rising flood may be as much as 5,000 ppm. To prevent heavy charges entering the canals, the Headworks have been designed to reduce the charge of suspended silt and bed-sand entering them. This has been further reduced by constructing 'ejectors' where there is still an undesirable amount. The ejectors serve the double purpose of disposing of the excessive bed material near the head, and evacuating much of the excess suspended silt that is induced to concentrate near the bed by regulating the velocity of flow to effect the optimum conditions for ejection. Where ejection is unsuitable, as in Sind where the slope of the river is very flat, resulting in an accumulation of ejected silt, excessive entry of silt has been prevented by designing the Headworks so that top-water is drawn from the outside of a permanent bend or by inducing favorable curvature of flow. In spite, however, of careful design and control, there are periods during the flood season, particularly during rising floods, when a heavy charge of suspended silt passes down a canal. Part of the silt deposits on the bed and mixes with the bed sand in the upper part of the canal system, with consequent smoothing of the bed ripples, and the rugosity coefficient falls from approximately 0.025 to 0.010, with a consequent drop in the water level required to maintain the same discharge. Nevertheless, regime is maintained because towards the end of the flood season the suspended silt charge decreases, and the finer silt is gradually washed out of the bed mix, with a consequent return to the N value of 0.025 and conditions similar to those a year earlier when, effectively, the ripple pattern of the bed, maintained by a very small charge of grains of bed material in movement, constitutes the channel's regime.

As pointed out by the authors, Kennedy's 1895 formula $V_o = 0.84\ D_b^{0.64}$, in which D_b refers to the bed portion only, permitted a wide and shallow or deep and narrow channel. But Kennedy realized that this was not so, and at the beginning of the century he published his "hydraulic diagrams for channels in earth" (61)[3] that were in general use in India for design purpose at that time. In 1909 R. E. Garrett (62) published tables and diagrams that, when properly used, indicated optimum dimensions, slopes, and velocities for values of the rugosity coefficient N from 0.018 to 0.030. These were a great advance, and, indeed, foreshadowed regime relations.

The writer, in order to link up canal conditions with those of rivers carrying widely varying discharges and charges, introduced the "dominant discharge" concept. This was facilitated by the fact that in Northern India the dominant condition was closely linked with "bank-full stage" when the formative discharge and charge were in equilibrium with the slope and dimensions of the rivers. Where this is not the case, great skill and judgment, based on experience, are required to assess the dominant condition of flow.

This is of importance in dealing with the point raised by the authors in connection with the investigation carried out by Leopold and Maddock (16) to assess "the applicability of the regime theory to American rivers." The authors say "The equations were of the same form as the regime equations, but did not have the same magnitude of constants and exponents. Hence, one might conclude that regime equations depend on the conditions on which they are based and are valid only within the range of the observed data." This conclusion is quite unacceptable. The writer greatly admires most of Leopold's work, but his use of average discharge in his analyses instead of dominant discharge,

3 Numerals in parenthesis, thus, (61) refer to corresponding items in the Bibliography.

however arrived at, renders his equations quite valueless for correlation purposes. Had the authors, on the other hand, compared the regime exponents with their own exponents for canals (see Figs. 3, 5, and 6, showing the relations they derived between P, R, A and Q and Fig. 8 between V: $(R^2\ S)^n$ they would have seen that the exponents of the Lacey regime equations were in close agreement with their own exponents, and the coefficients were also similar for their "sand bed and cohesive bank" condition. The Lacey regime equations are in fact basically sound and can therefore be amplified and extended to cover other bed materials and charges. The authors are therefore wrong in assuming that the equations "are valid only within the range of the observed data."

When the authors refer to "regime equations" they mean the Lacey equations of the late thirties in terms of P and R. These, strictly speaking, were only applicable to channels with beds of medium sand. Nevertheless, they have been found adequate in India for designing canals, even where they take off from rivers that, in flood, carry a heavy charge of debris, ranging from boulders and pebbles near the hills, to silt with a grain diameter less than 0.10 mm, in the plains. The explanation is that in India it has been considered bad engineering practice to allow sand coarser than medium sand to pass down a canal. Great skill and ingenuity have been used to achieve this end, by careful design of Headworks. As channels are generally in bank with water levels several feet above ground level and because they have to supply flow water for irrigation, the banks are preferably spaced much further apart than the regime width of the channel. This induces rapid siltation and the formation of wide, autogenous or self-generated berms.

Although the Lacey equations have generally been adequate, the effect of a constant heavy charge of bed material has not been overlooked. In this connection, the authors attention is drawn to a paper contributed by the writer (63) in 1947 (and a 1949 paper (12)). In this, a set of dimensional equations were derived that amplified the Lacey equations by showing the individual effects of "bed charge," "grain diameter," and "terminal velocity of fall of grains in water." These equations were based on experiments initiated by the writer at the Indian Waterways Experiment Station, Poona, in the early 1940's. Two series of experiments were carried out with 1.0 cu sec and 2.0 cu secs respectively, with "sand beds and banks" conditions. In each series, several experiments were run to equilibrium, with different bed charges, from 1/100,000 to 1/10,000 by weight, those with the heaviest charges being neglected, because meanders developed. The equations then derived were based on dimensionless relations of the type

$$\frac{W_T\ g^{1/5}}{Q^{2/5}} = \alpha \left(\frac{Q}{g^{1/2}\ m^{5/2}}\right)^{n_1} \left(\frac{X\ V_s}{(\nu\ g)^{1/3}}\right)^{n_2} \quad \ldots\ldots\ldots (61)$$

in which n_1 gave Lacey exponents, and n_2 agreed with the results of the Poona experiments. To simplify the use of the 1947 equations, now known as the Inglis-Lacey equations, they are now re-stated using g, m, V_s and ν as ratios to the standard values at 20 C for quartz grains. In this way it has been possible to incorporate Lacey coefficients as well as exponents. They are stated in terms of P and R, as the coefficients in terms of W_T and Dm = (A/W_T) are at present under revision.

$$P = 2.668 \frac{Q^{1/2}}{(g\ m)^{1/4}} \left(\frac{X\ V_s}{(\nu\ g)^{1/3}}\right)^{1/4} \quad \ldots\ldots\ldots\ldots (62)$$

$$R = 0.4725\ Q^{1/3} \left(\frac{m}{g}\right)^{1/6} \left(\frac{X\ V_3}{(\nu\ g)^{1/3}}\right)^{-1/3} \quad \ldots\ldots\ldots\ldots (63)$$

$$A = 1.26 \frac{Q^{5/6}}{g^{5/12}\ m^{1/12}} \left(\frac{X\ V_s}{(\nu\ g)^{1/3}}\right)^{-1/12} \quad \ldots\ldots\ldots\ldots (64)$$

$$V = 0.794\ Q^{1/6}\ g^{5/12}\ m^{1/12} \left(\frac{X\ \sqrt{3}}{(\nu\ g)^{1/3}}\right)^{1/12} \quad \ldots\ldots\ldots (65)$$

$$S = 0.00054\ Q^{-1/6}\ g^{1/12}\ m^{5/12} \left(\frac{X\ V_s}{(\nu\ g)^{1/3}}\right)^{5/12} \quad \ldots\ldots (66)$$

$$(S\ V) = 0.00043\ (g\ m)^{1/2} \left(\frac{X\ V_s}{(\nu g)^{1/3}}\right)^{1/2} \quad \ldots\ldots\ldots\ldots (67)$$

$$\frac{V}{(g\ R)^{1/2}} = 1.155 \left(\frac{X\ V_s}{(\nu\ g)^{1/3}}\right)^{1/4} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

and

$$\frac{V\ P}{\nu} = 2.12\ \frac{Q^{2/3}}{\nu} \left(\frac{g}{m}\right)^{1/6} \left(\frac{X\ V_s}{(\nu g)^{1/3}}\right)^{1/3} \quad \ldots\ldots\ldots (69)$$

It was pointed out in the same paper (63), that where the material was quartz sand and the grains were greater than about 0.15 mm and less than about 0.5 mm, the amount of material depositing in water was constant so long as the product ($X\ V_s$) was constant; for grains smaller than about 0.08 mm, Stoke's law held, and V_s was proportional to m^2. For this reason, the coefficient in the equation relating W_T to $Q^{1/2}$ in the Inglis-Lacey equation material-0.1 mm is about 2.0. This is in agreement with Sind experience and also with the data shown in Fig. 3 of this paper (63). Mr. Jethwani, in 1943, showed that shape in the silt range was much less clear-cut, and that a wide range of dimensions and slopes was permissible in the bed-silt range that was borne out

by Sind data. For designing for such conditions a paper by Peter Ackers (64), (65) will be found helpful.

Attention is also drawn to Mr. Lacey's paper (66) in which he has added a slope correction factor, E, to the Inglis-Lacey equations.

The addition by the authors, of two new types of flow, with (c) "sand bed and banks," and (d) "coarse non-cohesive material," is of great interest. Because, although these are "limit-of-erosion" rather than regime conditions, their equations have exponents that are basically similar to those in the Indian regime equations for (a) medium sand beds, and (b) non-rippling silt beds, the comparable coefficients in the $W = C\, Q^{\frac{1}{2}}$ relationship being (a) 2.668, (b) 2.0, (c) 3.7 and (d) 1.7. Therefore, all the regime exponents are available for use in applying the new conditions.

In conclusion, it can be said that whenever conditions have been comparable, the authors' data have confirmed the results of Indian work and vice versa. The main divergences have been due to the methods of approach being different. Thus, the authors have presented their data in terms of bed and bank materials giving the impression that these are known in advance and that the modifying effects of the several complex factors that will affect the design, can be worked out, provided a mass of data is collected about the individual effects of each factor. It is not suggested that the authors really hold this view, but merely that their presentation will give this erroneous impression to the great majority of those who will read the paper. Confusion will also arise due to the authors dealing simultaneously with two different problems, namely,

(1) the design of canals carrying clear water, and

(2) the design of canals taking off from rivers carrying a heavy charge during floods

Thus, the authors when commenting on Lane's Type 1 class of instability, "Channels subjected to scour that do not silt" say "This is the simplest and fortunately, also the one of primary importance, since most of the present and future canal problems are and will be clear-water problems." On the other hand they say "When it is realized that design must be confined to a range of small Froude numbers, it is immediately apparent that sediment load is also severely limited, usually to less than 500 ppm" so that "canals must utilize head works structures, that do not allow sediment concentration in the sand-silt range, to exceed 500 ppm for the dune form of bed roughness, and the concentration of sediment must be even less for ripples."

Although it is not uncommon for canals in India to carry almost clear water for a large part of the year, their design in such cases has generally been determined by local requirements, not by the material in which they are excavated, and has not presented any considerable difficulty.

Thus, as long ago as 1870, Fife designed the Mutha Canal very successfully, making the first ten miles down to Poona narrow and deep to store water and allow barge traffic, and constructing the canal relatively wider and more shallow further downstream, to maintain a nearly constant water level for irrigation.

Under such conditions, aquatic weed growth may have an important bearing on canal design. An investigation carried out at Poona (67) showed among other things, that a velocity exceeding 2 fps inhibited weed growth, irrespective of depth.

Although in the United States, the number of canals carrying a heavy discharge may be much fewer than clear-water canals, it is obvious from what

has been quoted that they are nevertheless, of great importance and present the gravest design problems, just as they do in India. There, the most difficult design problems occur where in addition to the parent river carrying a heavy charge of silt, the slope of the country is flat, as in Sind. Under such conditions, success depends almost entirely on the effectiveness of the Headworks design, and hardly at all on the material of the terrain through which the canal passes. It is therefore essential to select a site for a Barrage where favorable curvature of flow can be permanently maintained. This entails taking into account the changes that will occur upstream of the Headworks in years to come, and calls for deep knowledge of river behavior, particularly of the river in the vicinity of the proposed Headworks. Where this is overlooked, as once occurred when a carefully prepared design was altered to simplify construction and save capital cost precious "head" was squandered and very serious silting trouble ensued.

Finally, the writer cannot accept the authors' contention as to "the inadequacy of regime methods" or that "regime equations depend on the conditions on which they were based and are valid only within the range of the observed data." As already shown, the regime method is not restricted to the conditions of Lacey's medium sand equations of the late thirties. The regime method, as covered by an extension of the Inglis-Lacey equations, covers not merely other regime conditions than those of the Punjab and Sind, but also 'limit-of-erosion' conditions.

Lastly, it must be realized that hydraulic design is a fine art, depending on the knowledge, experience, and skill of the designer, and that textbook data is merely a valuable aid to engineering skill.

The attention of those interested in regime flow is called to a paper by the writer (68).

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RESISTANCE TO FLOW IN ALLUVIAL CHANNELS[a]

Discussion by H. A. Einstein, T. Blench,
Emmett M. Laursen and Gerald A. Zernial,
Dasel E. Hallmark, Thomas Maddock, Jr. and
W. B. Langbein, R. Hugh Taylor, Jr. and
Norman H. Brooks, and Lucien M. Brush, Jr.

H. A. EINSTEIN,[11] F. ASCE.—This paper is very valuable for the experimental data given, as well as for the excellent description of the various bed formations.

As far as the analysis of the data is concerned, it is most gratifying to see that the change of roughness is related to the geometry of the bed, but not to the "work spent on moving the sediment," one of the early concepts which has helped much to obscure the facts.

The experiments cover a large range of sediment rate and of slope values. They reach up into Froude numbers above critical and indicate that no particular changes occur in the friction law at that point. The water depth covers a rather small range. Only one sediment mixture was used.

In their analysis, the authors appear to use exclusively their own experimental results. If they had used these results to check previously proposed theories of friction along sediment beds, the paper would have been extremely valuable in its analysis. Unfortunately, that was not done. Instead, they developed a new relationship as given by Eq. 9 and in Fig. 11. This relationship is composed of rather complicated combinations of the variables and definitely needs an explanation.

This relationship in question is given in Fig. 11 in the form:

$$\frac{V}{V_*}\ \frac{\tau_o}{\Delta\gamma_s}\ \frac{S}{D} = f\left(\frac{w\,d\,S}{\nu}\ 100 + 5\,\frac{V^2}{g\,D}\right) \qquad (10)$$

in which the nomenclature is the same as that of the original paper and V is the average flow velocity; $V_* = \sqrt{\tau_o/\rho} = \sqrt{S\,D\,g}$ the shear velocity along the bed; τ_o is the shear stress at the bed; $\Delta\gamma_s$ is the specific weight of the sediment under water; S is the slope; D represents the water depth; w is the settling velocity of the sediment (average); d denotes the diameter of the sediment (average); ν is the kinematic viscosity; g represents the density of the fluid; and f is a function. Of these variables, $\Delta\gamma_s$, ρ, w, and d are constants for the entire set of experiments. Thus, Eq. 10 may be rewritten in the following form collecting all constants in brackets

$$\left[\frac{g\,\rho}{\Delta\gamma_s}\right]\left(\frac{V}{V_*}\right)\ S^2 = f\left\{S\left([100\,w\,d]\ \frac{1}{\nu} + [5]\ \left(\frac{V}{V_*}\right)^2\right)\right\} \qquad (11)$$

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[a] May, 1960, by Daryl B. Simons and E. V. Richardson.

[11] Prof., Mech. Engrg., Univ. of Calif., Berkeley, Calif.

With all the brackets constant for the range of the experiments, this is a relationship between (V/V_*), S and ν. A check of the range of temperatures shows that ν stays between the values of 1.03×10^{-5} and 1.22×10^{-5}. This represents a $\pm$ 10% maximum deviation from an average value, which is well within the scatter of the points in Fig. 11. For the sake of that discussion ν may thus be

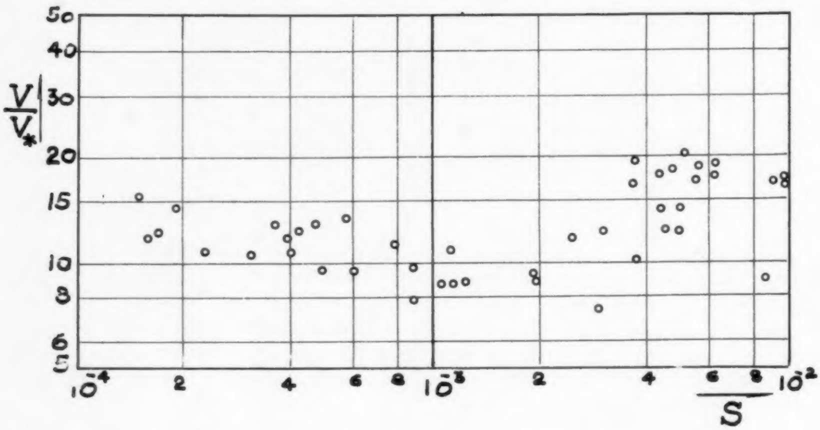

FIG. 12.—$\frac{V}{V_*}$ AS A FUNCTION OF S

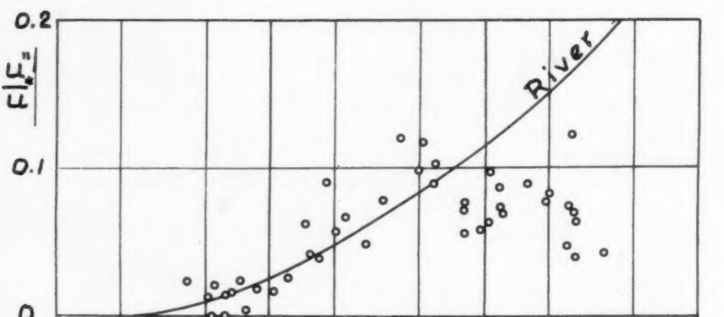

FIG. 13.—$\frac{u''_*}{u}$ AS A FUNCTION OF ψ'

assumed to be constant, too. Eqs. 10 and 11 are, therefore, a relationship between (V/V_*) and S. The values of (V/V_*) were thus computed for all experiments and plotted against the slope S in Fig. 12. It is seen there that this relationship is very weak with a minimum of (V/V_*) near $S = 10^{-3}$, with a slow increase towards $S = 10^{-4}$ and a more abrupt increase towards $S = 10^{-2}$. The

entire change of an average line is about from 9 to 18, or by a factor 2. This factor may be responsible for the slight deviation of the curve of Fig. 11 in the original paper from a slope 2. Otherwise, that curve is nothing but a plot of S^2 against S as is easily seen from Eq. 11.

In their analysis, the authors refer to another paper.[10] If they had used the entire procedure proposed in that paper, they would have found that their results (except Run 26, which appears to be far out of line) follow the river curve reasonably well for Ψ' - values below 10. (See Fig. 13). For higher Ψ' - values, u_*''/u deviate systematically from the river curve towards zero. This deviation is characteristic for all flume flows with coarser sediment than about 0.5 mm. It may be mentioned that the plotting of u_*''/u, instead of u/u_*'' as in the original paper, has been used here as it tends to emphasize the range in which the bar resistance is important rather than the range where it fades out. Fig. 13 shows a rather large scatter of the points which may be easily explained by the difficulty of defining and measuring the exact location of the bed in view of its large irregularities. It must be kept in mind, furthermore, that that method of analysis accumulates all accidental errors of the entire measurement into a part of the energy dissipation.

If the authors had been satisfied with checking the applicability of their measurements to various existing theories and formulas, they would have done the difficult field of river hydraulics a much better service than by trying to derive from the rather limited amount of information a new relationship which does not seem to have much physical significance.

T. BLENCH,[12] F. ASCE.—To see this starting attempt to repeat the outstanding work of Gilbert (5) with large equipment in the engineering department of a University is indeed a pleasure. To the "principal reasons why only limited answers..... have been developed" (Introduction) for the matter in hand the writer would add the lack of science instruction in schools followed by engineering curricula that also ignore basic science in favour of manipulation of uncritically accepted formulas; these added reasons seem necessary to explain the almost complete failure of engineers to study the information Gilbert collected and presented. The continued demonstration of transport phenomena in college, and the progressive measurement and critical analysis of quantities relevant to sediment-bearing flow in and beyond the Gilbert range in flumes, as the authors obviously intend, should help to inject a scientific outlook into engineering instruction and advance engineering information; extension to field observations will yield even more. The writer offers his congratulations and would like to profer some comments from his own experiences and analysis of Gilbert data[13,14] in the hope that they may be useful.

Experimental Equipment and Procedure

Recirculation of Sand.—The writer, after finding that the relatively modern method of circulating water and sediment saves time in qualitative work and produces very realistic river behavior even with "steady" discharge, has reverted to direct injection for exact quantitative purposes. First, steady pumping of sediment-laden flow is very difficult, and second, even with steady pumping there seems to be a tendency for oscillation of sediment charge with periods

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[12] Prof. of Civ. Engrg., Univ. of Alberta, and Pres., T. Blench & Associates, Cons. Hydr. Engrs.

[13] "Regime Formulas for Bed-Load Transport," by T. Blench, Proceedings, IAHR, 1955.

[14] "Regime Analysis of Laboratory Data on Bed-Load Transport," by T. Blench and R. Bryan Erb, La Houille Blanche, No. 2, 1957.

that may run from hours to days depending on the type of experiment. These oscillations seem largely responsible for the realistic sediment behavior of river models with steady discharge. The initiation of oscillation, in a flume, appears to arise from a disturbance, or error of adjustment, causing a backwater effect; then sediment either deposits or is removed from the downstream end and results in a starvation or excess of sediment supply at the upstream one. The consequent degradation or accretion that moves down the flume takes a relatively long time to travel, so the oscillations are of long period. In special experiments now in hand at the University of Alberta[15] where bed-load is about 20% by weight of the water flow, the pulsation of sediment-flow is rapid and clearly visible - presumably associated with the pump behavior. Oscillating discharge causes enhanced scatter of plotted points and might cause bias.

Measurement of Bed-Load Charge.—A special cause of scatter of the authors' plotted data, which the writer finds rather more than he expected from his own experiments, field canal observations, and regime analysis of Gilbert[13,14] might be the method of measuring charge. Charge (weight discharge of sediment divided by weight discharge of liquid) is not measurable generally by concentration found in a grab sample (weight of sediment found in such a sample divided by weight of water in it) unless the sediment moves at fluid speed.[16] An interesting analogue from physical chemistry is of anions and cations flowing in opposite directions between the poles of a battery; obviously, if a sample of their concentration showed them in the ratio of x/y this would not prove they moved in the same direction with discharge ratio x/y. Recent work at the University of Alberta[15] has shown discordance between charge measured from concentration and directly by collecting and analysing the water-sediment complex volume that is diverted for a considerable time into a tank. Perhaps the methods of measurement of charge, even in a nappe, merit check against a case where sediment is injected at a known charge rate.

Measurement of Depth.—The writer once had bed profiles observed in a glass-sided flume[17] with results that condensed to ". . . the mean of the mean depths observed on each side of the test reach differed by a very small, but statistically significant, amount from the mean found from that mean and the mean depth along the centerline in the small flume. Also the mean depth taken from the sides, by planimetering the bed-wave profiles, did not differ significantly, on an average, from that given by averaging the two extreme depths of each of the planimetered profiles . . ." Perhaps the mean bed elevation might be quite accurately found by smoothing out the dunes after a run.

Use of Orifice Meters.—It is possible that an orifice meter calibrated for clean water reads incorrectly for sediment-laden flow because (1) the kinetic energy correction factors at the measuring sections are different, (2) the sediment deposits against the orifice plate.

Natural River Sand.—Sand obtained from a river is not necessarily natural river sand; it may be a mix obtained from a dredging pile. Work at the University of British Columbia[17] showed that a mix of natural sands used in flume experiments alters during running and tends to become natural with a median diameter that may be different from that of the mix. The writer's observa-

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[15] Flume transport research under R. W. Ansley, M. Sc., sponsored by Cities Service Athabasca, Inc., 1960.

[16] "Regime Behavior of Canals and Rivers," by T. Blench, Butterworths Scientific Publications, Washington 14, D. C., 1957.

[17] Note on Studies of Bed-Movement in Flumes, 1949-51, Report of September, 1951 by T. Blench, to National Research Council (Canada) re Fraser River Model (Available Department of Civil Engineering, University of British Columbia).

tions[18] show that a natural river sand plots fairly straight between the 1% and 99% limits on log normal paper and has about 1% coarser than twice median size and 1% finer than half median size. Repeated observations over several years at the University of Alberta have shown that a natural sand used in a river tray does not change its constitution. Presumably an unnatural sand will split into bed and suspended load in different proportions at different discharges and cause considerable errors in assessing bed-load and what it would do in water free from suspensions.

Observed Flow Phenomena

Time of Development of Regime.—The writer agrees about the long time required for establishment of a new steady regime in sand-bearing flumes after even a small change in a factor such as bed-load charge. In flume experiments[17] aimed at relating bed-factor to charge (for designing a model) the mean depth, after a change, was plotted at intervals of several hours for periods of several days and was found to change asymptotically from some three to four inches and oscillate slightly about its limit; the results indicated that three days was a fair steadying time. The matter is important in measuring bed-factor, $F_b = V^2/D$, since discharge is usually known exactly, so F_b varies inversely as the cube of D.

Absence of Ripples or Dunes at Low Charges.—On page 81 is the statement "Other investigators have reported movement with a plane bed without formation of ripples. . ." This phenomenon is under investigation now at the University of Alberta[19] with 1 mm gravel. At discharge intensities such as in the present paper small charge flow is duneless, and if dunes have been previously induced they gradually vanish; but at higher charges in the subcritical range dunes form and persist. The behavior is important practically because, although a sand model can reproduce qualitative gravel river behavior[20] it is quantitatively imperfect. (Gravel may be defined as material have settlement velocity proportional to the square root of sediment size, so 1 mm is a convenient lower size limit (with some margin). In the sand range, less than somewhat smaller than 1 mm, settlement velocity does not follow a fixed index law; the index increases progressively from 1/2 till size is about 0.1 mm, after which it remains fixed at 2). The quantitative advantages of using a small gravel to represent the material of a gravel river are considerable, so the practical question arises whether, in a small model, some factor like Reynold's Number in terms of depth, or relative grain size[21] becomes important and prevents the model from behaving like the prototype. Unfortunately, Gilbert's notes on the appearance of dunes seem rather vague on whether they were absent in certain cases, or he did not verify, and his charges were mostly higher than those that affect engineering practice. The authors are paying attention to the engineering range of charges up to some 400 parts per million - so it is hoped that they will consider investigating this problem of gravel behavior and perhaps carry it to the field, aided by sonic sounding,[22] as an extension of their

18 "Normal Distribution Found in Sample of River-Bed Sand," Civil Engineering, February, 1952, with replies to discussion February, 1953.

19 Research at University of Alberta by A. N. Varzelioti under auspices of Alberta Government Highways Department (Bridge Branch) and Alberta Research Council, 1960. re certain river erosion problems.

20 "Regime Behavior of Canals and Rivers," by T. Blench, Butterworths Scientific Publications, London, England, and Toronto, Canada, 1957, Chapter 8.

21 Ibid., See 5, 18.

22 "Systematic Changes in the Beds at Alluvial Rivers," by Walter C. Carey and M. Dean Keller, Proceedings, ASCE, Vol. 83, No. HY 4, August, 1957, and discussion thereon by T. Blench, Proceedings, ASCE, Vol. 84, No. HY 2, February, 1958.

related paper[23] (Excessive saltation in a gravel river confuses sonic records).

Seepage Flow.—The effect of seepage flow, page 80, certainly deserves the attention given it. The writer has noted, in glass-sided flumes, that the phenomenon related to Figs. 5, 6 can be associated with piping failure of the bed due to formation of a hydraulic jump.

The Shock Antidune.—Sponsored research[15] being carried out at the University of Alberta by R. W. Ansley M.Sc. has recently shown an interesting new phenomenon that might be named the "shock antidune." With about 20% by weight sand bed-load (in supercritical flow) in a flume insertion of the hand causes a hydraulic jump and load deposits rapidly in its subcritical portion. The jump moves slowly upstream with sand continuing to deposit. Removal of the hand then lets a clear backwater curve form down the downstream side of the deposit. Erosion of this downstream side proceeds more slowly than deposition occurs just downstream of the jump, so a gradually lengthening super antidune combined with hydraulic jump (or "shock-wave") moves slowly upstream.

Ripples and Dunes.—Laboratory workers usually find the need for distinguishing between ripples and dunes, but the writer has not felt the need in canal practice since, even with very small charges, the bed-wave patterns observed during closure seem to be fairly described as dunes; also analysis of transport data do not seem to indicate that the change from ripples to dunes is associated with a discontinuity of formula such as Fig. 9 shows for the change from dunes to sheet flow (of course, data usually scatter considerably so may hide an effect). The question arises, in connection with the comments on "absence of ripples or dunes at low charges," whether ripples may be a small scale phenomenon. Would it be possible, now or later, to extend the authors' helpful descriptions to a detailed quantitative definition of ripples and dunes?

Analysis of Data.—It is a little surprising that the data have been analyzed in terms of "tractive force," which is not, to borrow a thermodynamic term, a function of state, and the rather peculiar concept of a laminar film thickness on a duned bed, when the authors' related paper[23] has found regime theory parameters so rewarding. As all the Gilbert data plus some of Dr. L. B. Straub and of the U. S. Waterways Experiments Station have been analyzed, using regime theory methods, the writer[13,14] has re-analyzed Table 1 accordingly and found some interesting agreements with and points arising from the authors' findings:-

(i) Regime slope relation and charge. The regime analysis just mentioned[13,14] led to the following:[24]

$$V^2/gDS = 3.63(1 + aC)(Vb/\nu)^{\frac{1}{4}} \quad \ldots\ldots\ldots\ldots \quad (12)$$

In this, for Gilbert sand and gravel, and subcritical flow, "a" was found to be 1/400 with C measured in parts per hundred thousand (not per million like C_T in the present paper). So, as an alternative to Fig. 9, regime theory would suggest plotting $(Vb/\nu)^{\frac{1}{4}}$ divided by V^2/gDS against C_T, and the expectation would be that a practically horizontal line would result (to the log scale) up to $C_T = 100$, with a slight dip up to 1,000 and a discontinuity of slope beyond about

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[23] "Uniform Water Conveyance Channels in Alluvial Materials," by Daryl B. Simons and Maurice L. Albertson, Proceedings, ASCE, Vol. 86, No. HY 5, May, 1960, and discussion thereon by T. Blench.

[24] Discussion by T. Blench of "Uniform Water Conveyance Channels in Alluvial Materials," by Daryl B. Simons and Maurice L. Albertson, Proceedings, ASCE, Vol. 87, No. HY 1, 1961.

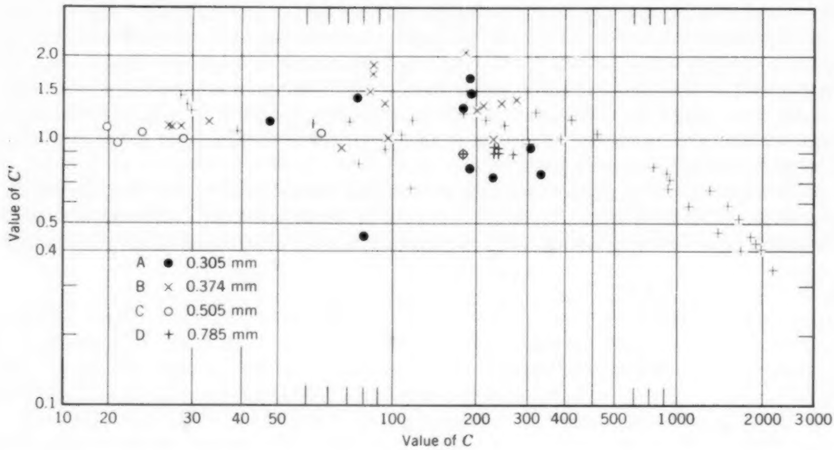

FIG. 14.—CHANGE IN REGIME SLOPE FORMULA WHEN DUNES VANISH

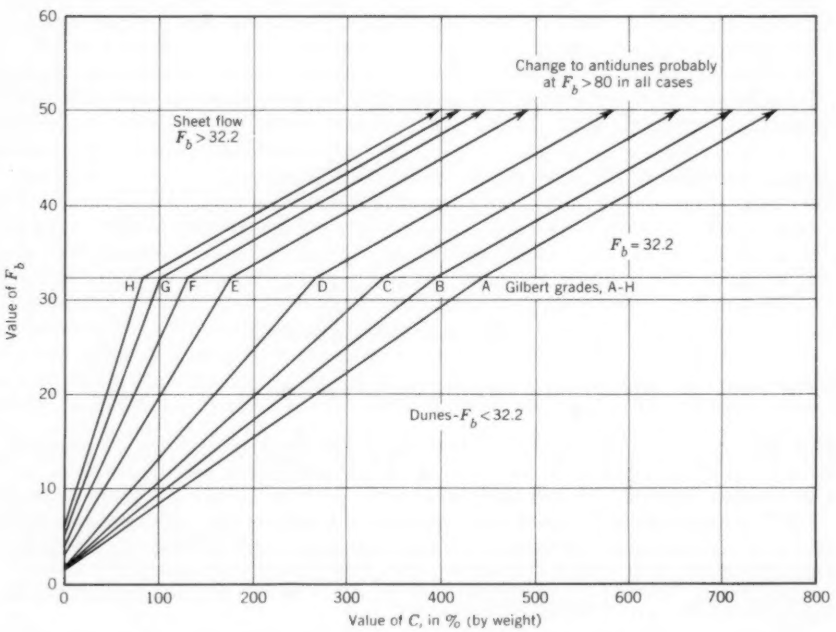

FIG. 15.—PROBABLE TYPE OF BED-FACTOR AGAINST CHARGE RELATION FOR ALL GILBERT SANDS AND GRAVELS

3,000. (Fig. 14 reproduces Fig. 4 of Ref. 13, in support; it tests, on logarithmic scale for convenience of publication, whether equation (12) with "a" = 1/400 does actually fit a good sample of Gilbert data in the subcritical range). The plot,[25] actually of $SQ^{1/12}/F_b{}^{11/12}$ gave a horizontal line with the scatter of points comparable with those of Fig. 9 and too much to show the expected dip; it showed a definite change of slope where expected; the scatter of points was more than would have been expected from Gilbert; the discontinuity of slope was as marked as that of τ in Fig. 9; the position of the line did not agree with the value of 3.63 in equation (12).

The discontinuity over which the writer and the authors agree is interesting because the well-known transport formula of Meyer-Peter[26] and the bed-load function of Einstein,[27] both resting partially on a selection of Gilbert data and

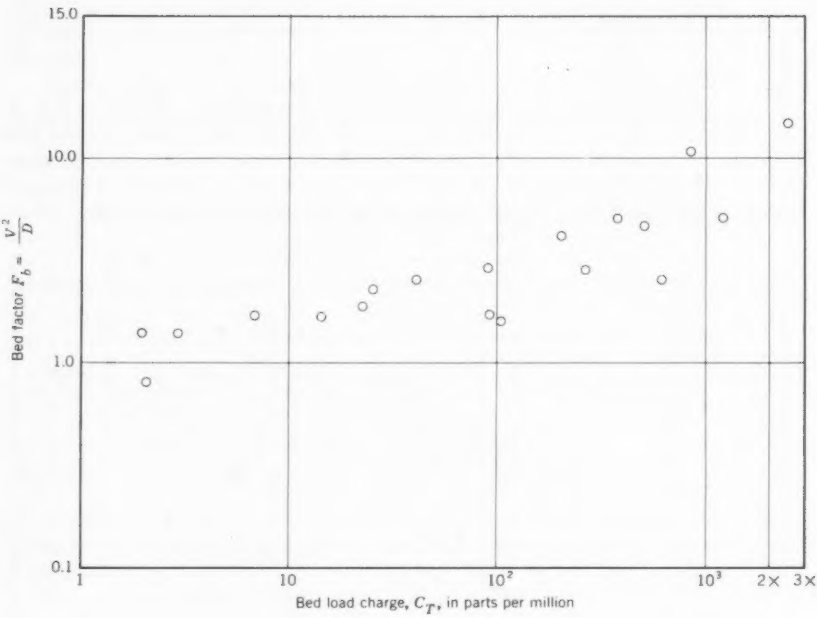

FIG. 16.—BED FACTOR CHARGE RELATION FOR DUNES: TABLE 1

both giving essentially the same relation,[28] are devised to show a continuous relation through ripples, dunes, sheet flow and antidunes. Obviously some reconciliation amongst authorities is desirable, perhaps by further observations.

The disagreement in coefficient between the writer and the authors might be due to measurement methods, see comments under EXPERIMENTAL EQUIP-

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25 "Regime Behavior of Canals and Rivers," by T. Blench, Butterworth's Scientific Publications, Sec. 1.7, Washington, D. C., 1957.

26 "Formulas for Bed-Load Transport," by E. Meyer-Peter and R. Muller, Proceedings, I.A.H.R., 1948, p. 39.

27 "Formulas for the Transportation of Bed-Load," by H. A. Einstein, Transactions, ASCE, Vol. 107, 1942, pp. 561-594.

28 "The Present Status of Research of Sediment Transport," by Ning Chien, Transactions, ASCE, Vol. 121, 1956, p. 833, Fig. 5.

MENT AND PROCEDURE. Possibly a re-plot to Meyer-Peter and Einstein parameters would help by showing whether there was disagreement with their data as well as with those of Gilbert and of major irrigating canals of India.[16]

(ii) Bed-factor, F_b, and charge. Regime theory[13,14] would replace Fig. 10 by a simple plot of V^2/D against charge, C, for each bed-material used. Fig. 15 is an idealized key diagram showing the type of relations obtained by doing this for all Gilbert data. Fig. 16 plots the subcritical ripple and dune data of Table 1 similarly and yields the kind of relation expected, but the slope of the line is considerably different from expectation from Gilbert and from work done with natural sands of 0.275 mm and 0.45 mm at the University of British Columbia.[17] Those last experiments, for the small but highly practical range of C = almost zero to C = 12 parts per hundred thousand were fitted by:

$$F_b = 1.9 \sqrt{d_m} \; (1+0.135C) \qquad (13)$$

in which d_m is particle median size in millimeters. As they made lengthy runs at vanishing C, the scatter at C = 0 was small, whereas the points at C = 1 in Fig. 16 scatter badly. So again the questions raised previously about natural sand and measurement methods come into prominence.

An important agreement about the type of fit to bed-factor data is that, at vanishingly small charge, V^2/d has a definite value. This contradicts theories that make bed-factor vanish when charge tends to zero. A further important showing is that bed-factor is a "function of state" in that it depends on the bed-material and the charge, but not on the discharge of the fluid. This contradicts hydraulics texts that publish tables purporting to give the velocities at which channels moving non-cohesive sediment along their beds will neither silt nor scour; such tables require that the channels of an ideal irrigating system that distributes sediment uniformly should be designed to run all at the same velocity whereas they would run all at the same V^2/D; that is, the large channels would run considerably faster than the small ones. These matters have been discussed in detail elsewhere.[16]

Need For Research.—Attention is drawn to the specific recommendations for research.[13,14] The Gilbert data, despite their great range and number, do not have uniform coverage and are for artificial sands and gravels. Their analysis[13,14] leads to a selection of specific questions, most of which could be answered in a couple of years by a few co-operating stations with large facilities, supplemented by large scale experiments in the field. A research not mentioned,[13,14] but likely to be fruitful because Canadian and American engineers have demonstrated its simplicity in the field,[22] is into the relation among dune parameters and those of the water-sediment complex and flow. Although individual dunes and ripples may be impermanent, the average number per unit length, the average heights and the average speeds of progression are definite, and regime theory[29] indicates that a suitable equivalent roughness height is given by $(\nu F_s)^{\frac{1}{2}} \; F_b(1 + a\,C)^2$. Perhaps the authors would consider the feasibility of such a program in terms of the canal facilities used for their related work[23] and, of course, their laboratory flume.

Nomenclature.—For several years the writer has adhered to ASCE nomenclature for his publications, including international ones.[30] It is suggested

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29 "Regime Behavior of Canals and Rivers," by T. Blench, Butterworth's Scientific Publications, Sec. 1.7, Washington, D. C., 1957.

30 "Civil Engineering Reference Book," Butterworths, London, England, 2nd Edition (under issue 1960). Chapter "Hydraulics of Canals and Rivers of Mobile Boundary," by T. Blench.

that the authors might also conform, for example in using a small d for depth and a large D for particle size; no system based on only 26 letters can be perfect.

EMMETT M. LAURSEN,[31] M. ASCE, and GERALD A. ZERNIAL,[32] A. M. ASCE.—Useful as dimensional analysis and dimensionless parameters can be in many instances, if not used with caution, their use can confuse and embellish rather than clarify and simplify. It seems to the writers that too great a faith in the power of dimensionless parameters has marred this report on an otherwise careful and worthwhile investigation.

Both Figs. 10 and 11 illustrate one of the traps waiting for the unwary, relating something to itself. The parameter of Fig. 10, for example:

$$\frac{V}{V_*}\ \frac{\tau_o}{\Delta\gamma_s\, d} \quad \text{versus} \quad \frac{V_*\, d}{\nu}$$

can be rewritten as

$$\frac{C}{\sqrt{g}}\ \frac{\gamma\, D\, S}{\Delta\gamma_s\, d} \quad \text{versus} \quad - \frac{\sqrt{g\, D\, S}\, d}{\nu}$$

since $V/V_* = C/\sqrt{g}$, $\tau_o = \gamma\, D\, S$, and $V_* = \sqrt{g\, D\, S}$. In this form, it is readily apparent that in the ordinate parameter there is a factor D S/d and in the abscissa parameter $\sqrt{D\, S/d}$, which by themselves would plot as a perfect line on a 2 to 1 slope on log-log paper. The data in Fig. 10 plot in a scatter band with a slight curve approximately a 2 to 1 line. Because most of the range in values of the parameters is due to the range in values of the slope, it can be concluded that the relationship indicated in Fig. 10 is primarily $D\, S/d = (\sqrt{D\, S/d})^2$. Presumably, the systematic deviation from this line should then be due to the residue of the parameters

$$\frac{C\,\gamma}{\sqrt{g}\ \Delta\gamma_s} = \phi_a \left(\frac{\sqrt{g}\, d^{3/2}}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

and the scatter due to other unincluded factors or to experimental errors.

Similarly, the parameters of Fig. 11

$$\frac{V}{V_*}\ \frac{\tau_o}{\Delta\gamma_s\, D}\ S \quad \text{versus} \quad 10^2\ \frac{w\, d\, S}{\nu} + 5\,\frac{V^2}{g\, D}$$

can be rewritten

$$\frac{C}{\sqrt{g}}\ \frac{\gamma}{\Delta\gamma_s}\ S^2 \quad \text{versus} \quad \left(100\,\frac{w\, d}{\nu} + 5\,\frac{C^2}{g}\right) S$$

and it is evident that the range of values of S, from 0.00015 to 0.0101, determines in large measure the manner in which the data plot. In this case, the residue of the parameters are still not completely independent when the S factors are removed.

$$\frac{C}{\sqrt{g}}\ \frac{\gamma}{\Delta\gamma_s} = \phi_b \left(100\ \frac{w\, d}{\gamma} + 5\ \frac{C^2}{g}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

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[31] Assoc. Prof., Civ. Engrg., Michigan State Univ., East Lansing, Mich.
[32] Assist. Instr., Dept. of Civ. Engrg., Michigan State Univ., East Lansing, Mich.

It would appear from Eqs. 14 and 15 that the dimensionless Chezy coefficient $C/\sqrt{g}$ is related to either $\sqrt{g}/d^{3/2}/\nu$ or $w\ d/\nu$. These parameters, however, only vary with the temperature insofar as this particular study is concerned. From a plot simply of run number (presumed to be in order of occurrence) against temperature, Fig. 17, it might be inferred that the first eighteen runs were made in the winter and the remainder in the summer. Because the conditions for the runs were more or less systematically varied, the runs with rippled beds (and rather small $C/\sqrt{g}$ values) happened to be at low temperatures, while the transition and antidune runs (with larger $C/\sqrt{g}$ values) happened to be at higher temperature. A suspicion that the apparent temperature effect is really a bed-form effect can hardly be avoided.

That the Chezy coefficient is a function of the bed form, or roughness, is shown in Fig. 18 where $C/\sqrt{g}$ is plotted against 2 D/h. The numbers beside each point are values of L/h. The values of L and h, the length and height of the dunes and ripples, were obtained from correspondence with the authors and are listed in Table 2. Considering the variety and complexity of bed forms, the relative height and length are not really sufficient to describe completely the geometry. Moreover, the surface texture (particle roughness) and the Reynolds number of the flow might also be expected to be influential in determining the resistance to flow. The data of the 0.18-mm sand used by Barton and Lin,[33] and of the 0.1-mm sand used in some experiments at the Iowa Institute of Hydraulic Research[34] also scatter about the Nikuradse line, Figs. 19 and 20, but with smaller L/h values. Qualitatively, the contribution to the total resistance of surface (particle) roughness would seem to be greater the coarser the sand.

As pointed out by the authors, the discontinuity in the relationship between τ_o and C_T in Fig. 9 is because of the relatively large tractive force associated with the dunes. The total tractive force is transmitted to the bed partly through a pressure distribution on the dunes and partly through the apparent shear on the sediment particles. If one assumes that the particle shear is the same as for a plane bed at the same velocity and depth of flow, this fraction of the total shear can be evaluated by the use of Manning's equation and Strickler's evaluation of n as:

$$\tau_o' = \frac{V^2\ d^{1/3}}{30\ D^{1/3}} \qquad (16)$$

The plot in Fig. 21 of this approximation of the particle shear against the concentration eliminates the discontinuity of Fig. 9.

The use of the particle shear was an integral part of the relationship for computing total sediment load, proposed by one of the writers:[34]

$$\overline{C} = \sum p \left(\frac{d}{D}\right)^{7/6} \left(\frac{\tau_o'}{\tau_c} - 1\right) f\left(\frac{\sqrt{\tau_o/\rho}}{w}\right) \qquad (17)$$

in which p is the fraction of sediment of size d, and the function, $f\ \frac{\sqrt{\tau_o/\rho}}{w}$ is in a figure presented elsewhere.[34]

Figure 22 shows a comparison between measured total concentration (C_{TM}) and concentration thus computed (C_{TC}). The symbols D, T, and A represent

33 "A Study of Sediment Transport in Alluvial Channels," by J. R. Barton and P.-N. Lin, Research Report, Civ. Engrg. Dept., Colorado A and M College, 1955.

34 "The Total Sediment Load of Streams," by E. M. Laursen, Proceedings, ASCE, Vol. 84, No. HY 1, February, 1958.

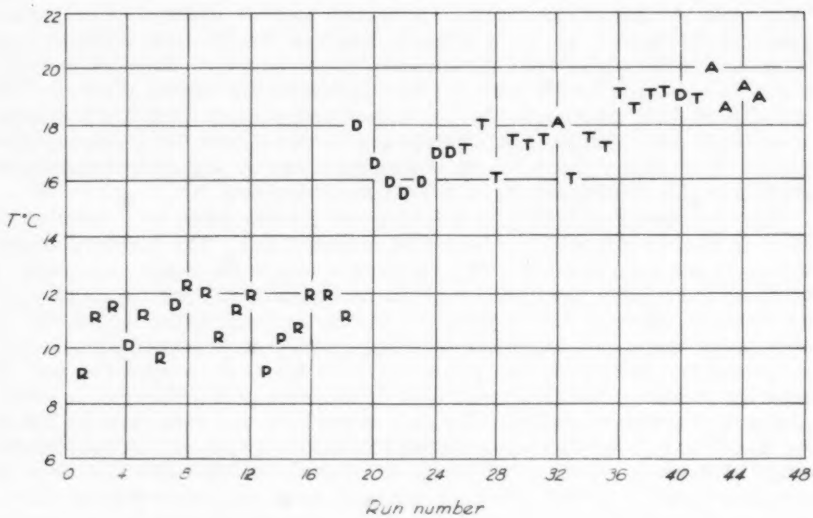

FIG. 17.—TEMPERATURE VERSUS RUN NUMBER

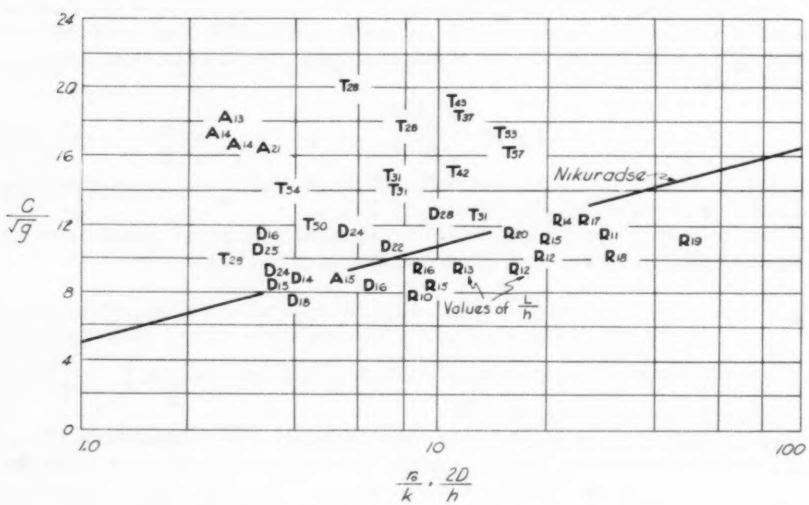

FIG. 18.—RESISTANCE AND ROUGHNESS, SIMONS AND RICHARDSON

dunes, transition, and anti-dunes, and the numbers beside each point are values of the Chezy parameter, $C/\sqrt{g}$. A systematic scatter about the line of perfect agreement $(C_{TM} = C_{TC})$ with the Chezy parameter is apparent. The large deviation in the lower concentration range can be partially explained by the factor $\left(\frac{\tau_0'}{\tau_c} - 1\right)$ used in the computational procedure. This factor should be the temporal mean of the integral of the positive values of $\frac{\tau_0'}{\tau_c} - 1$. The temp-

TABLE 2.—VALUES OF LENGTH AND HEIGHT OF THE DUNES AND RIPPLES

Run No. (1)	h, in ft (2)	L, in ft (3)	$\frac{C}{\sqrt{g}}$ (4)	$\tau_0' \times 10^2$, in psf (5)	C_{TC}, in ppm (6)	C_{TM}, in ppm (7)
14	---	---	14.9	0.294	---	---
17	0.04	0.77	11.2	.244	---	1
16	.06	.69	11.9	.254	---	2
13	---	---	14.0	.227	---	---
15	---	---	10.2	.255	---	1
18	.04	.71	10.2	.276	---	1
2	.07	1.21	12.3	.585	---	94
3	.09	1.36	11.2	.539	8.2	101
9	.06	0.73	10.4	.359	---	2
1	.10	2.00	11.8	.625	---	23
5	.07	1.00	12.4	.729	---	26
11	.06	.80	9.4	.264	---	4
4	.15	4.16	12.8	.890	---	92
8	.06	.73	9.4	.411	---	8
7	.20	4.41	10.8	.873	70	268
10	.08	.83	7.7	.310	---	20
6	.10	1.56	9.3	.564	---	42
12	.06	.89	8.5	.416	---	1
19	.26	6.42	10.7	.862	97	208
21	.31	4.82	8.4	.962	86	641
22	.52	7.50	8.5	1.09	113	710
25	.26	6.29	9.2	1.09	210	378
20	.36	5.37	8.6	1.26	240	508
23	.41	6.60	11.3	2.89	840	856
24	.31	5.51	7.31	1.38	346	1200
40	.31	7.35	11.8	4.48	1200	2460
39	.10	3.68	18.6	1.03	4354	3960
26	---	---	26.8	15.85	9171	4580
28	.05	2.57	16.2	6.38	3432	4230
29	.23	6.40	10.0	2.02	987	1850
31	.06	3.16	17.1	8.97	4998	4750
27	.19	10.22	13.9	4.92	3091	4100
36	.03	0.93	12.3	2.74	1977	1370
41	.14	3.93	17.7	11.85	6077	4340
30	.12	6.02	11.9	3.58	2558	3550
35	.07	2.20	14.0	4.71	3632	4610
34	.08	2.46	16.8	8.08	6451	5690
33	.10	2.76	19.9	12.40	11,798	6810
38	.09	3.81	17.0	13.85	9476	6230
37	.08	3.82	18.8	15.2	10,655	5570
32	.29	3.71	18.0	13.3	11,533	6180
45	.11	1.59	9.0	3.62	3700	9630
44	.21	2.95	16.8	13.28	14,876	15,100
42	.25	3.62	17.0	16.10	18,890	11,400
43	.27	5.81	16.5	19.17	23,500	11,500

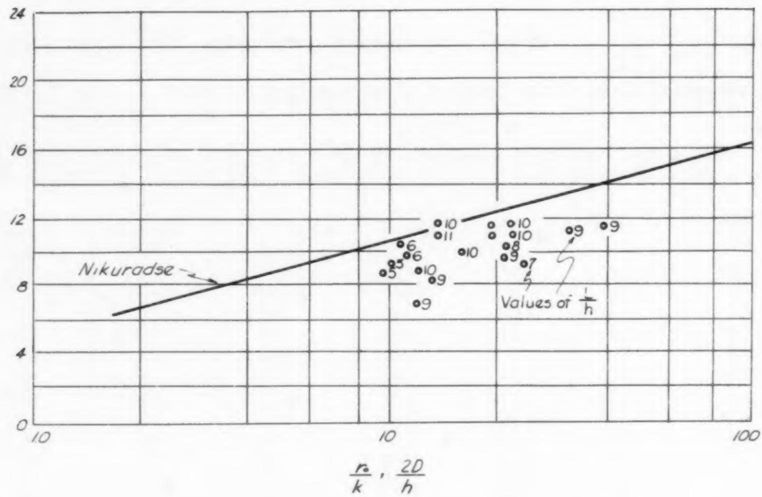

FIG. 19.—RESISTANCE AND ROUGHNESS, BARTON AND LIN

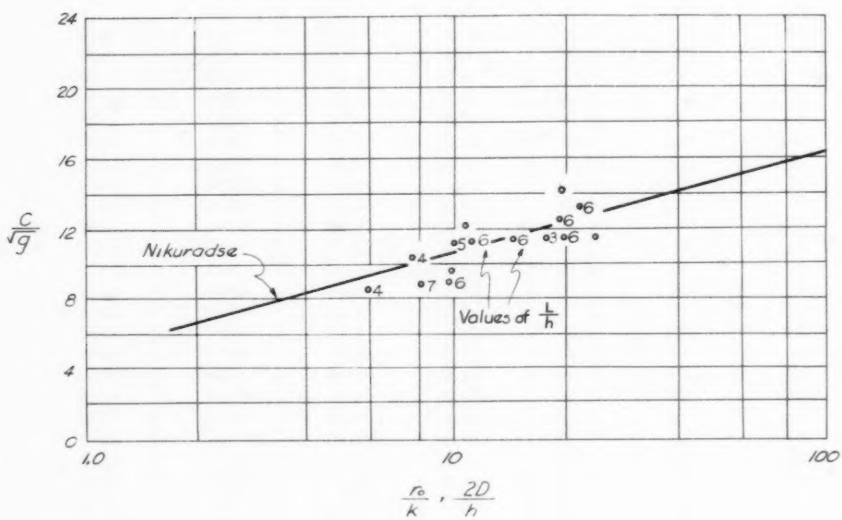

FIG. 20.—RESISTANCE AND ROUGHNESS, LAURSEN AND LIN

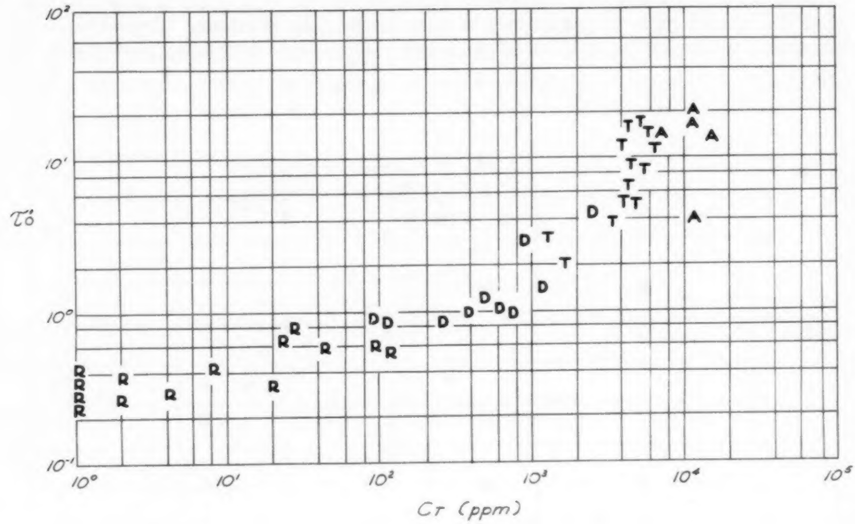

FIG. 21.—CONCENTRATION VERSUS PARTICLE SHEAR

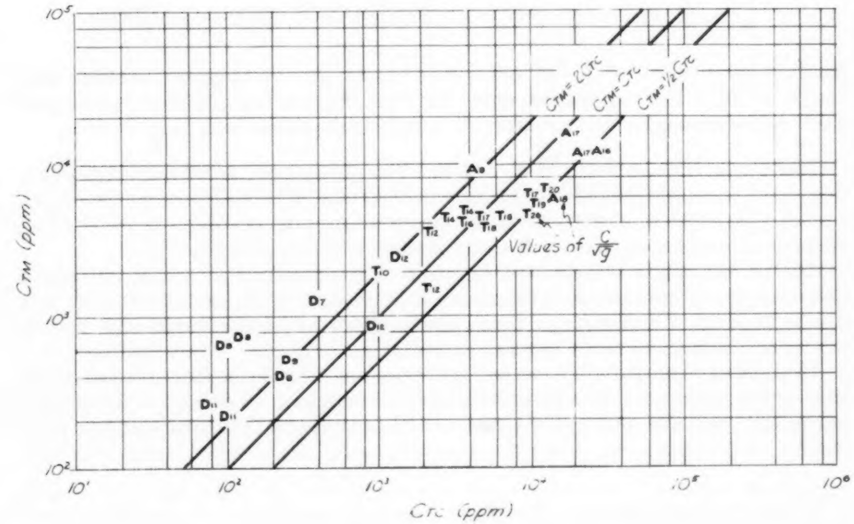

FIG. 22.—COMPARISON OF MEASURED AND COMPUTED CONCENTRATIONS

oral mean, obviously, would be much larger than the value used when τ_0' is almost equal to τ_c.

In Fig. 23 a comparison is shown between the dimensionless Chezy parameter $C/\sqrt{g}$ and ratio of measured to computed concentration. Concentrations computed by Eq. 17 tend to be high as the roughness, or resistance, to flow is

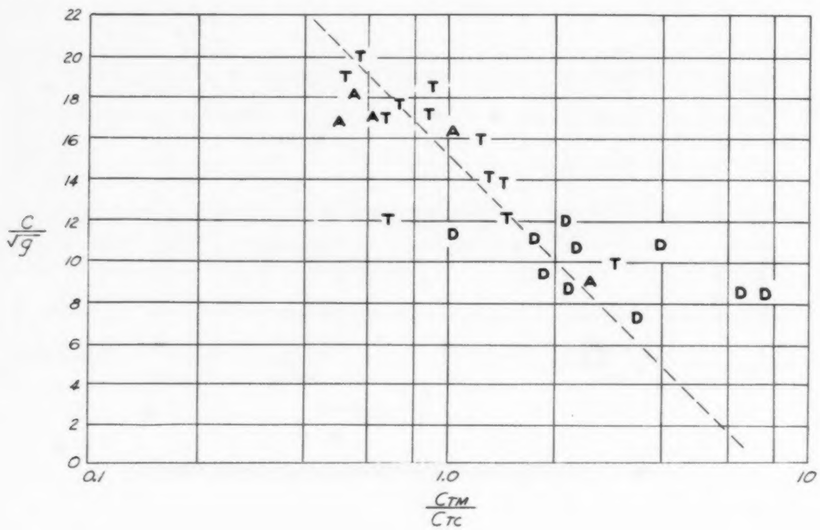

FIG. 23.—RATIO OF MEASURED TO COMPUTED CONCENTRATION

small (larger values of $C/\sqrt{g}$) and low as the roughness is great (smaller values of $C/\sqrt{g}$). It is not unreasonable that the chances for a particle to project itself into suspension are greater for a relatively rough than smooth bed.

DASEL E. HALLMARK,[35] M. ASCE.—The authors are to be commended for this paper in a field where only meager data exists. Alluvial streams have presented a problem to civilizations for many centuries; and even in the present century communities have been destroyed by alluvial streams.[36]

One of the major enlightening features of the paper is the ability to obtain data that are so consistent as demonstrated in Figs. 9, 10, and 11. This in part can be attributed to the large flume used, which eliminated side wall effects often encountered by most laboratory investigations.

In limiting this study to one bed material several of the difficult complexities are minimized. The relative standard deviation of the size distribution of the bed material σ of a given median fall diameter d influences movement of

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[35] Highway Research Engr., Div. Hydr. Research, Bur. of Public Roads, Washington, D. C.

[36] "Hydraulics Method to Fight Mud and Stone Carrying Streams," by M. S. Gagoshidze, Hydrotechnic and Soil Improvement Magazine, No. 9, Moscow, 1958.

the bed considerably. This is borne out by localized scour study[37] in which the d was nearly constant and σ was varied over a considerable range. Graphically this is illustrated in Fig. 24 in which the rate of scour parameter $\frac{h}{b}$ on the ordinate versus time parameter $\frac{QT}{H^2}$ clearly defines different rates of movement. The explanation of this phenomenon is in the sorting process of the fine parti-

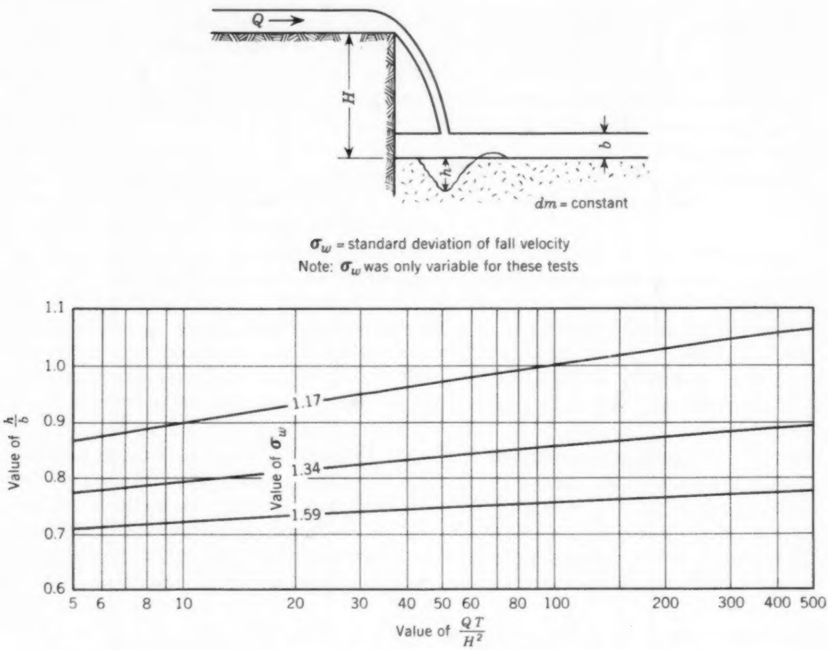

FIG. 24.—INFLUENCE OF σ ON RATE OF SCOUR

cles from the bed material, leaving larger particles which become interlocked and more resistant to motion. This sorting and interlocking process also is visible in streams where the bed material is well graded.

In the laboratory data presented here one would expect this phenomenon to appear in the ripple and dune formations. In fact, a plot of F_r versus R_e (Fig. 25) indicates that there is a definite relation between the type of bed roughness and the size of the bed material. In the low F_r - R_e range the viscous forces in the Reynolds number are important factors; where as in the higher Froude

37 "Influence of Particle Size Gradation on Scour at Base of Free Overfall," by Dasel E. Hallmark, thesis presented to Colorado State Univ., Ft. Collins, Colo., in 1955, in partial fulfilment of the requirements for the degree of Masters of Science.

numbers the relation of kinetic to gravitational forces predominate. Fig. 25 supports the discussion by the authors of the Froude number for classification of the bed roughness.

One of the perplexing elements in alluvial hydraulics is the tractive or shear force τ on the bed, γ D S. It is often used to make the data fall into a more consistent pattern but is it justifiable? With the major break-throughs in al-

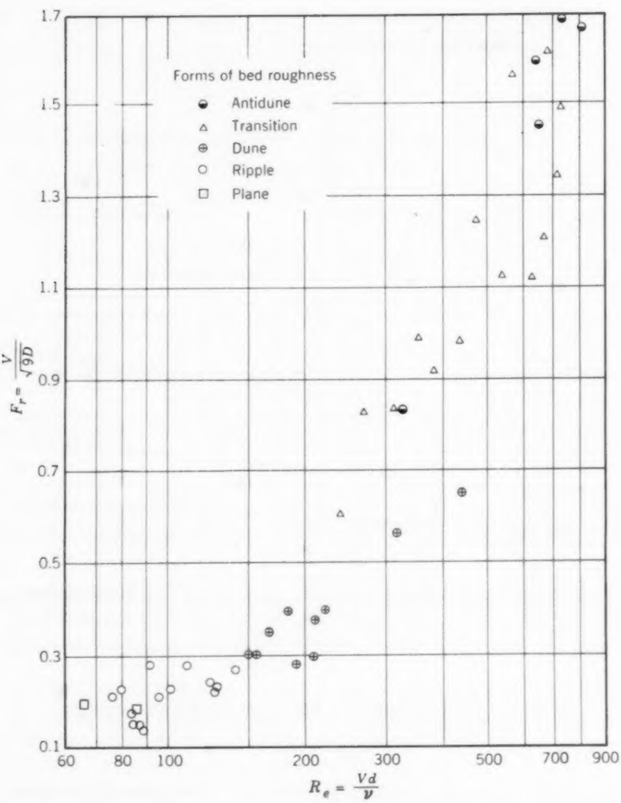

FIG. 25.—VARIATION OF F_r WITH R_e

luvial hydraulics in the last decade a better understanding of the shear force that is exerted on the bed and the force with which the bed opposes shear as related to median size d and standard deviation σ is becoming increasingly more important.

THOMAS MADDOCK, JR.,[38] F. ASCE, and W. B. LANGBEIN,[39] F. ASCE.—Messrs. Simons and Richardson have presented unusually detailed results of a careful study of the behavior of water and sediment in an 8-ft flume. As the authors note, their set of data contain the broadest range of flow conditions since those of Gilbert in 1914. Thus, these data are a considerable contribution to knowledge of the hydraulics of channels with movable beds. The authors are to be greatly commended for the care with which the work was undertaken and the clarity with which they describe what takes place in a flume of this size under different conditions of flow.

Data presented by Simons and Richardson are particularly of interest because of the unusual number of observations at very low rates of transport, that is <20 ppm or .002% by weight of water. (This is roughly about 1.2 cu ft of sand for each cfs day. At this rate, a stream flowing 100,000 cfs would move less than 5,000 cu yd of sand per day. Such a load certainly would not be a burden on the stream). It is to be noted that as previous investigators have shown, particularly Rubey,[40] in his analysis of the Gilbert data, and Casey,[41] the shear velocity, as described by $(\gamma\ D\ S)^{1/2}$ is not a constant but is a variable at the beginning of motion of the bed.

As a matter of fact, the Simons and Richardson data lead to the inescapable conclusion that there is no such thing as a critical or threshold shear in the sense that such a value is a constant for a given size and gradation of bed material. Rather, it would seem, there are ranges of threshold shears for each flume, size, and, perhaps, composition of material. This range appears to be a function of the possible variation in depth, as well as the possible range in slope of the water which can be made to flow without introducing sidewall effects. This is by no means a new conclusion but it is one which seems to be overlooked most frequently. Casey[41] made the following observation: "If the originally smooth bottom becomes riffled, then for constant discharge and slope the water surface is raised; and thus the tractive force is increased. The movement of the grains is, however, opposed by the greater resistance of the increasing bed roughness. The mean stream velocity decreases with increasing depth, the sorting becomes irregular and the sand movement decreases or vanishes."

While it may be true that there is a critical shear velocity at the beginning of motion on an artificially smoothed bed, such beds never exist in nature. Once deformed they can never again naturally become smooth. This is the reason why ripples are always found on a streambed, once flow has receded, if the sediment load of the stream declines more rapidly, relatively speaking, than the discharge.

The corollary to such a conclusion is, of course, that each value of shear velocity has a specific mean velocity which must accompany it. There is a certain value of the friction factor $\dfrac{\gamma\ R\ S}{\rho\ V^2}$ which must go along with each shear velocity. Thus "n" of the Manning equation would become a dependent variable.

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[38] Hydr. Engr., U.S.G.S., Washington, D.C.

[39] Hydr. Engr., U.S.G.S., Washington, D.C.

[40] "The Force Required to Move Particles on a Stream Bed," by William W. Rubey, U. S. Geol. Survey Prof. Paper 189E, 1938.

[41] "Ueber Geschiebebewegung, Mitteilung der Preuss," by Hugh J. Casey, Versuchsanstalt f. Wasserbau v. Schiffbau (VWS) Berlin, 1935, Heft 19.

At this point we are faced again with a question. If we would assume that roughness is wholly controlled by the bed material, this conclusion would be satisfactory. But hydraulic engineers for years have been judging the value of "n" at a given location for use in the Manning equation, by the appearance of the channel, and with a fair degree of success, too. Just what happens to the relationships at the beginning of motion once artificial roughness is introduced is unknown or unsure.

The Simons and Richardson data reveal a clear association of slope and depth at the beginning of (or at very low rates of) movement. Thus, the highest values of γ D S are associated with the lowest depths. Point 12 on Fig. 9 represents a discharge with depth of but 0.29 ft. Because of the relatively great width-depth ratios, it is doubtful if much higher values of γ D S would be associated with the beginning of movement in this flume. A narrower flume might yield different results.

TABLE 3.—COMPUTATIONS FOR FIGS. 26, 27 AND 28[a]

No. (1)	sin % (2)	D (3)	$\sqrt{s\,D}$ (4)	V (5)	$V\sqrt{S\,D}$ (6)	V_r (random) (7)	$V\,r\sqrt{S\,D}$ (8)
14	.015	.61	.09	.81	.073	.70	.06
3	.039	.85	.184	1.16	.214	4.78	.88
11	.049	.35	.13	.70	.091	2.04	.26
4	.057	.69	.20	1.44	.288	3.32	.66
10	.088	.33	.17	.75	.13	2.99	.51
22	.124	1.00	.36	1.70	.61	5.54	2.0
40	.301	.81	.49	3.32	1.63	5.38	2.64
26	.366	.34	.35	5.38	1.90	.75	.26
36	.446	.19	.29	2.05	.59	1.16	.34
27	.436	.33	.38	2.99	1.12	6.18	2.35
38	.619	.50	.57	5.38	3.08	1.70	.96
44	.898	.28	.50	4.78	2.39	5.38	2.69
43	1.01	.43	.66	6.18	4.08	2.80	1.85
37	.620	.43	.52	5.54	2.90	1.44	.75
35	.494	.25	.35	2.80	.98	.81	.28

[a] Data from Table 1, not all tabulated herein.

The smallest values of shear at beginning, or low rates, of sediment movement are associated with the greatest depths. Here again flume limitations came into play since point 17, the lowest value of γ D S after ripples developed, had a depth of 0.98 ft, which appears to be about as deep as Simons and Richardson cared to go.

Fig. 9 shows a trend in the relation between concentration and tractive force. However, the relationship surely must be more complex, depending upon the division in available energy between transport and frictional resistance. Fig. 26 shows the variation in concentration in terms of slope and depth (see Table 3).

Lines of equal tractive force that are also drawn upon Fig. 26 show the complex action of that factor on concentration. It is evident that the concentration is not uniquely related to the tractive force. The transport in this flume varies with each combination of depth and slope. For any given depth the concentra-

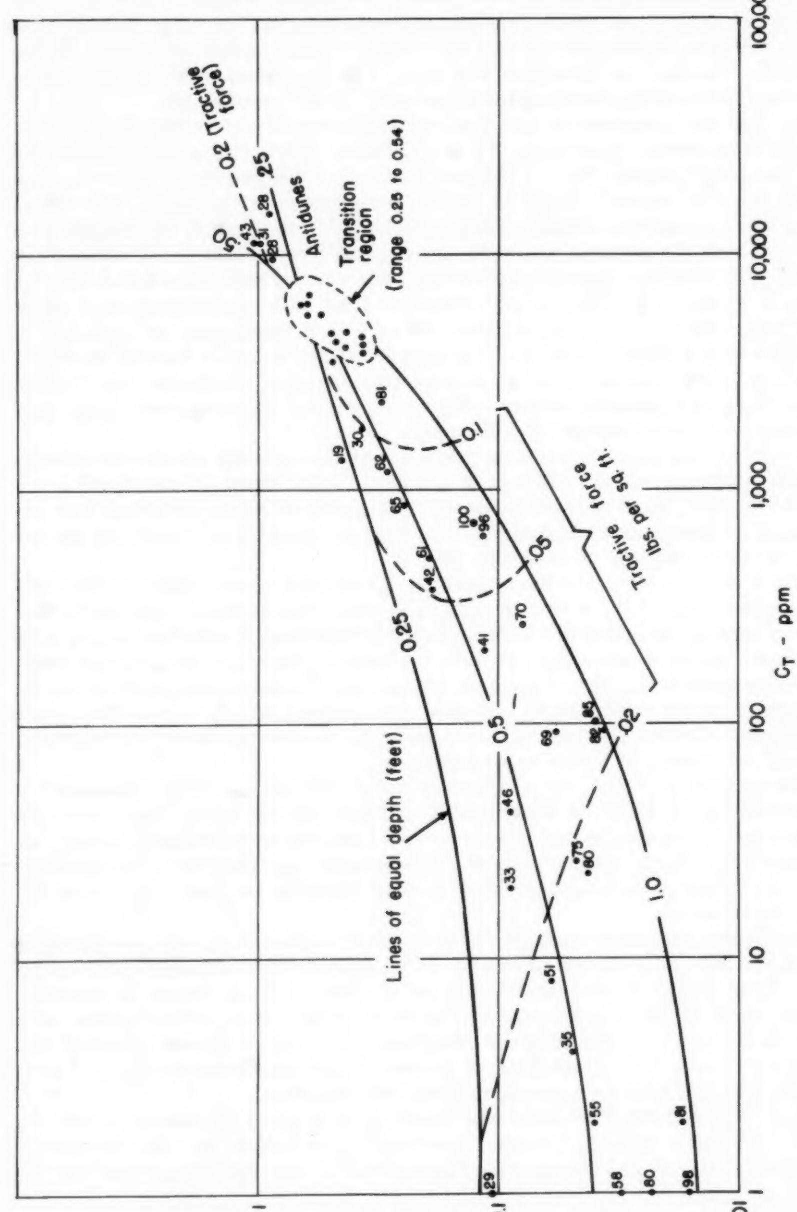

FIG. 26.—RELATIONSHIP OF CONCENTRATION, TO SLOPE, DEPTH, AND TRACTIVE FORCE

tion increases with slope, but note that the lines converge with increasing slope. There is evidence that the lines cross in the "transition region" so that in the antidune phase greater concentrations are associated with shallow depths. The region labeled "transition" on Fig. 26 contains those points which plot in the so-called "smooth" or transition region of Fig. 27. Seemingly, in this region the division of energy between transport and friction is unstable.

The authors' conclusions with respect to data presented in Fig. 9 appear to be open to question. They state, "It is significant to note that a sudden decrease in τ_0 occurred at about C_T = 2,000 ppm as the bed changed from dunes to plane bed or standing waves." Point 36 on Fig. 9 represents a discharge of 3.15 cfs with a depth of 0.19 ft. This is getting to the limit of feasible discharges in a flume because the accuracy of depth and slope measurements declines with low depths. Nonetheless, it would appear that with a lower discharge and a steeper slope, it would be possible to get transition flow with concentrations of about 1,000 ppm. On the other hand, point 40 on Fig. 9 represents a discharge of 21.41 cfs with a depth of 0.81 ft. This appears to be the upper limit of the flume capacity. There seems to be no reason why a larger discharge on a flatter slope, were this possible, would not give a dune bed with sediment movement at concentrations in excess of 4,000 ppm.

It would seem more likely that the phenomenon pointed out by the authors represents flume characteristics. If discharge is the same, or nearly so (runs 38, 39, 40, 43), there is little evidence of marked decrease in shear with increasing sediment load. Moreover, the break shown on Fig. 9 does not appear when the data are plotted as in Fig. 26.

Fig. 8 is of interest and even more so if sediment concentrations are plotted as a third variable. If this is done, it is clear that sediment load increases with increasing values of the Froude number. However, if two discharges have the same Froude number, the one with the largest slope has the greatest sediment concentration. The results of Simons and Richardson's work in effect confirm the work of Brooks.[42] Brooks' conclusions, which, at the time, were the subject of much discussion, now appear to be confirmed in their entirety, although not wholly by these experiments.

The authors approach the analysis of their data by a general statement of the mechanics of flow and dimensional analysis. In this study there were two independent variables--slope and discharge--and two pr possible three dependent variables, depth, total sediment concentration and possible size distribution. All other variables, such as viscosity, specific weights, etc., were essentially constant.

Therefore, the authors could improve their valued study by incorporating one of the few variables which can be measured, the total sediment concentration. Total sediment concentration is not an item in Eq. 5, where, as a matter of fact, most of the variables are irrelevant to this set of experimental data which deals with only one sand and one fluid. The authors appear to justify this omission by concluding that Figs. 8, 10, and 11 are an effective means of presenting inherent relations among some of the variables.

Fig. 8 is an attempt to order the modes of transport in relation to the velocity and depth without, however, developing a relationship. But the idea is perhaps similar to the treatment by Langbein.[43] Data (omitting those for "3-

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42 "Mechanics of Streams with Movable Beds of Fine Sand," by Norman H. Brooks, Transactions, ASCE, Vol. 123, 1958.

43 "Hydraulic Criteria for Sand Waves," by Walter B. Langbein, Transactions, AGU, Part II, 1942.

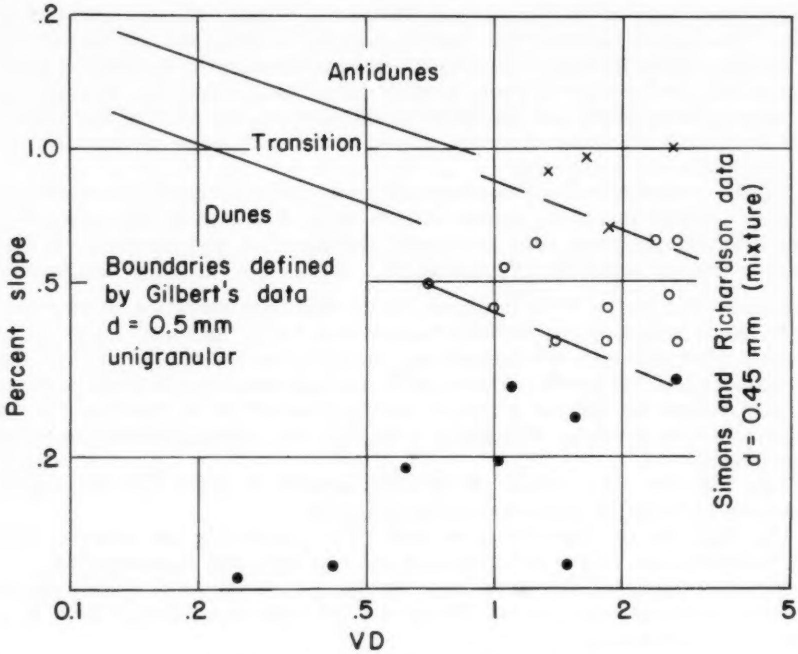

FIG. 27.—CRITERIA FOR REGIME OF TRANSPORT

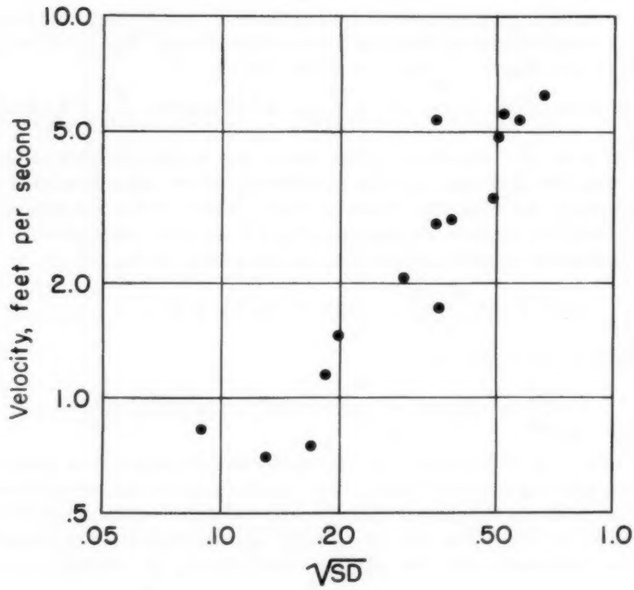

FIG. 28.—RELATION BETWEEN VELOCITY AND $\sqrt{SD}$

dimensional" flow) from Simons and Richardson (Table 1) have been plotted (Fig. 27) on an extension of Fig. 1 of the Langbein paper. The left side of Fig. 27 was defined by Gilbert's data, size 0.5 mm ungranular; the Simons and Richardson data are for size 0.45 mm, median diameter of a mixture. Considering the slightly finer grade and the effect of the mixture, the several data appear consistent with the older diagram, which, in terms of slope, perhaps merits consideration by the authors.

Fig. 10 is the kind of dimensionless diagram that also might be reconsidered because similar quantities appear on both axis. Eliminating the constant or near-constant quantities such as density, particle size, and viscosity, the diagram is actually a plot of VV_* against V_*. Since $V_* \sim \sqrt{S\,D}$, the diagram intrinsically is a plot of $V\sqrt{S\,D}$ against $\sqrt{S\,D}$. With this operation, it is possible to obtain relationships between random data, or to "improve" a poor relationship. For example, the authors say, in referring to Fig. 10, "It is of importance to note the precision with which the relation describes and systematically groups the various forms of bed roughness." It is clear that the relation cannot be general. Obviously, it applies only to the single series of the experiment.

Fig. 28 shows the intrinsic relationship between V and $\sqrt{S\,D}$; the scatter obviously reflects the variations in the Chezy C.

Fig. 29 shows the same thing, but with $\sqrt{S\,D}$ repeated in the ordinate. Note the "improvement" in the definition--an improvement that is misleading.

Fig. 30 is a plot of the same data, but the velocities were re-ordered so that they were entirely random. There is a definable trend, though there is no intrinsic relationship.

It should be made clear that the authors are not alone in their treatment. A more common illustration is offered by the sediment rating curve, which is a plot of discharge against the product of discharge and concentration.

Another type of artificial correlation results when products or quotients of variables are related against their sums or differences. Fig. 11 is an example of the former, and Fig. 3 an example of the latter.

Fig. 11 is essentially a plot of $\frac{V}{(\gamma\,D)^{1/2}}\,S^{3/2}$ against $\frac{V^2}{g\,D} + S$. Such a procedure would lead to a conclusion that there was some inherent relationship, if, for example, the Manning and the Colebrook-White equations were plotted against each other for a limited range of data. There is a considerable danger that the relations which look so good on Fig. 11 may be overhopeful.

The approximate equation of the line on the graph of Fig. 31 is

$$\rho\,V^2 + 110\frac{\gamma\,D\,S}{\rho\,V^2} - 1.24 = 89.7\,\gamma\,D\,S \qquad (18)$$

Rearranging this equation, we get

$$\frac{\gamma\,D\,S}{\rho\,V^2} = .01126 + .816\,\gamma\,D\,S - .00909\,\rho\,V^2 \qquad (19)$$

Notice that this is a division of two variables on one side and a subtraction of the same two variables on the other. Yet there is no fundamental relationship involved. The best we can say is that the value which results from the division of one variable by the other can be closely approximated by subtracting one variable from another, with the proper coefficients, of course, and adding a constant.

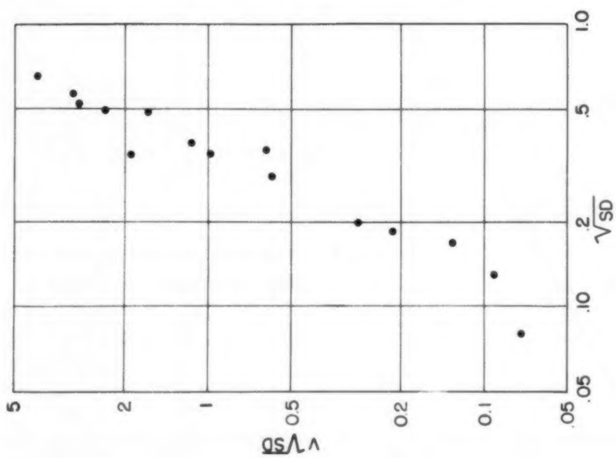

FIG. 29.—"IMPROVED" RELATIONSHIP BETWEEN V $\sqrt{SD}$ AND $\sqrt{SD}$

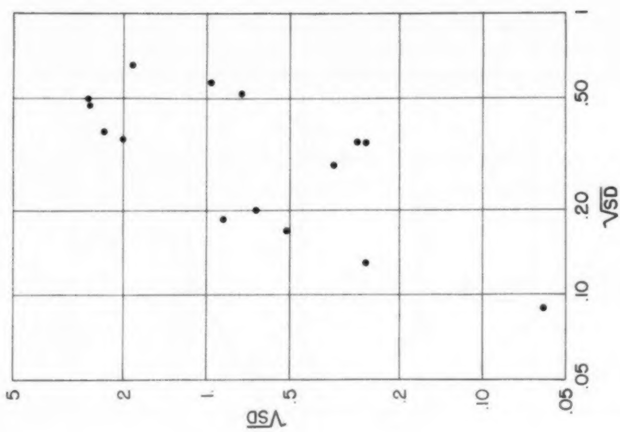

FIG. 30.—SPURIOUS RELATIONSHIP CREATED BETWEEN RANDOMLY RELATED V AND $\sqrt{SD}$

Because it is frequently possible that the use of non-dimensional parameters may mislead one to produce invalid correlations, the writers see danger in the

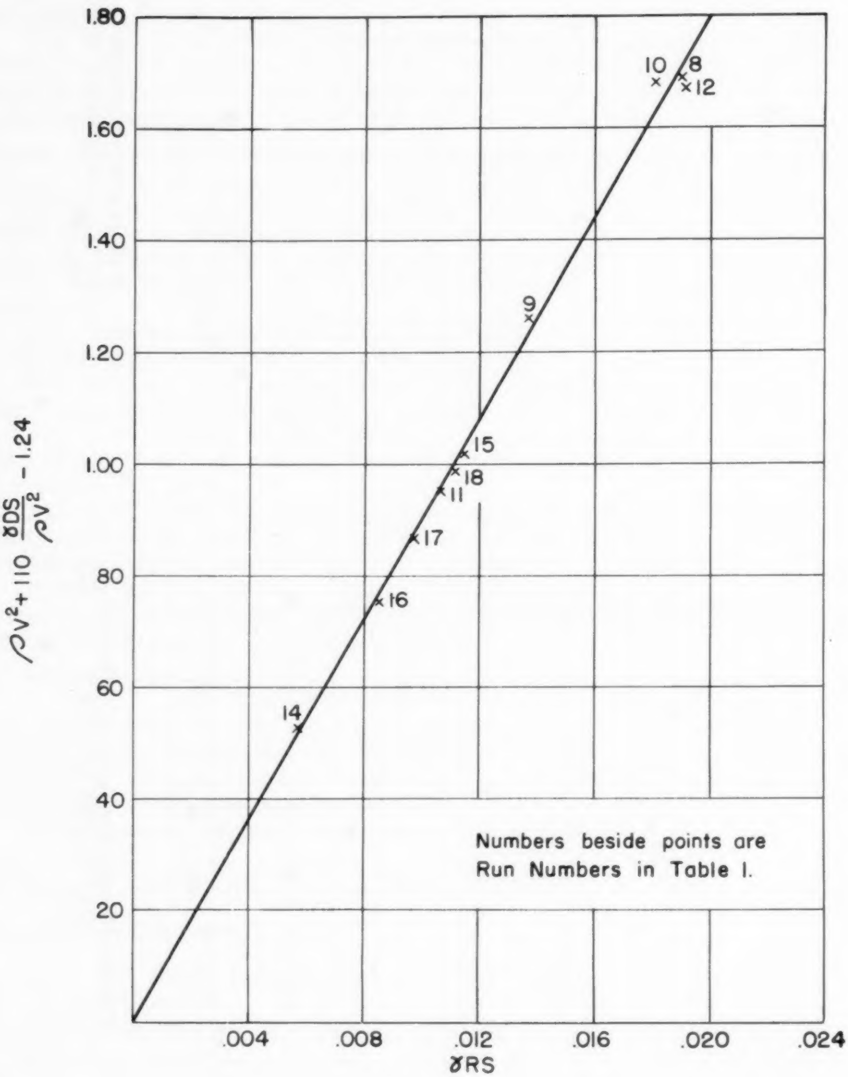

FIG. 31

use of dimensionless parameters where observations have been made of a limited number of variables.

Mention has been made of the unique amount of information made available by these studies at beginning, or very low rates, of sediment movement. If we take those 10 runs with no or very low rates of movement, excepting run 13 which does not fit the rest of the data in any type of analysis, we have only 3 variables, D, Q, and S, all of which are measured. Regarding the very low rates of sediment movement as being of no importance and assume that there is little change in the sediment composition or the density and viscosity of water.

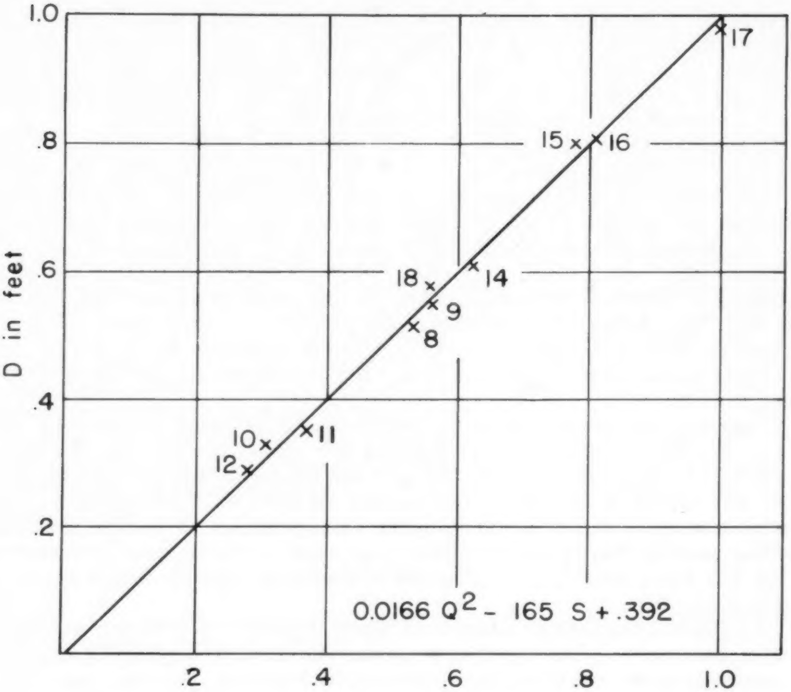

FIG. 32

The relationships existing among these variables can be closely expressed by the equation

$$D = .0166\,Q^2 - 165S + .392 \quad \ldots\ldots\ldots\ldots \quad (20)$$

Fig. 32 is a graph of this equation. Numbers beside the plotted points are the run numbers.

The correlation that exists in this equation is valid although the solution for D, if Q and S are known, will give much better results for small slopes and low discharges than the solution for either of the other two variables. This cannot be helped because D is the most difficult of all variables to measure precisely.

The equation can be expressed in the form

$$\frac{1}{V} = 1.062\ V\ D + \frac{1}{V\ D}\ (.392 - 165\ S) \quad \ldots\ldots\ldots \quad (21)$$

The result of this relationship is that $\frac{1}{V}$ has little variation even though the term .392 - 165S is positive. Note, however, that when Q is constant, V varies with S, there being a greater degree of variation for small values of Q than large.

It is to be doubted that this relationship would be found if V D were assumed to represent a Reynolds number. Who, at the outset, would assume an inverse relationship between V and V D at the beginning of motion? Whether the relationship is meaningful in terms of classical hydraulics is something else again, but it is unquestionably true, as Eq. 21 and Fig. 26 indicate, that γ D S is not a constant at the beginning of motion on the bed of a stream.

To summarize the results of the Simons and Richardson studies, it seems that once a certain combination of discharge and slope are given, the bed of the 8-ft flume, if it is originally smooth, will begin to deform. As deformation proceeds, the friction factor increases and the Chezy coefficient decreases. This adjustment will take place with no or extremely low amounts of sediment in motion after equilibrium is reached. With increasing discharge the sediment begins to move at increasing rates, until the dunes are washed out. During this stage the friction factor decreases and the Chezy coefficient increases with increasing discharge. With further increase in discharge, the sediment load increases rapidly, but the friction factor now increases and the Chezy coefficient declines.

It appears that differences in the bed configuration result in differing values of the shear velocity at beginning (or low rates) of sediment transport. There is a relation between mean velocity and discharge if slope is known, however, and without doubt other relations may be found with further study. The complete range of shear velocities under which sediment of the size and composition used by Simons and Richardson may begin to move cannot be measured in the 8-ft flume because of limitations of discharge capacity, flume depth, or width-depth ratio.

As a general comment on methods of analysis presented by Simons and Richardson, one may question whether non-dimensional analysis may be relied on to assure the maximum information being derived from the excellent data available. Considering the advanced state of work in the field of sediment transport, there should be little need to draw trend lines through clouds of points, thus obscuring many significant details. Non-dimensional analysis has its advantages, but when it is used to create artificial groups of variables for graphical treatment, something inherent is lost. The fault is not with graphical treatment, but with the forced association of variables. Some reconsideration could profitably be given to a method of analysis that resolves the separable and joint action of each independent variable, and thus achieves the kind of results that this excellent piece of research deserves.

R. HUGH TAYLOR, JR.,[44] A. M. ASCE, and NORMAN H. BROOKS,[45] M. ASCE.—The authors are to be commended for their comprehensive experimen-

[44] U. S. Geol. Survey, W. M. Keck Lab. of Hydr. Water Resources, Calif. Inst. of Tech., Pasadena, Calif.

[45] Assoc. Prof. of Civ. Engrg., W. M. Keck Lab. of Hydr. and Water Resources, Calif. Inst. of Tech., Pasadena, Calif.

tal program on sediment transport at the Colorado State University under sponsorship of the U. S. Geological Survey. Their experiments are valuable because they have been conducted in an 8-ft wide flume which is larger than any tilting flume used heretofore in sediment transport studies. The data which they are collecting, not only for the 0.45-mm sand reported on in the paper, but for all the sizes used in their program, will be of lasting value to the profession. For the record the writers ask in what form the complete record of their experiments will finally be published.

In the analysis and interpretation of the experimental results there are a number of points which warrant discussion. Under the section "Theory of Flow in Alluvial Channels," the authors have not actually presented a theory but only some dimensional analysis, leading to Eq. 6. To apply dimensional analysis sensibility to the roughness problem requires selection of the pertinent variables; but furthermore one must be careful to assume the correct number of functional relationships. In other words there may be more than just one dependent variable, as several relations may have to be satisfied simultaneously.

For example, the writers believe that for natural rivers the bed-material sediment discharge (Q_s) should be included as another independent variable, because field evidence shows that river behavior changes significantly with a change in Q_s supplied to a reach. The transport relation is strongly interlocked with the flow relation, and neither one can be properly considered without the other. Thus in Eq. 1 variables such as B, V, and S cannot be considered to vary at will as independent variables, but are constrained by other unmentioned relations such as a transport equation. Hence a simple listing of the variables is not as important as a careful consideration of the constraints, which at the present state of knowledge are none too clear.

However, one redundancy is apparent in Eq. 6. Because the drag coefficient for the particles C_D is the same as for spheres (because the fall diameter d is used for the characteristic length), C_D is a function of the particle Reynolds number, namely

$$C_D = f\left(\frac{w\,d}{\nu}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which f is the experimentally determined curve given in any elementary textbook. Rewriting,

$$\frac{w\,d}{\nu} = \frac{w}{V}\,\frac{d}{D}\,\frac{V\,D}{\nu}$$

$$= \left(\frac{w}{V}\right)\left(\frac{d}{D}\right)\left(R_e\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The last three parameters in parenthesis are given in Eq. 6, so C_D, a known function of the preceding product, is superfluous; that is, C_D is already a dependent variable by another relation not expressed.

Finally, dimensional analysis should give dimensionless combinations which are convenient. Since the paper is on resistance to flow, a dimensionless friction factor such as the Darcy-Weisbach f should be used instead of slope S in Eq. 6.

Furthermore, in the tabulation of experimental values (Table 1) it would be most helpful if the authors would give the computed roughness coefficients, and $C/\sqrt{g}$-values, upon which the subsequent discussion in the text is based. The arrangement of data in Table 1 according to increasing slope values is not a sequence with increasing roughness or shear because of the variable depth.

Bed Forms.—In the list of forms of bed roughness, "Plant bed and water surface" has been placed under the "Rapid flow regime, $F_r > 1$." While this may be true for their flume experiments (although none in the category are cited in Table 1) it should be made very clear that $F_r > 1$ is not a necessary condition for existence of a plane or flat bed. When there is no strong coupling between the bed and water surface profiles (as there is for standing waves or antidunes), the Froude number is a minor or irrelevant parameter because of the absence of a local gravitational effect on the flow pattern. For the plane bed condition it is only necessary to have velocities near the bed of such magnitude that a dune, if one were to form, would be "sheared" off. The stability of the sand surface depends on the magnitude of the local fluid forces relative to the resistance of particles to motion, and should not be sensitive to depth. Since the Froude number does depend on depth, and does not depend on the sand characteristics, it is an illogical parameter for describing bed forms which are not coupled with surface waves.

This brings us to the crucial weakness of the authors' entire method of description of the various bed forms in alluvial channels under "Observed Flow Phenomena." In attempting to make their description appear more generalized they have used the dimensionless quantities d/σ and F_r as criteria for separation of the various types of bed forms (for example, "transition" occurs when "$d/\sigma' > 2$ and $0.6 < F_r < 1.0$"). The citing of dimensionless values has merit, not a priori, but only if they are meaningful over a significant range of conditions. Because there is no physical reasoning to support the use of these parameters, and the cited values are easily proved to be lacking in generality, their use obscures the solution to the problem rather than clarifying it.

As indicated previously the Froude number has relevance only for the standing wave and antidune cases. It means very little to say that ripples change to dunes at $F_r = 0.28$; the authors would make their point more clearly by stating that dunes begin at a mean velocity of about 1.40 fps for their experiments. With change to another flow system of a different depth but with the same sand, the velocity at the point where ripples become dunes is more likely to be about the same actual value cited above than a new value computed from $F_r = 0.28$.

Consider now the parameter d/σ', the ratio of the grain size to the so-called thickness of the laminar sublayer. When the bed is covered with ripples, dunes, or other form resistance, σ' lacks physical interpretation, but still can be computed by the relation

$$\sigma' = \frac{11.6\ \nu}{V_*} \qquad (24)$$

Consequently

$$d/\sigma' = \frac{dV_*}{11.6\ \nu} \qquad (25)$$

Now for example the authors state that motion begins when d/σ' was between 0.48 and 0.53. By Shields' graph[46] the initiation of motion could begin at values of this parameter ranging from say 0.2 to 50 or more depending on the properties of the particles and of the fluid. (By comparison $\tau_o/\Delta\gamma_s$ d ranges from 0.035 to 0.07 as a function of d/σ'). Whether the boundary is hydrodynamically smooth or rough (as determined by the value of d/σ') is irrelevant; the question involves the balance of forces on the particles.

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46 "Sediment Transportation," by C. B. Brown, Engineering Hydraulics, ed. H. Rouse, Wiley, New York, 1950, p. 790.

The features which the authors call ripples have been called dunes in the past by the writers in describing laboratory experiments at the California Institute of Technology[47,48] (CIT). The larger features called dunes by the authors have not been observed at CIT, because of depths limited to 0.5 ft or less, and widths of 0.875 ft or 2.79 ft respectively in two flumes. They may be related, however, to what has been described as sand waves, which were features with long flat tops, and not three-dimensional in plan as shown by Fig. 4. Apparently there is no essential difference between ripples and dunes other than size, but it is curious that there is no gradual gradation from ripples to dunes. Still much larger dune-type features have been observed in rivers (as by Carey and Keller[49] on the Mississippi River). Perhaps for a given sand only certain ranges of dune (or ripple) heights and wave lengths are possible, with appearance of the large set(s) suppressed by inadequate depth or width. If this is so, it makes the roughness analysis and modeling of alluvial channels a very troublesome problem, which cannot be resolved by laboratory work alone.

The general description of antidunes and standing waves agrees well with recent research at CIT by John F. Kennedy[50,51] on the mechanics of these waves, sponsored by the Agricultural Research Service. It may be noted that antidune behavior is considerably influenced by the grain size, as well as the velocity. For example, Kennedy found that for 0.23 mm sand, the friction factor f consistently rose as F_r increased, due to heavy wave breaking, but that for 0.55 mm sand, f decreased as V increased because the amplitude of antidunes decreased, and wave breaking was less important than for the more easily transported 0.23 mm sand. Also for 0.55 mm sand, a flat bed regime was found following antidunes with increasing velocity (at $F_r \approx 2$); but for 0.23 mm sand this regime was not found.

In Fig. 8 there are a number of runs listed as "Standing Waves" (Runs 28, 31, 33, 34, 35, 37, 38, 39, 41), but in Table 1 these are all listed as "Transition." Clarification of the bed configuration is needed for these runs.

Analysis of Roughness by Authors.—At the end of the section "Analysis of Data," the authors present Fig. 11 (Eq. 9) which, it is claimed, can be used to compute $C/\sqrt{g}$ (or simply $\sqrt{8/f}$). Since $C/\sqrt{g}$ does not appear explicitly, one may instead seek S by trial and error, given depth, velocity, and sand and water properties. It must be noted that this curve is by no means general, but applies only for the particular sand and flume used by the author.

To see how well Fig. 11 allows the slope to be determined, trial-and-error solutions for S were attempted for several of the reported runs, assuming everything known by S. Fig. 33 shows the curve of Fig. 11, together with two other curves, each of which is obtained by assuming arbitrary values of S while using the depths and velocities reported for the runs indicated (runs 12 and 40).

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47 "Mechanics of Streams with Movable Beds of Fine Sand," by N. H. Brooks, Transactions, ASCE, Vol. 123, 1958, pp. 526-594.

48 "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by V. A. Vanoni and N. H. Brooks, Sedimentation Laboratory, Calif. Inst. of Tech., Pasadena, Calif., Report E-68, 1957.

49 "Systematic Changes in the Beds of Alluvial Rivers," by W. C. Carey and M. D. Keller, Proceedings, ASCE, Vol. 83, No. HY 4, August, 1957.

50 "Stationary Waves and Antidunes in Alluvial Channels," by J. F. Kennedy, thesis presented to the Calif. Inst. of Tech., Pasadena, Calif., in 1960, in partial fulfilment for the degree of Doctor of Philosophy.

51 "Study of Relations between Transportation of Sediment and Hydraulic Characteristics of Streams," by J. F. Kennedy, Keck Lab. of Hydr. and Water Resources, Calif. Inst. of Tech., Pasadena, Calif., Report KHWR-6, in press.

The assumed values range from 0.1 to 10 times the observed slope. For definitive trial-and-error solutions, of course, these curves should have well defined intersections with the given function. Unfortunately this is not the case.

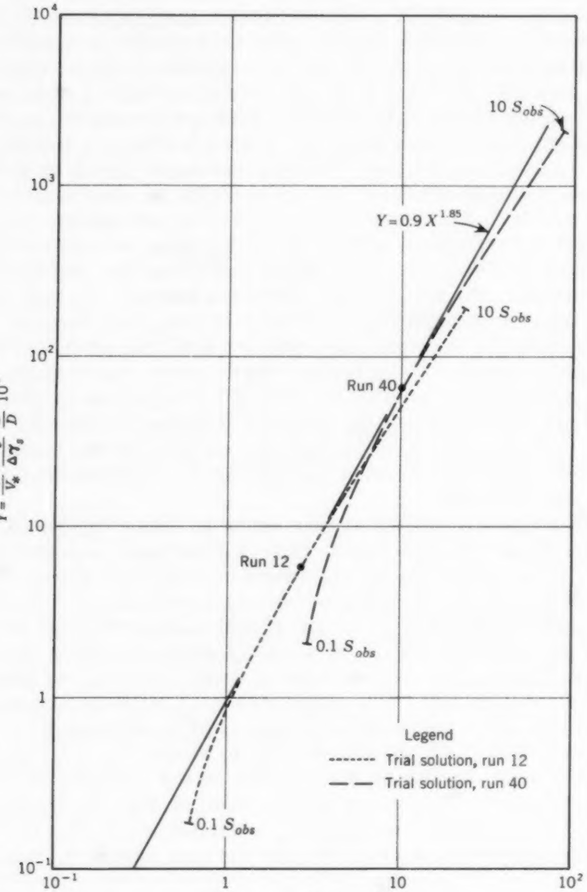

FIG. 33.—AUTHORS' FIG. 11, SHOWING IMPOSSIBILITY OF TRIAL-AND-ERROR SOLUTIONS FOR SLOPE

The trial curves shown represent a variation of a hundredfold in assumed S-values and still lie within the belt of scattering of the data points upon which the function is based. Several other runs were tried, with similar results. It must therefore be concluded that Fig. 11 is of no help in the estimation of slope

(or friction factor). The indiscriminate juggling of relationships may thus, as in this case, lead to relations more nearly approaching disguised mathematical identities than physical laws.

That the same conclusion holds for Fig. 10 is evident when it is noted that each of the variables contains slope to the one-half power. Thus the "trial" curves are straight lines which slope upward to the right at 45° and which likewise have rather unsatisfactory intersections with the zone within which the data lie.

An Alternate Analysis of Roughness.—In clear-water flow in pipes or in open channels with rigid boundaries, the slope of the energy grade line may be determined by means of the Darcy-Weisbach friction factor, f, which is a well-established function of the Reynolds number and of the relative roughness (and is usually expressed in a pipe friction diagram). If the flow is in an alluvial channel, however, the problem is complicated by the presence of additional causes of energy dissipation, such as the turbulence in the wakes of dunes and ripples on the moveable bed. Hence Einstein and others have chosen to apportion the total boundary shear stress τ_o into a part, τ'_o due to grain roughness and the additional part, τ''_o, due to form drag, so that $\tau_o = \tau'_o + \tau''_o$. Since $\tau_o = \gamma\, r\, S$, this partition requires some partition either of the hydraulic radius or of the slope (or of both). If the hydraulic radius is partitioned, following Einstein and Barbarossa,[52] the results are two virtual channels of the same slope and flow velocity but of different geometries, giving rise to a certain amount of intuitional difficulty. If, on the other hand, the slope is partitioned, as Meyer-Peter and Müller[53] proposed, one can write $S = S' + S''$, and visualize S' as that energy slope necessary for the given flow to overcome the resistance due to the grain resistance on a flat bed, where S'' would be the additional energy slope necessary to enable the flow to overcome the form drag caused by the bed configuration.

This being the case, the ratio of the true slope to that corresponding to the grain drag only (S/S') would be a simple measure of the energy dissipation arising out of the alluvial nature of the stream, as related to that expected in clear-water flow over a flat bed. Furthermore, to the extent that most of the additional dissipation arises from form drag, S/S' could be a useful description of the bed configuration itself. Since the Darcy-Weisbach friction factor, f, for open channel flow is defined as

$$f = \frac{8\, g\, r\, S}{V^2} \qquad (26)$$

in which r is the hydraulic radius, and V is the mean channel velocity, then a corresponding friction factor, related to the grain resistance only, may be defined as

$$f' = \frac{8\, g\, r\, S'}{V^2} \qquad (27)$$

Hence,

$$\frac{S}{S'} = \frac{f}{f'} \qquad (28)$$

and thus the ratio f/f' has the same physical significance as S/S'.

52 "River Channel Roughness," by H. A. Einstein and N. L. Barbarossa, Transactions, ASCE, Vol. 117, 1952, p. 1121-1146.

53 "Formulas for Bed-Load Transport," by E. Meyer-Peter, and R. Müller, Internatl. Assoc. for Hydr. Research, Second Meeting, Stockholm, Sweden, 1948, pp. 39-65.

According to the definition of S', f' is simply the friction factor for clear flow over a flat bed with grain roughness only. Hence a reliable value of f' may readily be found from the Moody pipe-friction diagram, using 4 r in place of the pipe diameter, and taking the equivalent sand roughness ϵ to be the mean grain size of the bed material. (Some investigators say that the equivalent roughness is bigger than the mean size (for example, the 65% -size is advocated by Einstein[52]); this may well be true, and further research is needed on this point, but it is of minor importance here). Namely,

$$f' = \Phi\left(R_e, \frac{\epsilon}{4\,r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

Thus to compute the value of the ratio f/f' from measured data one has only to divide the f of Eq. 26 by the f' from the pipe friction diagram.

TABLE 4.—FRICTION FACTORS AND f/f' FOR AUTHORS' DATA

Run No. (1)	f (2)	f' (3)	f/f' (4)	Form of Bed Roughness[a] (5)	Run No. (6)	f (7)	f' (8)	f/f' (9)	Form of Bed Roughness[a] (10)
14	.0308	.0208	1.48	Plane	20	.0927	.0193	4.80	Dunes
17	.0514	.0189	2.70	Ripples	23	.0537	.0187	2.87	Dunes
16	.0471	.0196	2.40	Ripples	24	.1288	.0192	6.71	Dunes
13	.0367	.0238	1.54	Plane	40	.0474	.0177	2.68	Dunes
15	.0630	.0195	3.23	Ripples	39	.0204	.0190	1.07	Transition
18	.0684	.0210	3.24	Ripples	26	.0101	.0211	0.48	Transition
2	.0435	.0190	2.29	Ripples	28	.0274	.0207	1.32	Transition
3	.0528	.0187	2.82	Ripples	29	.0740	.0223	3.32	Transition
9	.0657	.0210	3.12	Ripples	31	.0244	.0201	1.21	Transition
1	.0479	.0191	2.50	Ripples	27	.0384	.0213	1.80	Transition
5	.0435	.0191	2.28	Ripples	36	.0508	.0252	2.02	Transition
11	.0830	.0237	3.50	Ripples	41	.0222	.0190	1.17	Transition
4	.0420	.0192	2.19	Dunes	30	.0527	.0226	2.33	Transition
8	.0802	.0212	3.78	Ripples	35	.0380	.0230	1.65	Transition
7	.0585	.0192	3.04	Dunes	34	.0267	.0220	1.21	Transition
10	.1223	.0240	5.11	Ripples	33	.0187	.0222	0.84	Transition
6	.0816	.0216	3.77	Ripples	38	.0244	.0194	1.26	Transition
12	.1018	.0243	4.20	Ripples	37	.0201	.0200	1.00	Transition
19	.0624	.0215	2.91	Dunes	32	.0228	.0206	1.11	Antidune
21	.0907	.0178	5.09	Dunes	45	.0926	.0220	4.21	Antidune
22	.0886	.0177	5.01	Dunes	44	.0267	.0217	1.22	Antidune
25	.0850	.0211	4.00	Dunes	42	.0256	.0215	1.19	Antidune
					43	.0266	.0198	1.34	Antidune

[a] From Table 1.

Ideally one would expect f/f' to be near unity when the stream bed is flat, and much higher when the bed is dune-covered. In the situation where suspended load dampens the turbulence, one could even expect values of f/f' to be less than one.[48] These expectations are very clearly demonstrated by the authors' data. Table 4 shows the values of f, f', and f/f' computed from their data (Table 1). It will be seen that a value of f/f' = 2 very neatly separates the runs with bed forms (for example, dunes or ripples) from those either without bed forms or with only the gradual sinusoidal variations of the type associated with standing waves or antidunes.

There are two advantages which may be realized in the use of the f/f' parameter. The first is in the matter of terminology, and the second is in the problem of predicting roughness.

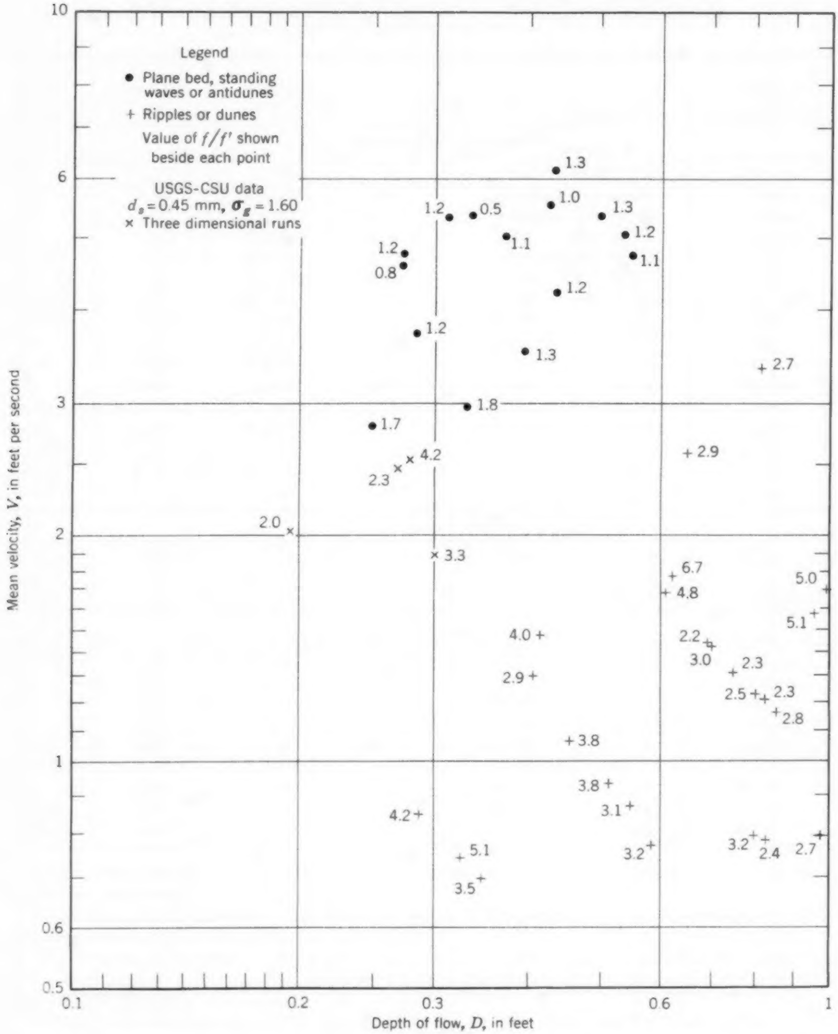

FIG. 34.—VARIATION OF f/f' WITH VELOCITY AND DEPTH FOR 0.45 mm SAND IN CSU FLUME ACCORDING TO AUTHORS' DATA

The present situation with respect to terminology for description of bed configurations is unsatisfactory. A list of all the terms which have been applied in

the literature is a long one, and in current use one can find a laboratory calling a ripple the same form that another calls a dune, while the latter uses the term "dune" to refer to something else again. Rather than depending solely on

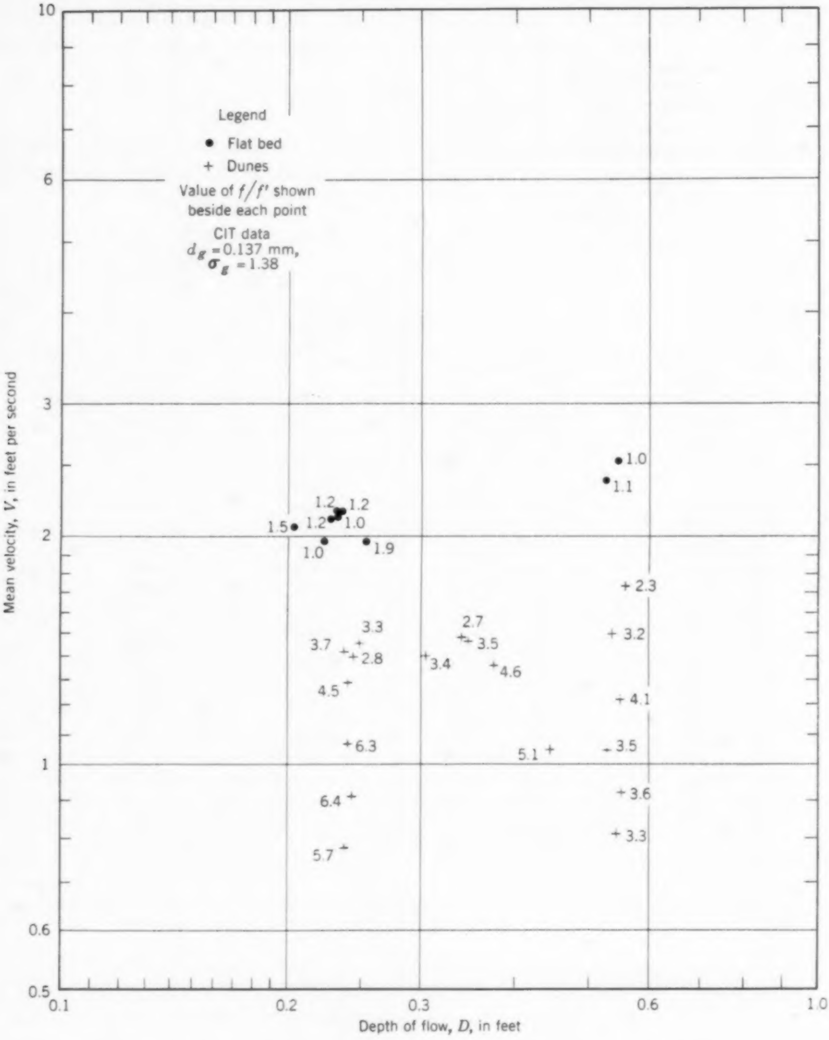

FIG. 35.—VARIATION OF f/f' WITH VELOCITY AND DEPTH FOR 0.14 mm SAND IN CIT FLUME

an agreement on terminology as a necessary step in the quantitative analysis of roughness, it would seem that a more fruitful approach would be to use a numerical quantity which can be objectively determined. In such a case the

use of curves for alluvial open-channel discharge (Liu and Hwang,[54] for example) would not require both a prediction of the bed forms to be expected and also a knowledge of what particular term had been used to describe them.

TABLE 5.—DATA FROM RUNS ON CALIFORNIA INSTITUTE OF TECHNOLOGY SAND NO. 4

Run No.	Velocity V fps	Depth D ft	f	f'	f/f'	Bed Configuration
(a) Vanoni and Brooks[b,c]						
2-9	0.77	0.238	.124	.0216	5.74	Dunes
2-3	0.90	0.243	.133	.0208	6.39	Dunes
2-8	1.07	0.240	.129	.0205	6.29	Dunes
2-1	1.28	0.240	.090	.0201	4.48	Dunes
2-7	1.40	0.237	.074	.0201	3.68	Dunes
2-6	1.44	0.249	.064	.0196	3.27	Dunes
2-17D[a]	1.39	0.302	.067	.0196	3.42	Dunes
2-17F[a]	2.07	0.203	.029	.0193	1.51	Flat
2-2	2.13	0.233	.0235	.0192	1.22	Flat
2-12	0.80	0.541	.061	.0183	3.34	Dunes
2-5	1.04	0.528	.063	.0178	3.54	Dunes
2-10	1.22	0.549	.072	.0175	4.12	Dunes
2-11	1.49	0.536	.055	.0170	3.24	Dunes
2-13D[a]	1.72	0.553	.038	.0169	2.25	Dunes
2-16F[a]	2.39	0.524	.0185	.0168	1.10	Flat
2-4	2.53	0.544	.0170	.0164	1.04	Flat
(b) Nomicos[c]						
H-2	2.13	0.233	.0215	.0207	1.04	Flat
H-3	1.97	0.223	.022	.0211	1.04	Flat
H-7a	1.38	0.243	.059	.0213	2.77	Dunes
(c) Kennedy[d]						
3-1	0.91	0.550	.068	.0187	3.63	Dunes
3-4	1.14	0.441	.096	.0187	5.13	Dunes
3-2	1.35	0.373	.086	.0189	4.55	Dunes
3-7	1.45	0.345	.067	.0190	3.51	Dunes
3-5D[a]	1.47	0.340	.052	.0192	2.71	Dunes
3-5F[a]	1.97	0.254	.036	.0192	1.87	Flat
3-6	2.15	0.233	.023	.0193	1.19	Flat
3-6A	2.13	0.235	.023	.0193	1.19	Flat
3-6B	2.21	0.228	.023	.0193	1.19	Flat

[a] D = dune section in run with long sand wave. F = flat section in run with long sand wave. No side-wall correction made.

[b] "Mechanics of Streams with Movable Beds of Fine Sand," by N. H. Brooks, Transactions, ASCE, Vol. 123, 1958, pp. 526-594.

[c] "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by V. A. Vanoni and N. H. Brooks, Sedimentation Laboratory, Calif. Inst. of Tech., Pasadena, Calif., Report E-68, 1957.

[d] "Study of Relations between Transportation of Sediment and Hydraulic Characteristics of Streams," by J. F. Kennedy, Keck Laboratory of Hydraulics and Water Resources, Calif. Inst. of Tech., Pasadena, Calif., Report KHWR-6, in press.

Because the principal reason for wanting to know what bed form will be found under given conditions is so that the friction factor can be estimated, it is use-

54 "Discharge Formula for Straight Alluvial Channels," by H.-K. Liu and S.-Y. Hwang, Proceedings, ASCE, Vol. 85, No. HY 10, October, 1959, pp. 65-98.

ful to have a means of estimating the friction factor directly. In connect with a roughness relation, moreover, it would seem unnecessary to distinguish between those features the authors call "ripples" and those they call "dunes." (While these situations are clearly different with respect to transport, they are similar as far as roughness is concerned). It is therefore suggested that an objective, numerical quantity is a necessary part of the description of a bed configuration, and that the value of f/f' is a measure suitable to this purpose. It is hardly necessary to say that such a value in itself is an incomplete description, in the same way that the verbal description, in itself, is also incomplete.

The second advantage which the use of f/f' may offer is in predicting the roughness which may be expected for given flow conditions. A number of interesting ideas are suggested by Figs. 34 and 35, which show f/f' as a function of depth and velocity. Fig. 34 uses data for the authors' experiments, d = 0.45 mm, (runs 13 and 14 have been omitted because there is no transport and the flat bed was artificially prepared). Fig. 35 uses data from experiments (2, 3, 6) made at the California Institute of Technology on a finer sand (geometric mean, $d_g = 0.137$ mm). (Tables 4 and 5 respectively.) First, it will be noted that the variation of f/f' seems fairly regular, especially when it is remembered that as a percentage of the actual friction factor a difference between f/f' values of 2 and 3 is much greater than between f/f' values of 6 and 7. Second, it is interesting to note that the separation between dune-type bed configurations and flat configurations occurs at about the same f/f' value in both sands.

The directness and simplicity with which f may be estimated for a given depth and velocity may be illustrated by the following example. Suppose it were desired to estimate the friction factor for a flow over the authors' sand if the depth were 0.8 ft and the velocity were 2.5 fps. From Fig. 34, f/f' would be about 3, and a rough (dune or ripple) bed would be expected. Taking the mean diameter for the effective roughness height and assuming $\nu = 10^{-5}$ sq ft per sec, the relative roughness, $\epsilon/(4\ r)$, is .00055, and $R_e = 4\ r\ V/\nu = 940{,}000$; for these values the pipe friction diagram gives $f' = .05$, approximately. The slope can be determined by substituting directly into the definition of f, Eq. 26; in this example, the result is about $S = .0015$.

Further research on the roughness problem is urgently needed. It may be hoped that continued flume studies like the authors' and more field investigations will make a comprehensive and logical roughness analysis possible in the future.

LUCIEN M. BRUSH, JR.,[55] M. ASCE.—This paper should be of interest to geologists as well as hydraulic engineers. Of importance to engineers is the accumulation of knowledge concerning resistance through controlled flume studies. The present-day inability to predict resistance in alluvial channels frequently has led to the inadequate design of channels and has greatly hindered progress in the field of sediment transportation. Geologists should find the description of bed forms and their associated sedimentary structures useful in attempting to evaluate past environments. Certain features of both the hydraulic and geologic implications of this paper deserve additional comment and will thereby form the basis of the discussion.

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Geologic Ramifications.—One of the aims of geologic investigations of sedimentary rocks is to infer the past history or environment of deposition. Admittedly this is difficult to do because the original fluid is no longer present in rocks. However, it is not uncommon to have sedimentary structures preserved in the rocks and these structures are often identical to the ones shown by the authors for dunes in Fig. 2. If it is assumed that physical processes involving the flow of water over alluvial channels obey the same laws of physics today as in the geologic past, much may be inferred from sedimentary structures found in rocks. For example, the basic sedimentary structure shown by the authors in Fig. 2 for dunes consists of thin horizontal strata of fine sediment overlain by coarser sediment arranged in parallel strata inclined at an angle approximately equal to the angle of repose of the sand. Geologists frequently encounter this structure in sedimentary rocks and denote it as planar cross-stratification or torrential cross-bedding. For rather poorly founded reasons, geologists usually associate this structure with torrents (high velocity streams on steep slopes) found in arid regions. Obviously, this interpretation is subject to question, because dunes and their associated structures are shown by the authors and others to vanish as the Froude number approaches one. Thus, planar cross-stratification would be the rule (subcritical flow) rather than the exception (supercritical flow) for flow in alluvial channels and would hardly be typical of high-velocity streams on steep slopes.

The preceding example is just one of many that may be derived from studies of dune structures and their associated hydraulic environment. For example, a statistical study of the spatial orientation of the planes of inclined strata would allow estimates to be made of the local, as well as the mean-flow direction of the stream. Furthermore, although the authors have not made this clear, the underlying horizontal layer of fine sediment is absent if there is no appreciable amount of suspended load present. The authors have failed to show the details of the bedding structures associated with ripples but it has been found by the writer that the ripple structure also consists of inclined strata similar to those formed by dunes. The mineralogical content of the underlying horizontal layer, which represents at least a part of the suspended load, and the mineralogical content of the inclined strata, which represent the bed load, are also of geological importance because proper identification of these minerals would enable deductions to be made as to the source areas of the sediment being transported by the streams.

Obviously, a complete and detailed reconstruction of the hydraulic characteristics of a stream is, most probably, beyond reach, but many possibilities appear encouraging and others remain to be explored. The authors have not incorporated the quantitative aspects of dune and ripple geometry into their analysis. Thus, one cannot tell whether or not the bed configuration as represented by sedimentary structures might yield still more information that the geologist could use in attempting to reconstruct the past.

Hydraulic Considerations.—The failure of the authors to employ parameters describing the dune geometry and to use them in subsequent plotting detracts somewhat from the problem at hand, but in all fairness it must be recognized that a detailed description and classification of bed regimes is a step in the right direction and that the paper serves to reemphasize the need for additional studies of dunes, in particular, their geometry and limiting size for various flow conditions and particle size distributions.

The relationship shown by Fig. 10 is by far the most important of those given regardless of the scattering of some of the points and overlapping of some

of the regimes. In fact, if this relationship should stand up for different particle size distributions, the need of worrying about regimes would be reduced vastly because all the pertinent flow characteristics usually needed for design purposes would be available. For example, given d, γ_s, γ, and ν and assuming any two of the remaining variables, slope, discharge, and depth are known, the remaining variable may be obtained by using Fig. 10. Furthermore, as is shown by the authors, the relationship is directly related to boundary-layer theory. Additional studies, of course, will be necessary to substantiate the validity of this relationship for different particle size distributions.

In contrast, the significance of Fig. 11, which may be expressed by the equation shown on the figure, is difficult to determine despite the fact that the relationship has much less scatter than the usual plots involving sediment. The authors' use of the Froude number as a term in the abscissa is difficult to understand in that more than half of the runs were made for subcritical flow. In addition, it is extremely difficult to grasp any physical significance to the combinations of the basic parameters used by the authors other than their dimensional homogeneity. If the authors could show a rational analytical development leading at least part way to this result, Fig. 11 would assume more meaning.

Although progress in alluvial channel roughness has been extremely slow, the authors have obtained data which should be of extreme usefulness to everyone in the field, both because of the wide range covered during the experiments and because the model channel is nearly stream-size in scale. Ultimately, the geometric details of the bed configuration must also be included in the analysis, and it is hoped that the authors will be successful in performing this task.

SEDIMENTATION ASPECTS IN DIVERSION AT OLD RIVER[a]

Discussion by Russell Woodburn and T. Blench

RUSSELL WOODBURN.[4]—Mr. Toffaleti has shown that for discharges below about 400,000 cfs the Atchafalaya River has about the same sediment transport potential from its head to Krotz Springs, Mile 41. For discharges above 400,000 cfs the transport capacity increases downstream. Under the conditions described there is little opportunity for increased flow downstream. Channel degradation would, therefore, be expected and was noted by the author.

Mr. Toffaleti is to be commended for his efforts in devising modifications to the Einstein Transport Theory to fit actual sediment concentrations as found in the Atchafalaya channel.

The author has made a significant contribution to potomology in applying sediment transport theory to explain the behavior of a river and to rationalize and forecast future activity.

The writer regrets that the author did not include details of the computations of sediment transport for the three reaches in question showing the modifications of Einstein's theory to fit actual sediment loads.

T. BLENCH,[5] F. ASCE.—The writer appreciates the desirability of estimating sediment-transporting capacity as part of a program for analysing an existing state of regime. He does, however, feel that available methods make it untrustworthy by itself, even in a case in which an offtake should obviously have developed into main river long ago but for site peculiarities and, presumably, major interferences by man. The writers objections are that (1) any method of computing has to rely on formulas based on experiments to small scale in the laboratory with straight uniform flumes and clean water, and (2) bed-load cannot be measured accurately in normal routine. The writers own rough rule[6] is that the slope of a meandering river is of the order of twice that of a canal of the same discharge carrying the same bed-load. That there is poor agreement between computation and very special observations has been demonstrated by Colby and Hembree[7] and although the use of the rough two-times slope rule brings their initially most discordant formula (Straub's) into relatively good agreement, on an average, daily comparisons scatter greatly

[a] June, 1960, by Fred B. Toffaleti.

[4] Dir., Sedimentation Lab., Agric. Research Service, U. S. Dept. of Agric., Oxford, Miss.

[5] Prof. of Civ. Engrg., Univ. of Alberta, and Pres., T. Blench & Assoc., Cons. Hydr. Engrs.

[6] "Regime Behavior of Canals and Rivers," by T. Blench, Butterworth's, London, Sections 6.28, 4.13, 6.27, 4.20.

[7] "Computations of total sediment discharge, Niobara River, near Cody, Nebraska," by B. R. Colby and C. H. Hembree, U.S.G.S. Water Supply Paper No. 1357, p. 57.

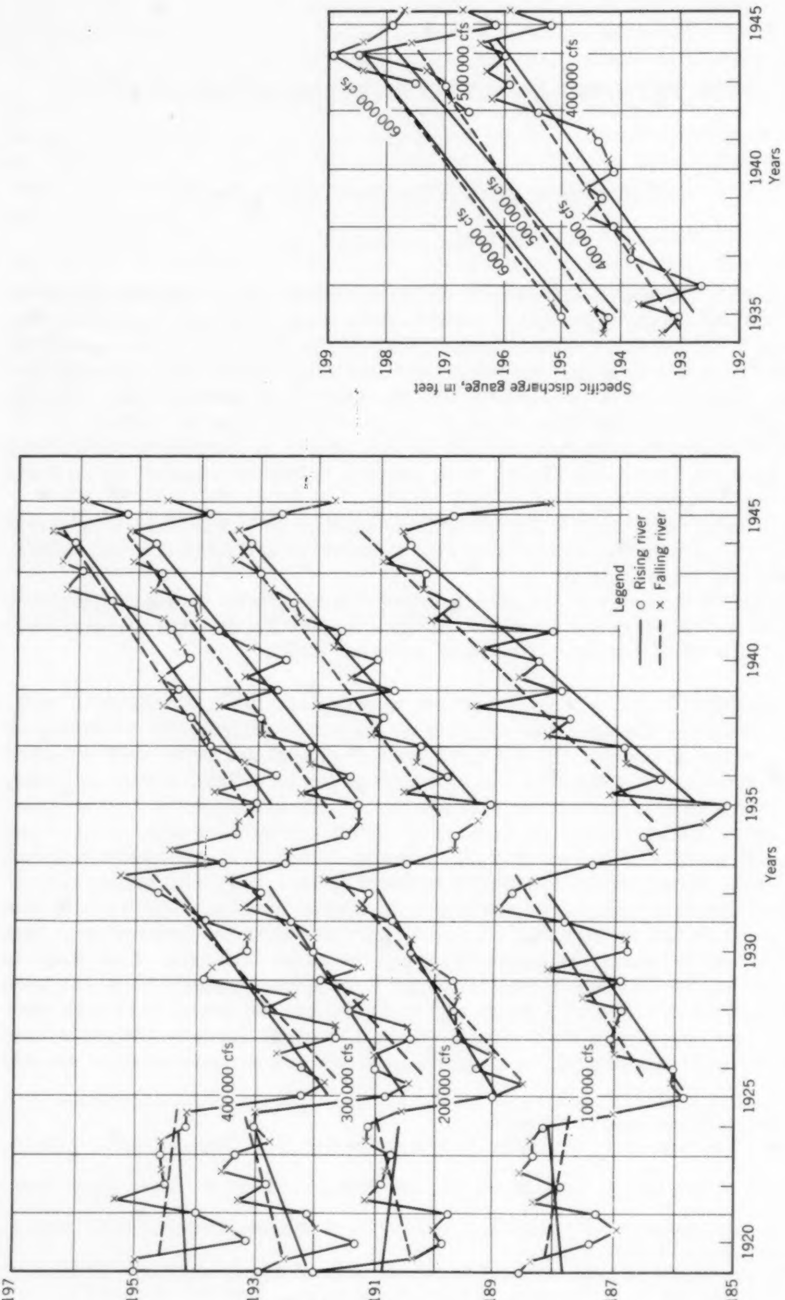

FIG. 7.—REGIME CHANGES DUE TO BARRAGE CONSTRUCTION (INGLIS)

by all formulas. Insofar as transport seems to vary roughly as the square of slope, the two-times rule indicates that use of slope taken directly from a length fully affected by meandering can give about 4 times the correct answer. A somewhat more precise way to assess the possibility of error from unadjusted slope is to start from the range of bed-load charge, C, in parts per 100,000 by weight. Using the fact that a cusec-year of water weighs 986,000 tons, 10^6 tons approximately, we can deduce that the average charge, C, for a yearly average discharge of Q cfs, and X million tons of bed-load transport is 10^5 X/Q. Using Table 1, period 1950, the average charges of Old and Atchafalya Rivers are then 5.699/1.95 and 8.531/2.97, both roughly equal to 3.0. At about this charge, f(C) curves[6] show that halving observed slope to obtain effective slope would drop computed charge to 1.0, and multiplying slope by 2/3 would drop computed charge to 2.0. This does not pretend to be an accurate computation, it rests largely on the same basic data and idealizations as rival formulas, but it suffices to indicate the possibility that agreements and differences among the curves of Fig. 3 might owe something to the differences in meander pattern among different reaches of the river at different stages. Apart from slope error the well known phenomenon of turbidities of even 1% by weight altering the apparent roughness of a river or canal appreciably should also cause some caution in interpreting the indications of Fig. 3. Further, the estimation of load when there is overbank spill might have some inaccuracies. Perhaps the author would explain how the Atchafalya, that seems to be enlarging its head reach, could have its greatest sediment charge capacity in its downstream reach at which the products of enlargement might be expected to be depositing. Such an explanation might throw considerable light on how engineering actions may have contributed to the river's present state.

Doubts about accuracy of measurement and formulas do not alter the need for an analysis such as presented, but the writer would like to draw attention to an Indian method of analysing, very simply and clearly, the actual changes of regime in a river system. This method, used with necessarily speculative methods, would enhance the utility of all. The method is to plot specific gauges of all river gauging sites as a routine. A specific gauge is the gauge corresponding to a specific discharge, 500,000 cfs for example. Usual Indian practice is to plot, for every year of record, at every discharge site, the gauges for several discharges during both rising and falling stages (although they all normally show the same regime trends). The method and its uses have been published by Inglis,[8] from whose work Fig. 7 is taken, and a principal use to analyse the regime changes due to barrages on the Indus system has been published by Foy.[9] Such records show aggradation or degradation, normal annual fluctuations due to sediment movements, effects of cutoffs, dikes, and dams at a glance.

It is suggested, for reasons obtainable elsewhere,[6] that the capacity analysis might be supplemented by that of the Mississippi and by an analysis of the distribution of bed-material median size throughout both rivers from just upstream of their junction to the sea.

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8 "The Behaviour and Control of River and Canals (with the aid of Models)," by C. C. Inglis, Government of India Central Waterpower, Irrigation and Navigation Research Sta., Poona, India, 1949, p. 179.

9 "Regime Level Changes in the Indus System, Part 1," by T. A. W. Foy, Punjab Irrig. Branch, Paper No. 16, Class B, Lahore, Pakistan, 1944.

DRAG AND LIFT ON SPHERES WITHIN CYLINDRICAL TUBES[a]

Discussion by Emmett M. Laursen and Egidio Indri

EMMETT M. LAURSEN,[7] M. ASCE.—The author deserves compliments for the simple ingenious equipment and technique developed for this investigation of the drag and lifts on spheres within cylindrical tubes. Not only did the technique permit the successful completion of his study, but with modifications of the equipment it may be possible to extend the investigation to the forces at the beginning of movement of sediment under even more realistic boundary conditions.

Although it is always dangerous to extrapolate empirical relationships such as those presented here, if it is understood that such extrapolation is strictly speculative, it can be interesting. At small values of d/D which would be more descriptive of the sediment problem, the definition of C_D and C_L on the basis of the shear velocity rather than the mean velocity in the tube would seem preferable. Similarly, a Reynolds number based on the shear velocity and the diameter of the particle could replace the Reynolds number of the tube.

$$\frac{D = C'_D \rho A V_*^2}{2} \qquad (16)$$

$$\frac{L = C'_L \rho A V_*^2}{2} \qquad (17)$$

$$\frac{R'_d = V_* d}{\gamma} \qquad (18)$$

The relationships for the newly defined drag and lift coefficients, equivalent to Eqs. 14 and 15, can easily be obtained as:

$$C'_D = \frac{R_d'^{4/3}}{32} \left(\frac{D}{d}\right)^{4/3} \left[94.5 \left(\frac{d}{D}\right)^{2.57} + 10.0\right] \qquad (19)$$

and

$$C'_L \quad \frac{R_d'^{4/3}}{32} \left(\frac{D}{d}\right)^{4/3} \left[50.5 \left(\frac{d}{D}\right)^{2.28} + 2.50\right] \qquad (20)$$

As d/D becomes small, the first term in the bracket can be neglected, simplifying the expression somewhat. It is also interesting to note that the apparent Reynolds number effect is changed by the new definitions, but that the ratio of drag to lift still has a limiting value of 4.

[a] June, 1960, by Donald F. Young.

[7] Assoc. Prof. of Civ. Engrg., Michigan State Univ., East Lansing, Mich.

Assuming for the sediment particle in Fig. 9 that the force F, the resultant of the drag and lift, is approximated by the relationships for a sphere, a sum-

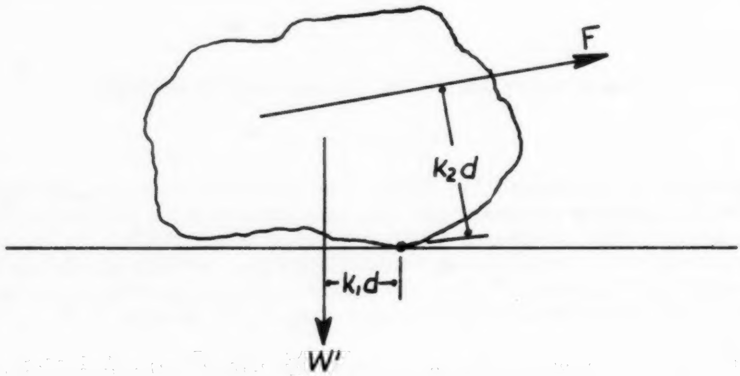

FIG. 9.—FORCES ON A SEDIMENT PARTICLE

mation of moments about the point about which the particle would roll results in:

$$W' \; k_1 \; d = F \; k_2 \; d \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

and

$$\left(\gamma_s - \gamma\right) \; k_3 \; d^3 \; k_1 \; d = \frac{\sqrt{17}}{4} \; \frac{10}{32} \; R_d'^{\,4/3} \left(\frac{D}{d}\right)^{4/3} \rho \; k_4 \; d^2 \frac{V_*^2}{2} \; k_2 \; d \quad \ldots\ldots\ldots \quad (22)$$

in which k_3 and k_4 are volume and area coefficients, respectively. In terms of Shields' parameters, Eq. 22 can be reduced to:

$$\frac{\tau_c}{\left(\gamma_s - \gamma\right) d} = \frac{k_1 \; k_3}{k_2 \; k_4} \; \frac{256}{10\sqrt{17}} \; \frac{(d/D)^{4/3}}{R_d'^{\,4/3}} \quad \ldots\ldots\ldots \quad (23)$$

Now, if d/D be considered a constant, the relationship between the critical tractive force and the particle diameter will be:

$$\tau_c \propto d^{-1/5}$$

However, if considering the critical tractive force for various sized particles in the same tube, the relationship is instead:

$$\tau_c \propto d^{3/5}$$

Which of these speculative interpretations is the more valid is impossible to say, neither agree with White $\left(\tau_c \propto d\right)$ or Shields[8] $\left(\tau_c \propto \delta'\right)$. Perhaps the only

[8] "Sediment Transportation," Ed., H. Rouse, Engineering Hydraulics, Chapter XII, John Wiley and Sons, 1950, pp. 790, 798.

conclusion that can be drawn is that the experiments need to be extended to small values of the ratio d/D.

For the comparatively large values of d/D used in this investigation, the contraction of the flow would influence both the drag and the lift, but especially the lift. For small values of d/D this contraction effect would disappear, perhaps resulting in a modification of the functional relationships indicated by Eqs. 14 and 15. It might also be noted that the lift force does not necessarily pass through the center of the sphere as shown in Fig. 1. This circumstance would modify Eq. 3 and could account for the lack of rotation observed.

In attempting to assess the findings of the investigation to the sediment problem, it would also be helpful to add a column of β values to Table 1.

EGIDIO INDRI,[9]—Experimental research was carried out in 1955 concerning the fall velocity of solid bodies (spheres and cubes) within cylindrical tubes of different diameters.

Analyses covered the effect of the walls delimiting the fluid (water) on the ultimate fall velocity of spheres and cubes of different size, as compared with the fall velocity of the same bodies in an unlimited space.

Relations were established that allow the computation of the actual fall velocity as a function of both the ratio of the diameter (or size of cubes) of the body to those of the tube and the absolute dimensions of the body itself that, contrary to what was stated in previous theoretical studies, affect the fall velocity in the limited space.

Further, it was found that the behaviour of bodies in still water within the tube differs from their behaviour when the water is in motion; that is, the fall velocity in still water is not equal to the ascensional velocity of the water that holds the body in equilibrium (stationary).

The experiments performed were not such as to permit general conclusions on a fact involving the whole technique of aero-hydrodynamic research. In the case of bodies falling into still liquids, their passage thru the medium is accompanied by a local and traveling turbulence. In the case of bodies held in equilibrium, the conditions of motion of the fluid, laminar as well as turbulent, are modified when the fluid hits the body.

From a formal standpoint, it would appear that the expressions relating to the fall velocity of bodies would be incorrect when this one is defined as the velocity of the current that holds the bodies stationary.

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[9] Prof. Engrg., Ricerche sperimentali sulla velocità di caduta di corpi solidi in acqua entro ambiente finito, ENERGIA ELETTRICA, No. 6, 1955.

MODELS PRIMARILY DEPENDENT ON THE REYNOLDS NUMBER[a]

By Melville S. Priest, R. C. Kolf and W. L. Reitmeyee, and W. P. Simons, Jr.

MELVILLE S. PRIEST.[11]—Judging from the title and text of the paper, it appears that the author considers the Reynolds number to be a basis for dynamic similarity of turbulent fluid flow in two or more pressure systems.

Although the Reynolds number may appear, along with other parameters, in the dimensional analysis of problems that encompass both viscous and turbulent motion, this does not mean that it can be singled out as the basis for dynamic similarity when the motion is turbulent. The writer will endeavor to clarify a limitation of the Reynolds number as a basis for dynamic similarity.

A ratio of the so-called "inertia force" to a shear force might be reduced to $\rho V^2/\tau$, in which ρ is mass density, V is a velocity and τ is an intensity of shear. Such a dimensionless expression is sufficiently general that it might pertain to either viscous or turbulent motion.

By substituting a Newtonian expression for intensity of viscous shear into the preceding expression, it may be rewritten as $\rho V L/\mu$, at which L is a length and μ is "absolute viscosity." This last expression is the Reynolds number. From the manner in which it has been developed, it appears that the Reynolds number pertains only to viscous motion. There is nothing to indicate that it pertains to turbulent motion.

The writer has the impression that most experienced investigators recognize that the Reynolds number is not a suitable basis for dynamic similarity of turbulent flow in two or more systems. In fact, if the Reynolds number were to serve as the basis for dynamic similarity, the requirements in many practical problems would likely be such as to preclude the use of small-scale models.

The author indicates that, for flow at the Reynolds number equal to or greater than a specified value, ". . . the data may be analyzed using the same relations as for models based on the Froude law." It is possible to arrive at apparent requirements for dynamic similarity that are the same as those based on the Froude number by assuming a high degree of mixing in all systems and by working from one of the well known expressions for head loss. For small-scale models with boundary surfaces of materials such as plastics, this would seem to be a dubious procedure. This is because of the question as to similarity of mixing in all systems. Although such an approach might suffice in some instances, the writer does not know of anything to justify it as a general solution for turbulent flow in pressure systems. In any event, such requirements for dynamic similarity are far removed from those indicated by the Reynolds number.

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[a] June, 1960, by W. P. Simmons, Jr.

[11] Head, Civ. Engrg., Auburn Univ., Auburn, Ala.

The only concern of this discussion has been an implication by the author that the Reynolds number has some bearing upon the requirements for similarity of turbulent fluid flow in two or more systems. Although the role of the Reynolds number in describing viscous motion has evidently been established, its role, if any, in describing turbulent motion has not been established.

MILTON A. CHAPPLE,[14] A. M. ASCE.—The author is to be commended for his paper that should be of great value to those engaged in hydraulic model studies.

A class of model studies that is worthy of consideration in the same context is that where the geometry of the turbulent eddies is significant, as in the design of deaeration chambers.

Eddies due to the physical shape of a conduit, projections, elbows, and other discontinuities, are generally considered to be geometrically sealed in the model and should, therefore, be correctly represented in an undistorted model. This is not so with turbulent eddies due to wall friction, because, in general, both the Reynolds number and the relative roughness of the wall differ between the model and the prototype.

It has been found[15,16] that velocities in these turbulent eddies are proportional to the shear velocity U_τ, defined by

$$U_\tau = \sqrt{(\tau/\rho)} \quad \ldots\ldots\ldots\ldots \quad (6)$$

in which τ is the intensity of shear force on the conduit wall, and ρ is the fluid density. The shear velocity may be usually more conveniently computed from

$$U_\tau = V\sqrt{(f/8)} \quad \ldots\ldots\ldots\ldots \quad (7)$$

in which V is the mean velocity in the conduit, and f is the friction coefficient in the Darcy-Weisbach formula. It is clear that, if the velocity scale in these turbulent eddies is to be the same as the general velocity scale, f must be the same in model and prototype, and the Reynolds number and wall roughness to be used in the model must be chosen accordingly.

Tests conducted by the Snowy Mountains Authority of the motion of air bubbles in turbulent water flowing through a straight conduit have indicated that the mean rate of rise of the air bubbles is independent of the degree of turbulence, but the scatter about the mean is approximately proportional to the shear velocity. This being so, the preceding restriction on the choice of Reynolds number and wall roughness for the model does not apply if the model is only required to indicate the mean paths of air bubbles; however, it does apply if an indication of the scatter is required.

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14 Fluid Mechanics Lab., Snowy Mts. Authority, Australia.

15 "The Structure of Turbulent Shear Flows," by A. A. Townsend, Cambridge Univ. Press, 1956, p. 197.

16 "The Structure of Turbulence in Fully Developed Pipe Flow," by John Laufer, N.A.C.A. Report No. 1174, Natl. Bureau of Standards, 1954.

R. C. KOLF,[12] and W. L. REITMEYER.[13]—The author writes in wide scope of a subject in need of both clarification and refinement. In the paper great care is advocated in achieving geometric similarity between model and prototype "reasonable similarity" in other respects is sought in the vague manner of running at such high Reynolds numbers that this parameter has little significance (in direct opposition to the basic proposition "viscosity remains as the predominant factor in the usual closed conduit, hydraulic flow problem"). The Froude number is then suggested, not because it is a more accurate criterion - though more convenient - but perhaps because it is no worse. This point of view is reenforced by a statement common to the usual development of the laws of dynamic similarity, but that generates confusion in their application. "The pressure ratio is usually regarded as the dependent quantity, and hence it does not play a controlling part in the following discussion of similitude techniques."[3]

In the design of an open channel model, the gravity force and the "inertia force" are considered as independent variables because they are determined by the choice of fluid, channel slope, discharge rate, and geometry. The pressure force can, in this case, be legitimately considered as dependent because the pressure variations existing in the system are not open to the choice of the investigator. The opposite is true in an enclosed system. The pressure differential, discharge rate, and geometry are predetermined by the investigator. The gravity force that acts on each fluid element in the system is dependent, as is indicated by the author's observation, "the flow direction will always be down the slope of the energy grade line and independent of the slope of the pipe itself." It is noted that in the paper to which the author refers for a discussion of similarity criteria,[2] it is stated that "the Euler number alone will determine the flow pattern" when effects of viscosity and gravity may be ignored, these being assumptions under which the presentation is posited. The statement, "viscous and inertia forces in the flowing fluids of such models, and hence Reynolds number, are predominant factors" should be amended to recognize the pressure force and inertia force. The Euler number must then be considered.

The problems involved in the use of this parameter are exactly the same as those encountered in open channel model design with the Froude number, neither case being "primarily dependent on Reynolds number." Without the greatly desired model fluid, with a viscosity significantly lower than that of water, it is impossible to operate the model so that both the Euler and Reynolds number will duplicate the corresponding values on the prototype, justifying the "direct geometric scaling of linear values." The ruse of operating the system at such high Reynolds numbers that viscous shear is not significant is the only presently available solution. In a closed conduit fluid system the Reynolds number is, as the author affirms, a defining relation for the state of the system. However, an extension of this as a similarity criterion - at high Reynolds number - to more than an approximation to the state of system in model and in prototype is unwarranted and serves to confuse rather than clarify the issue.

Although the Reynolds number is the ratio of inertial forces to viscous forces, one must keep in mind that the so-called "inertial force" term is but

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[a] June, 1960, by W. P. Simmons, Jr.

[12] Assoc. Prof., Dept. of Theoretical and Applied Mechanics, Marquette Univ., Milwaukee, Wis.

[13] Instr., Dept. of Theoretical and Applied Mechanics, Marquette Univ., Milwaukee, Wis.

the composite representation of all the other forces acting in the system. It cannot be properly said that "inertial forces assume such great significance that viscosity becomes relatively minor," but rather that compared to other forces in the system (pressure), the viscous shear force can be neglected. Neglecting this force then negates the utility of Reynolds number as a similarity criterion. (The similarity as defined by other criteria will not be altered by non-correspondence of Reynolds values in this range because relatively large variations in viscous force will not appreciably alter the state of the system). In the same manner, having affirmed that gravitational forces are not a factor affecting the flow, the Froude number - a ratio of "inertia force" to gravitational force - is not an applicable similarity criterion. The author states that at high Reynolds numbers the Froude relations may be used, and ". . . the model pressures would be multiplied by the scale ratio to obtain prototype pressures. Time intervals and flow velocities are obtained by multiplying model values by the square root of the scale ratio."

These relations are derivable from the Euler expression (not uniquely the Froude) but will be inconsistent with the Reynolds law if the same fluid is used in model and prototype. The confusion of Froude number with Euler number stems from their exhibiting similar dimensional terms, either implicitly or explicitly, both containing the velocity head $V^2/2g$ and a characteristic length. The latter is pressure head in the Euler number and elevation head in the Froude number. The author's model design and operating techniques are in no way jeopardized, for that he chooses to call Froude number is actually Euler number, pressure head having been chosen as the characteristic length (illustrated by the author's "dimensionless pressure factor relation"). Nonetheless, because there is great conceptual difference between the two, the inaccuracy of calling one by the name of the other is provocative only of chaos. An engineer's desire for usable expressions that define the relationships among observable phenomena is considered by some to be more essential than aesthetic correspondence with reality. The correspondence of the theoretical expressions with reality is not superfluous, however, for their consistency and comprehensibility are essential to successful extrapolation.

The writers agree with the author that it is appropriate "to state briefly the fundamental model relationships that must be observed." The Euler number is, in this paper, the fundamental relationship, both useful and realistic, and should be so designated and used.

DRAG FORCES IN VELOCITY GRADIENT FLOW[a]

Discussion by W. D. Baines

W. D. BAINES,[4] M. ASCE.—The authors are to be commended for their pioneering work in this engineering problem involving turbulent shear flow. Only recently have the practical problems in this field become apparent in meteorology, civil engineering, and wave mechanics. At first glance the variety of body shapes and velocity gradients to be considered leads to the conclusion that generalization may not be possible. However, the recent progress in analysis of pitot tube shift effect is a move in this direction. This apparent displacement of the pitot tube up the velocity gradient has often been noted in measurements made in a shear field. M. J. Lighthill has shown[5] that theoretically the magnitude of the pitot tube displacement is a function of the size only for a spherical tip tube; it is not a function of the steepness of the velocity gradient nor the Reynolds number of the flow. The geometry of this situation is similar to the conditions near the stagnation line of a blunt body. It might, therefore, be expected that the stream lines on the front of the bluff body would exhibit the same type of displacement in a turbulent shear flow. Evidence of this might be seen in the authors' results.

The writer is now engaged in a similar study of flat plates immersed in flows with velocity gradients. The study is general, and it is planned to start with linear and progress to more complicated gradients of the mean flow. Only exploratory results are now available. On the whole, these verify the authors' conclusions. However, the writer has made some observations not mentioned by the authors.

For a solid body immersed in a flow with a velocity gradient, the simplest theory describing the pressure distribution around the body is a linear relationship with the stagnation pressure. That is, the pressure variation around any cross section of the body can be determined from the pressure coefficients for the body in a uniform mean flow and the local velocity. In effect, this assumes that each section is independent of neighboring sections. The authors' note that this cannot be expected to be completely satisfied. The primary reasons for deviations from the linear theory is the existence of a boundary layer over the surface of the immersed object. At the surface the velocity is zero and hence there is a region of fluid in contact with the body in which the velocity is very low. Thus, this layer is easily disturbed if subject to pressure gradients. From the linear theory one can readily prove the existence of these gradients along the body. The resulting motion of the fluid in the boundary layer

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[a] July, 1960, by F. D. Masch and W. L. Moore.

[4] Assoc. Prof., Dept. of Mech. Engrg., Univ. of Toronto, Toronto, Ontario.

[5] "Contribution to the theory of the Pitot Tube Displacement Effect," by M. J. Lighthill, Aeronautical Research Council, Great Britain, Paper No. FM 2529, April, 1957.

of the body is the secondary flow observed by the authors. The writer has also observed these secondary flows as have investigators in the field of three-dimensional boundary layers.

On an upstream part of the cylinder, the boundary layer must necessarily be thin and the mean velocity comparatively strong. Thus, the secondary flows should be of a smaller order of magnitude, and a relatively small displacement of the stream lines should result. Comparison of the authors Figs. 3 and 4 should illustrate this effect. The stagnation pressures should be proportional to the square of the local velocities. This is definitely not the case. The pressures display approximately an eight fold variation between piezometers no. 2 and 12 whereas the square of the velocities taken from the third part of Fig. 4 exhibits less than a three fold variation. No physical explanation can be advanced for a steeper pressure gradient than that given by the linear theory. It can be noted however, that the two quantities would have the same variation if the slope parameter for results on Fig. 3 were 0.117. The writer therefore concludes that an error has been made and that the stagnation pressures follow the linear theory closely. From the results that the authors give a shift of the streamlines up the velocity gradient is not detectable. The writer also has not discerned this shift in flow around narrow flat plates. but this may be due to the crudeness of the experimental measurements. It is suggested that the authors review the original data to see if this phenomena is apparent.

On the downstream part of the cylinder there should exist a zone of separated flow similar to that with no velocity gradient. In this wake the mean velocity of the main flow is comparatively small. It would thus be expected that secondary flows would have a greater influence on the flow pattern. The authors' measurements in this region show a large deviation in the pressure distribution from that predicted by the linear theory. This is well demonstrated by the data shown on Fig. 3. At the center of the wake the pressure along the cylinder varies by a factor of two, thus, being about one-fourth that predicted by the linear theory. The writer has noted an even stronger effect on flat plates in which the pressure is uniform over the downstream face that is completely in the wake.

In summary, the flow about a bluff body in a velocity gradient closely follows the linear theory on the upstream side of the body, there being only a small displacement of the stream lines; on the downstream side of the body there is a wake in which pressure tends toward uniformity. These rules, when applied to the local drag coefficient, directly predict variations similar to those observed by the authors. However, a further assumption must be made if the reduction, in the value of the mean drag coefficient, is to be explained. One must know much more about the location of the separation line on the rear of the body and its variation along the body. Particular attention should be paid to this region in any future studies.

The authors have not mentioned the sides of the cylinder at which a different flow condition exists. Because of flow acceleration the pressure must be below ambient, and following the linear theory, a pressure gradient proportional to the ambient velocity squared should exist. This is similar to the wake, but because the mean flow is comparatively strong, the resulting secondary flow should be of a strength similar to that in the stagnation region. The writer would be interested to hear if this gradient has been measured, and if the secondary flow up the velocity gradient has been observed. Although this part of the cylinder contributes little to the magnitude of the local drag coefficient it is important for other reasons. The pressure distribution over the entire sur-

face of the body must be considered in many cases. For example, in buildings subject to a wind gradient, the magnitude of a local pressure will determine the required strength for windows or wall coverings.

An additional item that may cause complications in flow with a velocity gradient is the bounding surface at the end of the object. In flows with a uniform velocity these surfaces lead only to the development of a thin boundary layer along them. However, if a velocity gradient exists, the presence of such a surface inhibits the secondary flow along the body. The resulting deflection of the secondary flow often causes large changes in the pressure distribution on the body near the wall. The writer has found that for flat plates these effects are quite large if the width is relatively great. It might be expected that this effect would be much smaller for a cylinder, and the authors data should show the magnitude.

It is to be hoped that the line of study started by the authors will be continued, and that more quantitative data will be obtained for practicing engineers.

UNSTEADY FLOW OF GROUND WATER INTO A SURFACE RESERVOIR[a]

Discussion by M. E. Harr, Raphael G. Kazmann, W. T. Moody, and K. T. Sundara Raja Iyengar

M. E. HARR,[9] M. ASCE.—As noted in Fig. 2, the linearization of Eq. 6 (Eq. 7) produced the least satisfactory solution. This was to be expected. For considering the steady-state form of Eq. 7,

$$\frac{\partial^2 h}{\partial x^2} = 0 \quad \ldots \quad (24)$$

the equation of the free surface follows the linear relation

$$h = C_1 x + C_2 \quad \ldots \quad (25)$$

in which C_1 and C_2 are arbitrary constants.

Now considering Eq. 7 as

$$\frac{\partial^2 h^2}{\partial x^2} = \frac{\partial h^2}{\partial t} \quad \ldots \quad (26)$$

the steady state solution becomes

$$h^2 = C_3 x + C_4 \quad \ldots \quad (27)$$

demonstrating the free surface to be of parabolic shape, a form more consistent with observations and known steady state solutions (Dupuit's parabola). Shifting the origin of reference to the reservoir surface the second method, in normalized form reduces to

$$\frac{h}{H} = \sqrt{\Phi} \quad \ldots \quad (28)$$

that is merely the square root of the first linearization method. This solution is the solution IV of the paper that represents the best approximation to the experimental data.

In their development of the linear equation (Eq. 7), the authors state that h can be discarded from Eq. 3 if d is much larger than H. This is not only inconsistent with the results given in Fig. 2 in which d is taken as zero, but ignores a fundamental assumption. Inherent in their derivation, although not

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[a] July, 1960, by William Haushild and Gordon Kruse.

[9] Assoc. Prof. of Civ. Engrg., Purdue Univ., Lafayette, Ind.

stated as such, is the requirement that the free surface varies but little from some constant value (Dupuit - Forchbeimer assumption), say $\tilde{H}$. Hence Eq. (1) can be written as

$$Q = k \tilde{H} \frac{\partial h}{\partial x} \quad (29)$$

and Eq. 7 becomes

$$\frac{\partial h}{\partial t} = \alpha \tilde{H} \frac{\partial^2 h}{\partial x^2} = a^2 \frac{\partial^2 h}{\partial x^2} \quad (30)$$

For computational purposes $\tilde{H}$ can be taken as an average height of the surface such as D in Eq. 2.

RAPHAEL G. KAZMANN,[10] F. ASCE.—This paper deals with an interesting practical problem, namely, bank-storage. This phenomenon is not confined to the operation of reservoirs, but is also encountered in aquifers hydraulically connected to streams.

The equation known as the error-function is shown as Solutions I and III, and is stated in ground-water terms elsewhere.[11] This relationship, that deals with non-steady stage ground-water flow, was first adapted to ground-water studies by Rorabaugh in his 1948 mimeographed report on ground-water investigations in the Louisville area.[12]

Solutions II and IV are modifications based on the error function equation (Solutions I & III). These modifications, the only contribution of the paper, are defended by comparing the results of computations with cited laboratory data. Reference to the details of the Keller & Robinson experiments reveals that verification of these non-steady state, approximate equations is based on steady-state tests of flow through sand! These tests, performed in the laboratory for another purpose, do not possess the basic assumption of the authors' equations that the drawdown shall be small compared to the thickness of aquifer.

In the following quotation from the paper the statements in parenthesis are the writers.

"All the solutions are approximations based on the assumption that d (the thickness of aquifer below the lowest reservoir level) is very large compared to H (the drawdown). This assumption is least valid (it's not valid at all) for the case of complete drawdown. Therefore (it necessarily follows that . . . ?) the results from using this solution with partial drawdown should be good." So, the very discrepancy between experiment and assumption is used as an argument to assure the engineering profession that the results may confidently be applied in the field!

Let it be noted, in passing, that when the assumptions are met, the error-function relationship, in the form given by Rorabaugh, is a good basis for predicting changes in the ground-water levels that are associated with river fluctuations.

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10 Cons. Engr., Stuttgart, Ark.

11 "Ground Water in Northeastern Louisville, Kentucky," by M. I. Rorabaugh, USGS WSP 1360 B, Washington, 1956, Eq. 21.

12 "Ground-Water Resources of the Northeastern Part of the Louisville Area, Kentucky," by M. I. Rorabaugh, 1948.

The writer submits that engineering is the application of scientific principles to the control of the environment. Consequently, any equations or procedures proposed for use in the field to determine the flow of ground water in response field. Common sense would indicate that it is not desirable to base verification of an engineering paper on laboratory tests that, admittedly, do not meet the assumptions of the equations proposed by the authors.

W. T. MOODY,[13] F. ASCE.—The authors have made a valuable comparison between experimental data and mathematical solutions that give the approximate shape of the drawn down water table.

The difficulty of solving the nonlinear partial differential equation describing the shape of the water table for the general case in terms of known functions is unquestionable. However, for any particular case in which a parameter relating two independent characteristic depths of the aquifer has a specific value, an exact numerical solution can be obtained. In the following development, because the authors' notation is followed wherever possible, reference to Fig. 1 will be helpful. Additional symbols are defined where used.

The differential equation of the drawn down water table (Eq. 6) is

$$\alpha \frac{\partial^2 h}{\partial x^2} + \frac{\alpha}{D}\left(\frac{\partial h}{\partial t}\right) = \frac{\partial h}{\partial t} - \frac{\alpha}{D} h \frac{\partial^2 h}{\partial x^2}$$

Using the Boltzmann[14] transformation that introduces the new independent variable u defined by

$$x^2 = 4 \alpha t u^2 \quad \text{(31)}$$

carries Eq. 6 into the ordinary differential equation

$$\left(1 + \frac{h}{D}\right)\frac{d^2h}{du^2} + \frac{1}{D}\left(\frac{dh}{du}\right)^2 = -2u\frac{dh}{du} \quad \text{(32)}$$

Replacing h in Eq. 32 with the new dependent variable defined by

$$y = \frac{h}{H} + \frac{1}{2} \quad \text{(33)}$$

and introducing the characteristic depth parameter m, in which

$$H = m D \quad \text{(34)}$$

results in

$$\left(m y - \frac{1}{2} m + 1\right)\frac{d^2y}{du^2} + m\left(\frac{dy}{du}\right)^2 = -2u\frac{dy}{du} \quad \text{(35)}$$

The original and transformed boundary conditions are

$$x = 0, \quad t > 0, \quad h = -\frac{1}{2} H: \quad u = 0, \quad y = 0, \quad \text{(36a)}$$

[13] Head, Experimental Design Analysis Sect., Tech. Engrg. Analysis Branch, Div. of Design, Bur. of Reclamation, Denver, Colo.

[14] "Zur Integration der Diffusions gleichung bei variabeln Diffusionscoefficenten," by L. Boltzmann, Ann Physik, Leipzig, 1894, Vol. 53, pp. 959-964.

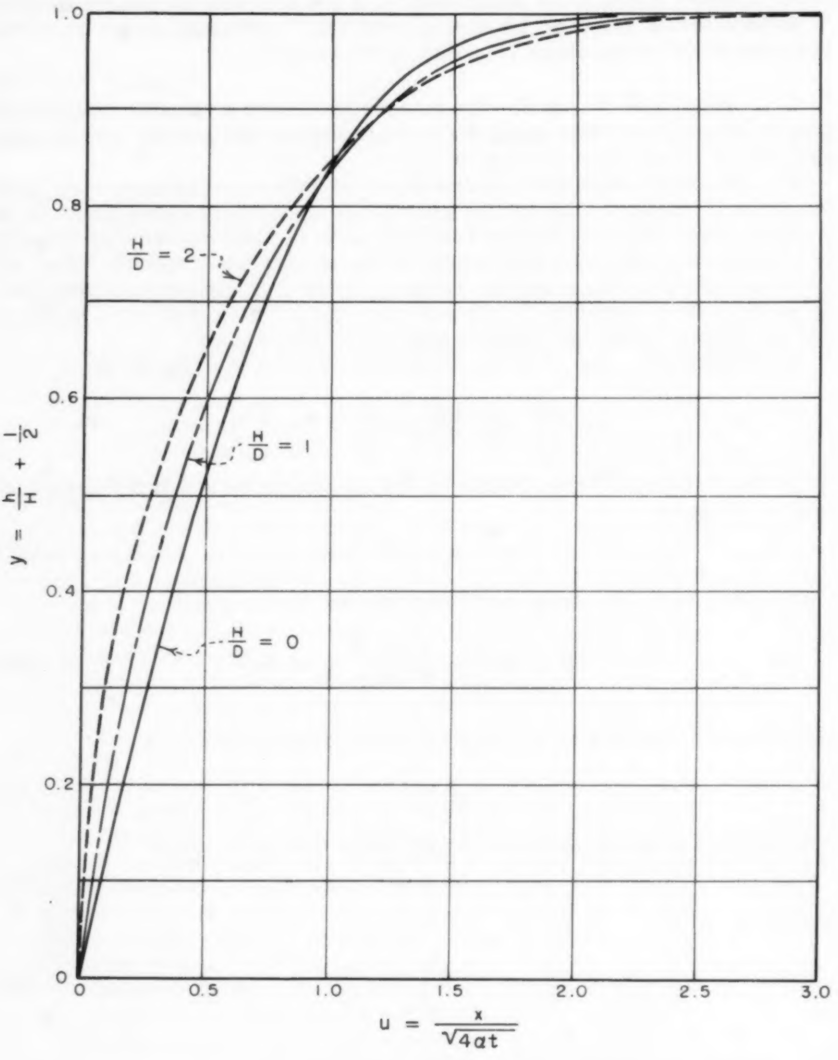

FIG. 4.—SOLUTIONS OF THE DIFFERENTIAL EQUATION OF A DRAWN DOWN WATER TABLE

TABLE 2.—SOLUTIONS OF THE DIFFERENTIAL EQUATION OF A DRAWN DOWN WATER TABLE

u	y		
	m = 0	m = 1	m = 2
0	0	0	0
0.1	0.112	0.178	0.305
0.2	0.223	0.312	0.428
0.3	0.329	0.423	0.518
0.4	0.428	0.516	0.592
0.5	0.520	0.595	0.653
0.6	0.604	0.662	0.705
0.7	0.678	0.720	0.750
0.8	0.742	0.769	0.788
0.9	0.797	0.811	0.821
1.0	0.843	0.846	0.850
1.1	0.880	0.875	0.874
1.2	0.910	0.900	0.895
1.3	0.934	0.921	0.913
1.4	0.952	0.938	0.929
1.5	0.966	0.951	0.942
1.6	0.976	0.962	0.953
1.7	0.984	0.971	0.962
1.8	0.989	0.978	0.969
1.9	0.993	0.984	0.976
2.0	0.995	0.988	0.981
2.2	0.998	0.994	0.988
2.4	0.999	0.997	0.993
2.6	1.000	0.998	0.996
2.8	1.000	0.999	0.998
3.0	1.000	1.000	0.999

and

$$\left.\begin{array}{l} x \to \infty \\ x > 0,\ t \to 0 \end{array}\right\} h = \frac{1}{2} H: \ u \to \infty, \ y = 1 \qquad (36b)$$

Following the general method described by Crank,[15] it can be shown that the solution of Eq. 35, including boundary conditions Eq. 36a and b, is given by values of y that satisfy

$$y = \frac{\int_0^u Y^{-1} \exp\left(-\int_0^{u''} 2\, Y^{-1}\, u'\, du'\right) du''}{\int_0^\infty Y^{-1} \exp\left(-\int_0^{u''} 2\, Y^{-1}\, u'\, du'\right) du''}, \ 0 \leq m < 2 \qquad (37)$$

in which, for brevity,

$$Y = m\, y - \frac{1}{2} m + 1 \qquad (38)$$

15 "The Mathematics of Diffusion," by J. Crank, Oxford, 1947, pp. 149-152.

The exclusion of m = 2 in Eq. 37 is necessary to avoid indeterminacy at the point y = 0. This case can be treated separately, however, and a similar solution is obtained in the form.

$$y^2 = \frac{\int_o^u \exp\left(-\int_o^{u''} y^{-1}\, u'\, du'\right) du''}{\int_o^\infty \exp\left(-\int_o^{u''} y^{-1}\, u'\, du'\right) du''}, \quad m = 2. \qquad (39)$$

For this case it can be shown that

$$\underset{u \to 0}{\text{Limit}} \left(y^{-1}\, u\right) = 0 \qquad (40)$$

so that the apparent indeterminacy of the inner integrand is resolved.

Numerical solutions of Eqs. 37 and 39 are obtained by an iterative procedure in which successive approximation to y are substituted into the right-hand members thereby generating new and closer approximations. When the n-th and (n+1)-th approximations agree to the desired accuracy, the solution is completed.

The entire range of aquifer dimensions is represented by the variation from m = 0 (infinite depth of aquifer below the reservoir water surface) to m = 2 (reservoir water surface at the bottom of the aquifer). For m = 0, the solution of Eq. 37, is seen to reduce to the familiar error function. The cases in which m = 2 (solved approximately by the authors) and m = 1 have been computed from Eqs. 39 and 37, respectively. Results for all three cases are given in Table 2 and are shown graphically on Fig. 4. The insensitivity of the solution to variations of the parameter m, and the near uniformity of spacing of the curves, suggests that results for any value of m can be obtained from Table 4 by second degree interpolation with sufficient accuracy for most practical purposes.

K. T. SUNDARA RAJA IYENGAR.[16]—The non-linear differential equation occurring in this problem has been solved by the authors by using two approximate solutions. The method of perturbation offers a good approach to such non-linear equations. This method of approach, originally due to Poincarè, has been developed by Lighthill and Kuo, and a detailed treatment of this method under the name 'PLK METHOD' is given in a recent literature.[17] It is interesting to see that the solution given by the perturbation method coincides with the solution II given by the authors, who attribute this method to Picard.

The differential equation to be solved is

$$\alpha \frac{\delta^2 h}{\delta x^2} + \frac{\alpha}{D}\left(\frac{\delta h}{\delta x}\right)^2 = \frac{\delta h}{\delta t} - \frac{\alpha}{D}\, h \frac{\delta^2 h}{\delta x^2} \qquad (41)$$

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[16] Asst. Prof. of Civ. Engrg., Indian Inst. of Science, Bangalore 12, India.

[17] "The Poincarè-Lighthill-Kuo Method," Advances in Applied Mechanics, Vol. IV, Academic Press, 1956, pp. 281-348.

with the boundary and initial conditions stated in the paper. Following the notation of Collatz,[18] we can introduce a perturbation parameter ϵ and write Eq. 1 in the form

$$\alpha \frac{\delta^2 h}{\partial x^2} - \frac{\delta h}{\delta t} + \epsilon \left[\frac{\alpha}{D} \left(\frac{\partial h}{\partial x}\right)^2 + \frac{\alpha}{D} h \frac{\delta^2 h}{\partial x^2} \right] = 0 \qquad (42)$$

when $\epsilon = 0$, Eq. 42 reduces to the linear equation whose solution is given by Carslaw and Jaeger (that is Eqs. 8 and 9). When $\epsilon = 1$, we have Eq. 41. The differential equation with $\epsilon = 0$ has been called the "undisturbed equation" and that with $\epsilon = 1$ the "disturbed equation."

Assuming that the solution $h(x, t, \epsilon)$ may be expanded in powers of ϵ we have

$$h(x, t, \epsilon) = h_o(x, t) + \epsilon\, h_1(x, t) + \epsilon^2 h_2(x, t) + \ldots + \ldots \qquad (43)$$

The first term $h_o(x, t)$ is the Carslaw and Jaeger solution. Substituting for h in Eq. 42 from Eq. 43 and equating the coefficients of powers of ϵ to zero, we get the following equation for h_1:

$$\alpha \frac{\delta^2 h_1}{\partial x^2} - \frac{\delta h_1}{\delta t} + \frac{\alpha}{D} \left(\frac{\partial h_o}{\partial x}\right)^2 + \frac{\alpha}{D} h_o \frac{\partial^2 h_o}{\partial x^2} = 0 \qquad (44)$$

whose solution is given in Eq. 13 of the authors' paper, in which p_1 and p_2 are the particular integrals for $-\frac{\alpha}{D}\left(\frac{\delta h_o}{\partial x}\right)^2$ and $-\frac{\alpha}{D} h_o \frac{\delta^2 h_o}{\partial x^2}$, respectively. Their values are given in Eqs. 10 and 12. By continuing this process, we get differential equations for h_2, h_3 and so on, are more laborious to solve.

18 "The Numerical Treatment of Differential Equations," by L. Collatz, Springer-Verlag, Berlin, 1960, p. 187.

ERRATA

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p. 63. In line 38 change "expansion of public enterprise" to "expansion of private enterprise."

June 1960

p. 17. In line 32 and in line 36 change "Permeabilite" to "Perméabilité." In line 33 change "gravite periodiques, deuxieme . . . Blance" to "gravité périodiques, deuxième . . . Blanche." In line 37 change "gravite periodiques" to "gravité périodiques."

p. 19. In line 40 change "Technique" to "Technik."

p. 117. In the first line of text the name WILLIAM G. HOLT should be changed to WILLIAM G. HOYT.

August 1960

p. 25. In Table 1, Col. 6, Oct. 11, 1949, change 63 to .63.

p. 26. In Table 1, Col. 6, Mar. 31, 1953, change .25 to .24.

p. 27. In the caption for Fig. 3 change CLAYPEN to CLAYPAN.

p. 29. In Table 2, in the columns headed "runoff Used in Computing Unit Hydro." add a horizontal line above the words "Peak" and "Amount" and delete the horizontal line below those words. The vertical rule separating these two columns should then be extended upward to meet the new horizontal line.

p. 32. In Table 3, in the column headed "Time in Minutes from Peak to—" add a horizontal line above the series of words "Before" and "After" and remove every other vertical rule that extends above this horizontal line.

p. 32. In the caption for Table 3 remove the word WATERSHED.

p. 54. The material on this page should be moved to p. 56.

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The technical [illegible] published [illegible] the past year [illegible] number below. Technical-[illegible] sponsorship is indicated by [illegible] abbreviation at the end of each [illegible] number, the symbols [illegible] Transport (AT), [illegible] Construction (CO), Engineering Mechanics (EM), [illegible] draulics (HY), [illegible] Drainage (IR), Pipeline (PL), [illegible] Sanitary [illegible] Mechanics and [illegible] Structural (ST), [illegible] (SU), and [illegible] (WW), divisions. [illegible] by the [illegible] Practice are [illegible] bols (PP). [illegible] "Civil Engineering." [illegible] with Volume [illegible] papers were published in Journals of the various Technical Di[illegible] locate papers in the Journals, the symbols after the paper number are followed by a numeral [illegible] the issue of a particular Journal in which the paper appeared. For example, Paper 2703 is [illegible] 2703(ST1) which indicates that the paper is contained in the first issue of the Journal of the Str[illegible] vision during 1961.

VOLUME [illegible] (1960)

JANUARY: [illegible](EM1), [illegible], 2333(EM1), [illegible](EM1), [illegible], 2336(HY1), [illegible] 2339(EM1), 2340(HY1), [illegible](SA1), 2342(EM1), [illegible], 2345(ST1), [illegible] 2348(EM1)[c], 2349(EM1), [illegible](ST1), [illegible], 2354(ST1).

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AUGUST: 2564(SM4), [illegible](ST8), [illegible], 2568(PO4), [illegible](PO4), [illegible](HY8), [illegible] 2572(EM4), 2573(EM4), [illegible], 2575([illegible]), [illegible], [illegible](HY8), 2578(EM4), [illegible] (EM4), [illegible]

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OCTOBER: [illegible], 2617(ST10), 2618(SM5), 2619(EM5), [illegible] 2623(EM5), [illegible], 2626(EM5), [illegible] 2630(ST10), [illegible](PO5)[c], [illegible]

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DECEMBER: 2668(ST12), [illegible](EM6), [illegible] (EM6), [illegible](HY4), 2677(HY10), [illegible], [illegible](ST12), 2681([illegible]), [illegible] 2684(SM6), [illegible], [illegible](EM6), 2688(EM6), 2689(EM6), 2690([illegible]), [illegible] (ST12), [illegible], [illegible], 2697(ST12)[c].

VOL[illegible] (1961)

JANUARY: [illegible], [illegible](HY1), 2700(HY1), 2701(SM1), [illegible], [illegible](ST1), [illegible] (HY1), 2707(HY1), [illegible], 2709(PO1), 2710(HY1), 2711(HY1), 2712(ST1), [illegible] (ST1), [illegible], [illegible](SA1), 2719(SU1), 2720(SA1)[c], [illegible](ST1), 2722(PO1)[c], [illegible] (HY1)[c], 2725(ST1)[c].

c. Discussion [illegible] by divisi[illegible]